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JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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[ENTERED AT STATIONERS' HALL.]

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Sir George Francis Hardy K.C.B.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Memoir.

Sir George Francis Hardy, K.C.B.

WITH the greatest regret we have to record the death, which occurred at his residence in Kensington on the 5 October 1914, of Sir George Francis Hardy, K.C.B., F.R.A.S., F.I.A., F.F.A., F.A.S., Past President of the Institute of Actuaries. Consequent on an attack of rheumatic fever in his youth, he had for many years been the subject of heart complaint which had rendered it necessary for him to avoid exposure or any undue strain, but had not otherwise interfered with his ordinary life. Some months ago, however, the malady took a more acute form, and though it was hoped this would be only temporary it later on became evident that complete recovery was impossible. The end came suddenly and peacefully.

By his death the actuarial profession loses one of its brightest ornaments, and many of its individual members one of their dearest friends. His position in the profession and his influence on its scientific development were perhaps unique, and it is fitting that some extended memoir of his life and work should be attempted, though the writer feels that his pen is altogether inadequate to do justice either to Sir George Hardy's great intellectual gifts or to his personal charm and singular beauty of character.

George Francis Hardy was born on the 14 December 1855 and was educated in a private school of which his father was the head. From an early age he showed the accurate and

retentive memory and remarkable powers of mind that ultimately carried him to the top of his profession, and would have made him eminent in any sphere of mental activity. At one time it was proposed to send him to Cambridge, but this was not found practicable. Had it been otherwise, it can hardly be doubted that actuarial science would have been robbed of one who was destined to rank high among its ablest exponents, while Cambridge would have absorbed into her life one whose clearness of vision and instinctive mathematical gifts—combined as they were with that power of scientific imagination which has so often been associated with the greatest mathematical work—could hardly have failed to make a deep and enduring mark in the world of pure mathematics and physics.

In June 1871, at the age of $15\frac{1}{2}$, he was appointed a clerk in the British Empire Mutual Life Office, and three years and a half later, in December 1874, he passed the first examination of the Institute of Actuaries, being placed second in the list. In the following month he gave evidence of his general knowledge by passing the London University Matriculation Examination, in the First Division of the Ordinary Pass List. In December 1876 he passed the second examination of the Institute, and in April 1880 the third, in both cases being placed first on the list, and on 12 October 1880 he was admitted a Fellow of the Institute of Actuaries. It is not a little interesting to note that, in spite of his great powers, over three years elapsed between his passing the Intermediate and Final examinations. Probably this was partly due to that thoroughness which did not allow him to present himself until he was assured that he had completely mastered the work, but partly also it was no doubt due to the fact that he was never a man of one idea, for he had wide and varied interests. From a comparatively early age he had taken a special interest in astronomy, and his copy of Herschel's "Outlines of Astronomy" was dated in his own hand when he was between fourteen and fifteen. He took up the subject originally on the side of practical observation and he was in this way led into the study of the mathematical theory. That the process was not reversed is a matter of some interest, because, similarly, in all his actuarial work mathematics was a means to an end rather than an end in itself.

One of the first things he did with his earnings after joining the staff of the British Empire Office was to buy a

good second-hand $2\frac{1}{2}$ -inch telescope. This he fitted up at the window of a darkened room, and thus he got a two-foot image of the sun, and made sketches of the sun's spots. Very soon he wanted a larger instrument and, as the cost of buying one was prohibitive, he set out to build one for himself. Accordingly when he was between 17 and 18, with his father's help he put together an 8-inch equatorial reflector, mounted on a concrete base in the garden of his house, with its own hut and revolving roof. In February 1874 he contributed to the "English Mechanic and World of Science" (volume xviii, page 533) an article on a new method of determining the sun's distance by means of measuring the duration of the occultation of a fixed star by one of the exterior planets. Not only was this method new in itself but he added largely to its value by an ingenious suggestion for increasing the accuracy of the measurement by means of spectroscopic observation. He spoke of the method as "far exceeding in accuracy even the method of the transit of Venus", and he added: "The phenomenon is evidently, however, less rare than transits of Venus; and combining as it does the double advantage of giving measurements of the distance of the sun and the diameter of the observed planet with an accuracy utterly unapproachable by any other method, I believe it is well worthy of the attention of Astronomers." When one reflects that at this time he was little more than 18 years of age, one cannot but feel amazed both at the ingenuity of the conception and at the clearness and power with which it was expressed in an article which gives the impression of having been written by a man of mature powers. About two years after the article appeared, he saw that attention had been called, at a meeting of the Royal Astronomical Society, to a similar proposal made by a French astronomer, who, however, made no suggestion as to improving the accuracy of the observations by means of the spectroscope. Hardy had no difficulty in proving his claim to priority, and as a result he was elected a Fellow of the Royal Astronomical Society in 1876, on an application supported by, among others, Dr. (afterwards Sir) William Huggins and Captain William Noble, both well-known astronomers. The names of his sponsors are sufficient to indicate the value of his ideas.*

To use a finely-tempered blade to cut a block, or a delicate loom to weave a coarse fabric, can hardly be satis-

* It is understood, however, that the method has never been applied in practice.

factory either to the instrument or in the result, and it is not surprising that the routine work that he had to perform as a clerk in the British Empire Office was uncongenial to him. While his colleagues universally liked and respected him they regarded him as somewhat of a dreamer. When the present writer joined the same Office there was a tradition which was not without significance, though probably not strictly true, to the effect that letters or documents which were missing in Hardy's day were not infrequently discovered at the back of his drawer, covered with abstruse mathematical investigations not always strictly related to the Office business. In short, his mind was not and could hardly be centred in routine work, and it was no doubt with a feeling of relief that in November 1880 he left the British Empire Office to take up actuarial tuition and private practice. It must have been about this time that he was appointed by the India Office to investigate and report upon the rates of mortality among the natives of India, as deduced from the Census Returns of 1881. This was the beginning of a long connection, lasting the whole of his life, with that and other Public Departments. The work was one of no little responsibility for a young man of 25, but the result was a masterly report which afterwards formed the basis of a paper read before the Institute on 27 April 1884. This paper (*J.I.A.*, vol. xxv, p. 217) still richly repays perusal on account of the difficulties of the problems involved and the ingenious methods which Mr. Hardy brought to bear on their solution.

His mind was now at work on original research. So early as January 1880, Mr. Hardy and Mr. George King had brought before the Institute a joint paper entitled "Notes on the practical application of Mr. Makeham's Formula to the Graduation of Mortality Tables" (*J.I.A.*, vol. xxii, p. 191) in which new and greatly improved practical methods were developed and the theory of the subject was greatly extended. The two years following his leaving the British Empire Office were remarkably fruitful in independent scientific work of the highest order. In March 1881, he brought before the Institute the first contribution from his own pen alone. This was a paper entitled "On the Mortality observed amongst various classes of Bonus Policies in the British Empire Mutual Assurance Company" (*J.I.A.*, vol. xxiii, p. 1) in which he measured the power of the policyholder's selection against the Office in a most interesting and original way. This paper is still often referred to for its valuable incidental discussion

of the relative advantages of lives, policies, and amounts as the basis of a mortality-experience investigation.

This paper was followed in the year 1882 and the early part of 1883 by a series of investigations of great power which now form an essential part of the literature of the science and are read as the work of an acknowledged master, though at the time he wrote them the author was a young man of only 26 or 27. In the paper on the determination of "The Rate of Interest in Annuities-Certain" (*J.I.A.*, vol. xxiii, p. 266) he finally replaced the existing cumbrous and unsatisfactory algebraic solutions by others which are based on finite differences and are at once more accurate in result and simpler in application. Concurrently with this he presented his memoir on "An improved Method of approximating to the Value of Annuities involving three lives" (*J.I.A.*, vol. xxiii, p. 274) in which he showed from purely scientific considerations how to secure great improvement in accuracy by the simplest means, involving little or no additional labour.

Another remarkable piece of work was his simplification of Woolhouse's method of adjusting mortality and other tables. As originally devised, this method involved a large amount of intricate calculation, a fact that Woolhouse himself recognized by putting forward in 1878 an improved method which, however, was still both lengthy and complicated. Woolhouse was probably a more highly-trained pure mathematician than Hardy, but he had not perhaps the same remarkable power of seeing things in their simplest forms and relations—a power which enabled Hardy to reduce Woolhouse's complicated process to a form which was theoretically more simple and instructive, and practically very easy to apply by a series of summations. This discovery was announced on the 24 April 1882 (*J.I.A.*, vol. xxiii, p. 351) in the discussion on a paper by Mr. J. A. Higham in which that gentleman developed a method similar indeed to Woolhouse's in its result, but expressly designed to replace it on account of its cumbrousness. It is impossible now to say whether this paper led Hardy to his own discovery, or whether it was one that he had previously made but not announced. It would not be surprising if the latter were the case, for in the course of his work he was continually throwing off the most ingenious methods which it was difficult, if not impossible, to get him to publish. Shortly after the reading of Mr. Higham's paper, Mr. Hardy deduced a number of similar graduation formulæ in a simple way

but it was not until 1896 (*J.I.A.*, vol. xxxii, p. 371) that he published his investigation, which has since done so much to help the systematic study of the subject.

On the 29 January 1883, Mr. Hardy read his paper "On Some Formulas for Approximate Summation" (*J.I.A.*, vol. xxiv, p. 95). This paper was followed by no regular discussion, but the President, Mr. (afterwards Dr.) T. B. Sprague, said, "For myself, I should like to say that I have a very high opinion of the value of the paper. It seems to me to be one of remarkable interest and to show great originality and ingenuity, and I shall anticipate that it will be practically found useful to actuaries." The nature and value of this most important paper were even more fully recognized in the following remarks by Mr. George King (*J.I.A.*, vol. xxvi, p. 277): "Mr. G. F. Hardy published a paper, the ultimate result of which will be to revolutionize our methods of dealing with complicated problems. These have hitherto been solved by resorting to a variety of expedients more or less reasonable, but the precise effects of which can never be foretold; or else the problems have been looked upon as insoluble, and mere guesses have been substituted for calculation. But Mr. Hardy, by an original and brilliant analysis, has furnished us with a number of formulas, simple, accurate, and brief, and which are applicable to functions of almost every kind. Nothing better seems to be required, and I feel sure that when these formulas come to be known and appreciated, they will be universally adopted." Mr. King's anticipations have been verified almost to the letter, largely no doubt owing to his own work in showing how Mr. Hardy's formulæ, and others derived therefrom, could be most conveniently applied, and because he included a full discussion of them in the Institute Text Book, Part II.

This paper contained some of the most characteristic, as it is now some of the most widely known, of Hardy's work. The mathematical processes involved were fundamentally not new, but it was left for him both to bring out how effectively they could be applied to actuarial problems, and to modify them so as to give the best results in that particular connection. In his original memoir the mathematical work was largely on the lines originated by Jacobi. These were somewhat advanced, but were no doubt more familiar to Mr. Hardy than they were to the profession at large, and it was natural that he should adopt them in his paper. When,

however, the formulæ were incorporated in the Text Book, Mr. Hardy produced original alternative demonstrations of a very much simpler description for the benefit of students to whom Jacobi's methods would be too difficult. As the result, the formulæ are now universally adopted, as Mr. King predicted they would be, and they form part of the everyday practice of the scientific actuary.

After leaving the British Empire Office Mr. Hardy had devoted a great deal of attention to Friendly Society work, both as an understudy to the late Mr. Ralph P. Hardy* and also on his own account. In August 1885 he was appointed Assistant Actuary of the General Reversionary Company on the recommendation of Mr. Ralph P. Hardy, and in that position he was able to continue his coaching as well as his Friendly Society work and other private work. In June 1887 he was appointed the official tutor of the Institute of Actuaries for Part II of the Examinations. In February 1888 the President of the Institute announced that the Messenger Prize offered for an essay on Friendly Societies had been awarded to "Mr. G. F. Hardy, a Fellow of the Institute, who had done great service to the Institute in connection with the class of which he was tutor, and who had distinguished himself by the valuable papers which he had contributed to the Journal." Up to this time the very brilliance of Mr. Hardy's mathematical work had perhaps produced a slight tendency among the senior members of the profession to regard him as a purely theoretical actuary with his head well in the clouds. After the publication of his Friendly Society essay† no such feeling was possible. Rather a complete treatise than a mere essay, and the more remarkable from the fact that nothing of the same kind had previously been attempted, its largeness of scope, breadth of treatment, and the practical way in which many difficult problems were handled, showed very clearly that it was the work of a man having a wide grasp of affairs and an intimate personal knowledge of the subject on its practical as well as its theoretical side. The essay, in fact, was adopted by the profession as a text book, which still forms an essential part of the education of the student; while men who had spent their whole life on the everyday work of friendly societies were struck by its extremely practical character, of which they were the best judges.

* Ralph P. Hardy and G. F. Hardy were not related.

† *J.I.A.*, vol. xxvii, page 161.

From this time forward, his progress to the head of the profession was steady and inevitable, in spite of the fact that his great and perhaps excessive modesty entirely precluded any attempt or desire to push himself to the front.

Mr. Hardy resigned the tutorship for Part II of the Institute Examinations in May 1893, and in the following June he was for the first time elected a Member of the Council, on which he served in all for 18 years. In the following February he left the General Reversionary Company to become the Actuary of the English and Scottish Law Life Assurance Association, a position which he held until April 1898, when he was appointed Actuary and Secretary (Principal Officer) of the Universal Life Assurance Society. About three years after he found it desirable to arrange for the union of that Office with the North British and Mercantile, and on the conclusion of the necessary arrangements he retired from official life, retaining, however, a wide and growing consulting practice, largely amongst the Government Departments. It was shortly after this time that he was first elected as one of the Vice-Presidents of the Institute.

Mr. Hardy was a prominent member of the *Joint Committee of the Institute of Actuaries and the Faculty of Actuaries in Scotland on Mortality Investigation*, which dealt with the British Offices' Experience of Assured Lives and Annuitants for the period 1863 to 1893. When the compilation of the experience had been completed and the results published, Mr. Hardy was invited by the Committee to undertake the highly responsible duty of preparing adjusted or graduated mortality tables in respect of the principal sections of the experience. This was a task which was at once highly onerous, congenial to his mind, and well fitted to display his powers. He had for some time previously been attracted by Professor Karl Pearson's new methods of curve-fitting, and he was the first to bring them to the notice of the actuarial profession.* These methods naturally appealed to Hardy's mind by their happy combination of elegant mathematical form and practical applicability to a variety of problems, and it is largely owing to his work that they have since become much more familiar to the actuarial profession. Mr. Hardy did not, however, find it desirable to adopt Pearson's methods in

* See his Review of Pearson's "Chances of Death" (*J.I.A.*, vol. xxxiii, p. 530 (1898)).

their entirety, since, although they might doubtless have produced a highly satisfactory graduation, they would not have had the same practical advantages to the actuary that might be obtained by other methods. So far back as 1894 (*J.I.A.*, vol. xxxi, p. 359), Mr. Hardy had announced his important discovery that Makeham's Formula, which had up to that point been regarded as applicable only to Aggregate Tables, could, by an ingenious extension and adjustment, be successfully applied to the graduation of Select Tables; and in 1898 (*J.I.A.*, vol. xxxiii, p. 493) he had given an example of its application to Sprague's Select Tables. This was the main idea underlying the whole of Mr. Hardy's graduation of the British Offices' Tables. In its practical working out he evolved many new methods, and while he adopted Pearson's basis of fitting by means of moments, he introduced what is a great practical improvement when a large mass of figures has to be dealt with, namely, the calculation of the moments by a simple process of successive summations, instead of by Pearson's actual methods.

The results of Mr. Hardy's graduations were presented to the Committee in a series of most valuable Reports, one of which is reprinted in the *Journal* (vol. xxxviii, p. 501), while in the final volume issued by the Committee (Account of Principles and Methods), he gave a full explanation of the whole of the work of graduation. An admirable review of this by Mr. Todhunter was given in the *Journal* (vol. xxxviii, pp. 359-363), and it is unnecessary here to say more than that the graduation was a brilliant piece of work, both from the elegance and power of the novel methods employed and the general excellence of the results. Speaking generally, it may be said that the $O^{[M]}$ Select Tables, the $O^{[NM]}$ Select Tables, the Male Annuitants' ($O^{(am)}$) Select Tables and the $O^{M(5)}$ Aggregate Table were all graduated on a strictly Makeham basis, thus securing the most important advantages in the practical calculation of Joint Life Annuities and other functions. The Female Annuitants' Select Tables ($O^{(f)}$) were adjusted by a combination of two Makeham graduations which secured many of the same advantages in a modified form,* while the important O^M (aggregate) Table bore a simple and definite functional relationship to the $O^{M(5)}$ Table.

* See the Memorandum as to the two-life Tables appended to the Annuitants' Tables; also an important paper by Mr. T. G. Ackland (*Transactions of the Faculty of Actuaries*, Vol. III, p. 285) which is perhaps not sufficiently widely known.

In a course of Lectures delivered before the Institute in the Session 1904-5 and published in 1909 (see Mr. Todhunter's review, *J.I.A.*, vol. xliii, p. 471), Mr. Hardy dealt systematically with his own and other methods of graduation and curve-fitting, and with the general theory of the subject, and he introduced the Pearsonian methods to actuarial students in a simplified form. At the same time he suggested also other and more elementary methods of his own, which he had long been in the habit of using with great success in his practical work, and which have perhaps not yet received the attention they deserve.

In June 1908 Mr. Hardy was elected President of the Institute, and thus became the titular head of the profession which he had done so much to adorn. His Presidential Address (*J.I.A.* vol. xliii, p. 1) will long be remembered and re-read for its originality of thought, power of expression, and stimulating quality; and probably its modest author alone had any doubt as its high place in a long series of such addresses.

During the term of his Presidency, the Government took up the question of National Insurance, with the view to preparing the scheme ultimately to be embodied in the National Insurance Bill of 1911, and it was natural that they should go to the President of the Institute of Actuaries for the advice which they required as to the financial cost of the scheme. With him was associated Mr. F. B. Wyatt, the ex-President. Mr. Hardy was exceptionally well fitted to take a leading part in such a matter, by reason of his unique actuarial skill and fertility of method—which enabled him easily to overcome difficulties that to others would be insurmountable—his wide grasp of affairs, his familiarity with the working of sickness assurance by the friendly societies, and his great sympathy with all well-founded measures of social reform.

The main Report of Messrs. Hardy and Wyatt, dated 20 May 1911, will be found in the *Journal*, vol. xlv, p. 406, and a supplementary Report, dated 28 November 1911, as to the revision or extension of the original estimates caused by changes effected in Committee, in vol. xlvi, p. 64. Mr. Hardy's hand is very plainly to be seen in these Reports, and it is a liberal education to any actuary to read or rather study them, and to note the ingenuity with which he and his colleague dealt with problems that, essentially of the most difficult character, were made much more so by the unfortunate absence of reliable data and the

inconvenient form and out-of-date character of a great part of the data that were available.

When the measure became law and the great scheme had to be put into operation, the National Health Insurance Joint Committee appointed an Actuarial Advisory Committee consisting of Mr. Hardy as Chairman and four other actuaries. Some idea of the intricate and difficult questions which were submitted to this Committee may be gathered from their earlier Reports, which are reprinted as an Appendix to the first Report of the Joint Committee (Cd. 6907). But only the members of the Committee themselves can fully appreciate the remarkable knowledge and skill with which their Chairman placed these complicated questions before the Committee, reduced them to their simplest and clearest form, and, in many cases, indicated new and ingenious methods for their solution.

In recognition of his distinguished services to the British and Indian Governments, Mr. Hardy's name was placed in the New Year Honours List of January 1914 as a K.C.B. Rarely can such an honour have been more richly deserved or more modestly received. It was a source of the greatest pleasure to his friends and admirers in two hemispheres and, as has been well said, his own pleasure arose more from the joy of his friends than from any personal pride, though he naturally appreciated very highly this public recognition of his work.

Sir George Hardy was a mathematician by instinct, but though few were better fitted than he to "ride a whirlwind and direct a storm" of complicated analysis, the bent of his mind was not in the direction of theoretical enquiries tending to no practical result. Mathematics has been defined as the science of measurement—often in its higher applications of *approximate* measurement—and it was from this standpoint that most of his original work proceeded, so that nearly all of it leads to some practical result or process. It will be found too, though this may be contrary to the general belief, that ingenious and original as his work was he rarely found it necessary to use any really advanced mathematics. The secret of his work, in fact, lay in his remarkable power of seeing things in their simplest relations, and accordingly of stating a difficult problem in such a way that its difficulty seemed to disappear and comparatively simple means were sufficient for its solution. The essential qualities of his mind were its extreme quickness of perception, its clearness and precision of view, its instinctive logic, and its capacity to pierce

through unessentials to the core of the subject. His mind had, in fact, a solvent quality, which enabled it to disperse the mists concealing the truth and to reveal the truth itself in a clear light. It was this quality, combined with his evident sincerity and intellectual power, that rendered him always an effective and attractive speaker; and though his style of speech was quiet and conversational, and almost wholly free from ornament, it had the essential quality of rhetoric—the power to impress his personality and views upon his hearers.

The direct influence of Sir George Hardy's work is of course very great, as will be obvious from our somewhat detailed though far from complete record, and it has been well said that his writings had "that classic quality which renders the best work suggestive and stimulating long after its results have become part of the common stock of scientific knowledge" (*J.I.A.*, vol. xliii, p. 471). But perhaps the indirect effect of his work may be even more important and enduring than the direct. He created what we may fairly call the "Hardy touch", which could scarcely fail to be transmitted in some degree to his pupils, and, through them, to pupils of their own, and though he was cut off at a comparatively early age he enjoyed the rare privilege of seeing the widespread influence of his work. It has already been remarked that he acted as official tutor for Part II of the Institute Examinations from 1887 to 1893, and it may be added that during the greater part of that time he had also private classes for the final examination, so that many of his pupils had the great advantage of reading with him continuously for both Parts II and III. None of them will soon forget his luminous, patient and suggestive teaching, or the trouble that he would take with individual pupils to remove their difficulties or assist them to pursue an enquiry. His pupils were indeed fortunate in their master, and perhaps also he was fortunate in having to train a band of pupils of something more than average ability, who were qualified both to appreciate his work and to diffuse and transmit it. In this way he virtually founded a new school of actuarial thought. He lived to rejoice in the fact that his pupils were selected for many of the principal posts in the profession; while they, on their side, can never forget what they owe to their tutor. Many of them became his personal friends, and it would be difficult to over-rate either the privilege or the charm of association with him in his ideally happy home life. His

union with the gracious lady whom he married in 1883 was so peculiarly complete in every way that the subject can hardly be more than touched upon.

It has been natural to dwell chiefly upon Sir George Hardy's work for and in the actuarial profession, but only a very imperfect estimate of him would be formed if it were supposed that this work, great as it was, absorbed the whole of his energies. So powerful a mind could never be satisfied with so limited a field, and in fact he was a man of wide culture. Music, Art and Literature were to him both a study and a recreation. He took a keen and appreciative delight in the beauties of nature and art, and he travelled widely in search of them. A great and quick reader, he had that selective memory which throws away the dross but never forgets anything of real importance, and his was a singularly well-stored mind, which made it a pleasure and an education to converse with him on any subject. His early interest in astronomy was always kept up, and he followed all the latest developments both of practice and theory. He was also a student of Egyptology, in which he was deeply versed, and he made important original investigations and suggestions as to a means of determining the age of the great Pyramid, and hence the period of the Fourth Dynasty, from astronomical considerations.* He took, also, great interest in all matters of social reform.

To his great qualities of mind, Sir George Hardy united a noble character, an equable temperament and a most lovable disposition. His uniform courtesy and consideration for others, his gift of quiet wit and kindly humour and his wide culture made him the most charming of companions in any surroundings. He had, in fact, a singular and magnetic attraction. To meet him was to wish to know him: to know him was to love him, and the more deeply as the knowledge increased. During a long and intimate friendship, lasting over 20 years,

* In a letter printed in the *Academy* (1069) 29 October 1892 and reprinted in the *Insurance Record* of 15 January 1915 he presented "a chain of astronomical evidence proving the commencement of the IVth Dynasty to have been very approximately 3700 B.C." The present writer does not know whether the evidence and results have ever been properly considered by Egyptologists; but it is interesting to note that the date given, *circa* 3700 B.C., is nearly midway between the widely-differing dates given by the most modern investigators, namely, 2700-2840 B.C. by one set of authorities and 4731 B.C. by Flinders Petrie (see *Encycl. Brit.*, 11th Edition, volume ix, p. 79). Hardy's letter shows his intimate knowledge both of practical Astronomy and of Egyptology, and his most interesting line of argument seems to the non-expert very convincing, especially in view of the consistency of the results derived from independent data.

the writer never heard a single unkind or uncharitable remark from him, nor noticed the slightest sign of failure of temper. Intolerant of nothing but intolerance, injustice and oppression, he was yet a man of strong character, capable of righteous indignation on behalf of others when the cause arose. In spite of his great powers, of which he can hardly have been quite unconscious, one of the charms of his character was a remarkable modesty as regards his own work, combined with a constant readiness to see good in the work of others.

Looking back on what has been written, the writer is conscious that to those who did not know Sir George Hardy it must wear the appearance of a too unrestrained panegyric. Such, however, will not be the opinion of those who knew him best; to them he will always remain a brilliant example and a very dear memory.

G. J. L.

Opening Address by the President, ERNEST WOODS, ESQ.

[Delivered 27 November 1914.]

GENTLEMEN,—It is my duty this evening to deliver the Presidential Address which is the prelude to our Session. It is a duty which falls very heavily on one who is not gifted with the pen of a ready writer, and it is a doubly difficult task to perform in this time of crisis. Occupied as all our minds must be with the wide issues at stake and the personal interests involved, it is practically impossible to restrain our thoughts within the narrow limits of actuarial science, strictly so-called. Touching, as our profession does, in a variety of directions, other sciences—law, medicine, finance, and political economy—it does not in itself appeal to the popular imagination. We have no tales of adventure in the Arctic or Antarctic, like those of Nansen and Scott, to unfold; no glorious doings in the field of war, as of Joffre, French or Jellicoe, or discoveries for the benefit of humanity, such as are associated with the names of Lister, Kelvin, and Rayleigh. Nevertheless, our work is of surpassing value in its sphere, concerning itself as it does with the safeguarding of the provision made by the provident for the welfare of those dependent upon them, whether in the form of premiums paid to Assurance Companies or of the contributions to those great Friendly Societies which have been one of the features of our national life, and we, too, by following our

vocation with earnest diligence, have our share in furthering the welfare of mankind.

During the year we have had to regret the loss of one Corresponding Member and three Fellows of the Institute—Monsieur Paul Guieyette, Mr. Jas. Chisholm, Sir George F. Hardy, K.C.B., and Mr. T. B. Winser.

Monsieur Guieyette, who was born in 1841, died on 19 May last. He was one of the founders of the Institut des Actuaire français, and its first President, being re-elected to that office continuously until his death. His ability in many fields of work was recognized by his fellow countrymen who elected him for twenty years as a Deputy for Lorient. In 1895, Monsieur Léon Bourgeois nominated him Minister for the Colonies, and he was filling that high position when the French Government annexed Madagascar. Those of us who were present at the third International Congress of Actuaries in Paris remember how ably he presided over our meetings, and the innate courtesy of his bearing to those with whom he was brought into personal contact. He was a practical as well as a theoretical actuary and was a Member of three very important French bodies—the Comité Consultatif of Railways, the Conseil Supérieur of Friendly Societies, and the Comité Consultatif for Workmen's Accident Assurance. In addition to his strictly mathematical work, he wrote valuable papers on such subjects as Deep Sea Soundings and Tidal Rivers, and his researches into the funeral rites of the ancient Egyptians and cognate matters were of such an important character that they won for him a high position at the Sorbonne.

Of the career of Sir George F. Hardy, who died on 5 October last, it is difficult to speak without emotion, his loss is so recent, and his personal relations with us so dear. His intellectual talent, his modesty, his charm of manner, and his kindness to those less gifted than himself are known to us all. He has been taken from us at the height of his career, when his great abilities were beginning to be recognized outside the limits of our profession. He was our President in 1908-10 and was in June last elected an Honorary Fellow of the Faculty of Actuaries in Scotland. We have only to recall his work as a Tutor, his graduation of our latest Mortality Experience, his paper on Formulas for Approximate Summation, his Prize Essay on Friendly Societies, not to mention his numerous other contributions to our *Journal*, to show the debt which our science

owes him. The ability which he displayed in connection with the preparation of the National Insurance Act 1911 led to wider appreciation of his talents, and the high opinion entertained of him by His Majesty's advisers was shown in the Knighthood of the Bath conferred on him last January. Like Monsieur Guieyesse, he was a worker in other scientific fields than those of the Actuary, and it is a curious coincidence that they should both have concerned themselves in their leisure hours with questions connected with the ancient civilization of Egypt. In astronomy also he took great interest, and was among the first to make calculations as to the age of the Great Pyramid based on the relative positions of the various passages and the stars.

Mr. Chisholm, who died on 5 July last, was by heredity a member of our profession, being the son of Mr. David Chisholm, the first Actuary of the North British and Mercantile Insurance Co., and became a Fellow (by examination) of the Faculty of Actuaries in Scotland in 1882, and of our Institute in the following year. He was a Member of this Council for twenty years, and served the offices of Honorary Treasurer and Vice-President.

Mr. T. B. Winsor, who died on 21 October last, at the age of 91, became a Fellow of the Institute under our Charter as a Member of the Actuaries' Club. His versatility was shown by his skill with the military rifle, which brought him to the fourth place in the competition for the Queen's Prize at Wimbledon in 1864, by his facility in the use of the lathe, and in other ways.

My friend, Mr. Schooling, in his Presidential Address delivered two years ago, was not content with reviewing the circumstances of the time, and endeavoured to draw lessons from the history of the "directions the curve produced by the rate of interest has taken during the last hundred years or so." Like him, I propose to dive into the past, and submit for your consideration some facts and comparisons in regard to the financial burdens of the Nation—its liabilities, its expenditure, and its revenue, but I propose to leave to you the task of drawing lessons therefrom, for any attempt to do so would involve the expression of political opinions, which would be irregular in this place. It is too soon to forecast the cost of the mighty war in which we are engaged, and the method which will be adopted to liquidate it. The burden will be heavy, but not intolerable. During the short lifetime of this Institute there have been wars of no small magnitude—in the Crimea,

in Abyssinia, in Ashantee, in Zululand, in South Africa, on the frontiers of India and other parts of the world; but we have to go back to the time of Napoleon if we desire to find a parallel for the crisis through which we are passing, and as the country triumphantly emerged from that time of stress, so it will assuredly do again when the present war is over, for it is now in a far better position. The Total Debt on 1 February 1816 was £840,758,781, as compared with £707,654,110 on 1 April 1914, while the net produce of the Revenue in 1815-6 was £79,939,670, as against a total Revenue in 1913-14 of £198,242,897. The population of the United Kingdom in 1816 was about 18,640,000, and was 46,035,570 as estimated on 30 June 1913, so that the position at the two dates may be summarized as follows :

	1816	1914
Liabilities to Income	10·5	3·6 years' purchase.
Liabilities to Population	£45 2 1	£15 7 5 per head.
Income to Population	£ 4 5 9	£ 4 6 1 per head.

It has been estimated that the direct cost to this country of the Napoleonic wars was about £830,000,000. and that about £600,000,000 of that amount was met by an addition to the National Debt, for which the annual charge was £42,000,000, including the sum of £12,968,000 for the support of the Sinking Fund. According to modern ideas, too large a proportion was left to be borne by posterity, but the provision for repayment was in the circumstances of the time comparatively large, and I have seen it stated that had the provision been maintained, the whole of the National Debt would have been paid off by the year 1845.

It would have been interesting to review the progress of the National Debt from the end of the Great War to the present date, but time does not permit this to be done, and I propose to limit my remarks to a brief review of the National Finances during the last quarter of a century. Twenty-five years ago the influence of Mr. Gladstone on the fiscal arrangements of the country was still supreme, and it seems only the other day that he appealed to the country with a promise to abolish the Income Tax. In the short time which has since elapsed, the change in practice with regard to the methods of raising revenue, and the objects upon which it has been expended, have been revolutionary.

Mr. (afterwards Viscount) Goschen, who was imbued with the Gladstonian spirit, was responsible for the six Budgets 1887-8 to 1892-3. During the administration of that expert financier, whose ability was undoubted, the country's expenditure increased by £10,165,923, from £87,423,645 to £97,589,568 and his term of office was remarkable for the conversion of the National Debt, which he carried out with brilliant success, but he reduced the fixed charge for cancelling the Debt from £28,000,000 to £25,000,000—a doubtful precedent for his successors. As a result of the Debt conversion and careful supervision of the Estimates, notwithstanding an increase in the expenditure on the Navy, he was able to reduce the Total Debt of the Nation by £65,033,130, from £736,153,067 at the date of his first Budget to £671,119,937. It is not a little remarkable that with all his skill and foresight, understanding as he undoubtedly did the defects in our fiscal arrangements, he did so little to reform them. He must, however, be credited with good work, in that, without throwing too heavy a burden upon the taxpayer, he provided the foundation for a reformed system of Local Government and for Free Education, and by his Naval Defence Act 1889 controlled the expenditure on Contract Work, enabling the Admiralty to retain unexpended balances, which conduced to economy, and prevented the absurd practice of voting money twice over for the same object. He also was the pioneer in that movement which has since made such great strides in the direction of the graduation of death duties.

Mr. Goschen was succeeded by Sir Wm. Vernon Harcourt, who produced three Budgets, 1893-4, 1894-5, and 1895-6, and his term of office was historical, as he was the first to bring into practice some of those more modern theories of taxation which have since been so widely adopted. Mr. Gladstone, when at the height of his financial powers, declared himself to be perfectly impartial as between direct and indirect taxation and described them as “two attractive sisters,” each with an ample fortune, to both of whom he, as Chancellor of the Exchequer, had always thought it not only allowable but even an act of duty to pay his addresses. It had become a tradition to maintain as far as possible an equal numerical proportion between these two branches of taxation, balancing relief in one direction by a like relief in another, but from the time of Sir Wm. Vernon Harcourt it began to be the object of legislation to relieve the poorer classes by increasing the direct taxation

paid chiefly by the richer members of the community. The reform of the death duties, which had been foreshadowed by Mr. Goschen, was taken in hand by Sir Wm. Harcourt in his second Budget, and the two characteristics of the scheme were the endeavour to bring personalty and realty into line, and the adoption of a graduated scale in the rate of duty, taking into account the whole aggregated value of the property passing on the death of the deceased. It is not necessary to enter into the details of the scheme, but their financial effect may be noted. The net receipts from death duties were in 1887-8 £8,284,203, and in 1894-5 £8,754,343. In 1895-6, the year the reform was brought about, they brought in £11,639,900, while in 1914-5 they are estimated to produce £28,750,000. Another feature of this Budget was the extension of the graduation of the Income Tax, the limit for total exemption being increased from £150 to £160 (so as to include the recipient of wages of £3 a week) and abatements being allowed of £160 upon incomes up to £400, and of £100 on incomes between £400 and £500. The rate of income tax, which had been 6*d.* in the £ for the preceding five years, was raised successively to 7*d.* in 1893-4 and 8*d.* in 1894-5 at which rate it remained until the expenditure on the war in South Africa caused it to be raised to 1*s.* in 1900-1. The net result of Sir Wm. Vernon Harcourt's Budget was to reduce the Total Debt of the country by £18,833,571, to £652,286,366. The increase of £10,165,923 in the total gross expenditure in Mr. Goschen's six years was succeeded by an increase during Sir Wm. Vernon Harcourt's three years of £7,540,906, making the total expenditure £105,130,474, and in this connection the warning given by the last-named is worth recalling at the present day. He said that the expenditure had "already reached the limits of tolerable taxation." "There is a universal demand for more and more expenditure every year for every conceivable object, all of them excellent objects, but all of them pursued absolutely without any regard to their cost. Besides these demands for additional expenditure for every possible object, there are continual proposals to cut off first one and then another item of the public revenue. In private establishments you endeavour to regulate your outlay with some regard to your income, but in public administration you have to make your taxation keep pace with your profusion. I am not going to preach a sermon on this subject, because economy (not only political economy) has become a lost art

“at the close of the century. It is a despised and unfashionable idea, and I do not know whether, under any circumstances, it will ever come into fashion again. Everyone grumbles if money is not spent on his favourite fancy, but he grumbles still more when he is called upon to find the means of paying for it.”

Sir Michael Hicks-Beach (now Viscount St. Aldwyn) carried through seven Budgets (1896-7 to 1902-3) and during those years a wave of national prosperity in commerce and industries was at its height, and fortunately so, for it made the burden of the South African War less heavy to bear. The total of the direct expenditure on military operations for the four years ending 31 March 1903 was estimated to be £217,000,000, and the indirect loss was very large. In a paper which Mr. Edgar Crammond read before the Institute of Bankers in April 1910, he compared certain groups of figures during the three years 1900, 1901, and 1902 with those for the three years 1906, 1907, and 1908, when the effects of the war were practically overcome. He showed that the values of the exports of British Manufactures averaged £284,879,000 in the former period and £392,908,000 in the latter, while the average amounts of capital offered for public subscription in London for foreign and colonial investments were £47,167,000 and £104,921,000 respectively, and he deduced from his calculations that “two years’ savings”, which would otherwise have been available for investment abroad, had been absorbed, with the inevitable result of depressing our export trade. The Total Debt of the country during the first three years of Sir Michael Hicks-Beach’s term of office was reduced by £16,892,632 to £635,393,734 on 1 April 1899, the lowest level that it has attained in modern times, but on 1 April 1903, it had again been increased by the huge sum of £162,955,456 to £798,349,190. At the present time, when we are face to face with a position similar but inevitably far more serious, it is instructive to review the steps taken to raise revenue to meet this expenditure. It would not be in accordance with the object of this review, which is to record financial results actually achieved, if more than allusion is made to the controversial views taken by prominent politicians and parties as to the methods adopted, and it will suffice to note that during Sir Michael Hicks-Beach’s years of office the question of “fair trade” or “tariff reform” began to be more or less seriously mooted. In 1899-1900 wine and stamp duties were increased, and in 1900-1

tea, tobacco, beer and spirit duties were raised. In 1901-2 new duties were imposed on sugar and coal, and in 1902-3 on imported corn and grain products. The new duties on corn and coal, about which there was much controversy, were eventually taken off, the former on 1 July 1903, having produced only £2,448,030, and the latter after six years having produced on the average £1,847,775 a year. In the same years the Income Tax which was previously 8*d.* in the £ was successively raised by 4*d.* in 1900-1, 2*d.* in 1901-2, and 1*d.* in 1902-3. The effect of these changes was to increase the receipts into the Exchequer by £31,514,505, from £129,804,566 in 1899-90 to £161,319,071 in 1902-3.

Mr. Ritchie's only Budget, 1903-4, and Mr. Austen Chamberlain's Budgets 1904-5, 1905-6, introduced no great changes save the abolition of the corn tax, and although 4*d.* in the £ was taken off the Income Tax in 1903-4, an extra 1*d.* was imposed in the following year, leaving it at 1*s.* in the £. The total aggregate gross liabilities of the State were reduced in these three years by £9,359,003, to £788,990,187. Perhaps the most important financial legislation of this period was that contained in Mr. Wyndham's Irish Land Act of 1903 (amended in 1909), which has enabled more than one half of the agricultural area of Ireland to be sold, at a total price of £112,839,088, of which £67,839,088 had actually been paid by November 1911. The discussion of Mr. Ritchie's Budget was concerned to an important extent with the incidence of the war taxation. Just before the South African War the proportion between direct and indirect taxation was 47·9 per-cent and 52·1 per-cent respectively, while for the additional war taxation it was 55·5 per-cent and 44·5 per-cent, the proportions for the whole Budget in 1903-4 being 49·3 per-cent direct and 50·7 per-cent indirect. Previously to this time the Income-Tax had been regarded as a "special implement of war," as in practice it had been found that business was less disturbed by its increase than by any increase of an old tax or imposition of a new one, and experience showed that in war times it is most difficult to devise new taxes on commodities, when a proper enquiry into their consequences is, in fact, impossible. The total gross expenditure which, as a result of the war in S. Africa, had increased in 1901-2 to £205,236,305, was reduced in 1905-6 to £150,413,245, and the aggregate gross liabilities by £9,359,003 from £798,349,190 on 1 April 1903 to £788,990,187 on 1 April 1906.

Mr. Asquith introduced three Budgets, 1906-7, 1907-8, and

1908-9, although he had become Prime Minister and ceased to be Chancellor of the Exchequer when his latest came before the House of Commons. He was face to face with representatives of the people, who were prepared for heavy expenditure of all kinds on social reform, and desirous of reducing naval and military expenditure. He nevertheless insisted on the absolute necessity of reducing the national indebtedness and made reforms in our fiscal arrangements. He was opposed to the practice of paying by means of loans for capital expenditure incurred by public departments (even though their repayment was provided for by their own sinking funds), and to the system under which revenue was collected by the State, and paid direct to the local taxation authorities without being included in the revenue of the nation. He introduced a valuable administrative change which had the effect of leaving the management of indirect taxation to the Commissioners of Customs and Excise, and of direct taxation to the Commissioners of Inland Revenue. He reduced the Tea Duty by 1*d.* per pound, and the sugar duty from 4*s.* 2*d.* to 1*s.* 10*d.* per cwt., and increased the rates of Estate Duty. He carried the reform of the Income Tax a step further. The principles of differentiation and graduation had been for long dear to the more modern school of financiers. Differentiation, in that it takes into account the character of the income as permanent or precarious, unearned or earned, and graduation, because it adapts itself to the ability of the taxpayer to bear its burden, so far as that can be measured by the amount of his income. In fact, Mr. Asquith reduced the Income Tax from 1*s.* to 9*d.* in the £ on earned incomes. The final result of his three years' management of the Nation's finances was to reduce the total debt by £34,868,878 to £754,121,309 on 1 April 1909, the gross expenditure being increased by £1,879,150 to £152,292,395 in 1908-9.

Mr. Lloyd George, the present Chancellor of the Exchequer, has introduced six Budgets, commencing with that for 1909-10, and his general policy may be briefly summed up as forsaking the older views as to the objects and limitations of taxation, and the adoption of the principle that our fiscal arrangements should be employed for the purpose of social reform. This, however, is not the place for the expression of any personal opinion as to the wisdom or otherwise of this change, as to whether its objects will or will not be attained,

and if attained whether their results will be for the real welfare of the people or whether they may in the end be the reverse.

In his first Budget he introduced one new duty, that on motor spirit, and increased the duties on spirits, tobacco, and motor car licences, and he revised and increased the liquor licence, stamp and estate duties. He increased the income tax from 1s. to 1s. 2d. (less certain new allowances) and he introduced the super-tax and the land value duties. He made no further change of any outstanding importance until this year, when he brought forward proposals which will be in your recollection, but the political questions which occupied Parliament during the earlier part of the Session and the legislation which had to be hurriedly carried through to meet the exigencies of the war prevented that full consideration of the Budget which it required. The financial result of Mr. Lloyd George's tenure of office has been that the Total Debt of the Nation was reduced by £46,467,199 to £707,654,110 on 1 April last, but the gross expenditure increased by £45,200,574 to £197,492,969 for the year 1913-14, with a further increase of £9,528,031 to £207,021,000 (as estimated before the war) for 1914-5, a total increase of £54,728,605. Of this great increase £2,045,000 is for the Army, £19,362,000 for the Navy, £21,619,000 for such social services as Education, Labour Exchanges, National Insurance, Old Age Pensions, and Public Health, £14,596,891 for the Customs, Excise, Inland Revenue, Post Office, and Civil Service, with a reduction of £4,500,000 in the National Debt Service.

The general result of the changes in the period under review has been to increase the gross expenditure of the Nation by 136 per-cent, and to change materially its incidence. In 1887-8 the Army and Navy accounted for 35·2 per-cent of the total expenditure, as against 38·8 per-cent estimated for 1914-5, while the proportions for Social matters were 5·6 per-cent and 20·2 per-cent respectively. The service of the National Debt absorbed 30 per cent of the country's income in 1887-8, and only 11·4 per-cent was to be set aside for that purpose in 1914-5, notwithstanding that the gross liabilities of the country have only been reduced from £736,153,067 to £707,654,110, or by 3·87 per-cent. If the change in the incidence of the expenditure has been great, the change in the incidence of taxation has been even greater, for whereas in 1887-8 the receipts into the

Exchequer from Customs and Excise amounted to 50·4 per-cent of the total, they were estimated (before the war) to amount in 1914-5 to 36·4 per-cent only, while on the other hand the proportion of receipts from Death Duties and income-tax has increased from 25·3 to 39·9 per-cent.

The mere recital of measures adopted since the declaration of War for the preservation of credit and the regulation of commercial transactions with the enemy, would be useless, and to review them would occupy too much time, so I will confine myself to referring to the unfortunate error in a Treasury letter as to the application of the Moratorium to the payment of Life Assurance Premiums, and to expressing the opinion that it was better for the assured that the Moratorium did not apply. Otherwise many might have been tempted to avail themselves of its provisions, for later on, if the strain of the war begins to be more seriously felt, there may be difficulty enough to find the necessary means to pay premiums, and the longer the non-payment is deferred the better for the assured.

I fear that you will have found it difficult to follow the large array of figures which I have submitted to you, but they are perhaps not unworthy of attention, for it is to the yield of the normal working of the country's fiscal system that the war expenditure will have to be superadded. What that expenditure will amount to must depend on the development and duration of military operations, and, with no data to build upon, it is futile to attempt to make any forecast. It will be for Parliament to decide how the final cost shall be eventually provided, whether by the methods of Gladstonian finance, or the later methods of the present Chancellor of the Exchequer, or by some compromise.

Before leaving the subject of Finance, I must ask you to let me refer to the *Times* of 23 July 1914, in which it is reported that the Chancellor of the Exchequer had said in the House of Commons that "Every Budget which increased Taxation increased the business of Insurance Companies. The Budget of 1909 had had the effect of quickening the business of insurance very considerably, and he had no doubt at all that the present Budget would have a still greater effect in the same direction." I cannot but think that the Chancellor must have been badly advised by someone who based his opinion on the number of Death Duty circulars and advertisements issued by the

various Companies rather than on the essential facts of the case. I assume that he had in his mind "ordinary" and not "industrial" business, and it is true that the premium revenue of ordinary Life Companies has steadily increased year after year, as have also the means of the classes who are assured by them. Has the increase in the premiums kept pace with the means of those who pay them? Economists have divided the population into three classes: (1) Those who pay Income-tax, (2) the manual labourers, and (3) those who are not engaged in manual labour, but do not pay Income-tax, and in my opinion it is a fair assumption to make that those who pay premiums to the ordinary Insurance Companies may be taken to correspond with the first of these three classes. If, then, the Premium Revenue of the companies be compared with the taxable income of those who pay Income-tax, it will be found that premiums show the following proportions of incomes—in 1900-1, 3·1 per-cent; 1902, 3·2 per-cent; 1903-4, 3·3 per-cent; 1905-6-7-8, 3·4 per-cent; in 1909 and 1910 (combined), 3·5 per-cent; and in 1911, 3·4 per-cent. These figures do not appear to support the dictum that increased taxation quickens the business of insurance to any material extent. In confirmation of the foregoing figures, it may be of interest to note that from the year 1889, when the percentage was 2·6, there was a steady rise to 3·3 per-cent in 1895—the year of Sir Wm. Vernon Harcourt's revision of the Death Duties and thereafter a steady decline to 3·1 per-cent in 1900. These results are such as would be expected, seeing that increased taxation diminishes the means of that vast body of assured (which is the mainstay of Life Assurance business) who have only limited incomes out of which to pay premiums, and any increase of assurance among the richer classes, who are more particularly affected by the Death Duties, is not sufficient to counterbalance the loss of new business received from those of more limited means. In this connection we sometimes hear of the relief given to policy-holders by means of the allowances from Taxable Income in respect of Life Assurance premiums, and it seems probable that we sometimes measure the size of this relief by the amount of the premium revenue of the companies as summarized in the Annual Returns to the Board of Trade. This amount was £27,122,977 for business within the United Kingdom, in the Accounts published this year, but, in fact, the actual allowances made for the year ended 31 March 1913 only

amounted to £11,882,213, or 43·8 per-cent of the former figure, and that on a gross income of £1,070,142,343 brought under the review of the Department.

The taxation of Life Offices in the near future is not a pleasant subject of contemplation. It is true that a higher yield from interest may be anticipated, but apart from any change which may be introduced in the methods of taxing Assurance Companies, there will have to be faced a substantial increase in the income-tax for many years, besides the provision out of income for permanent loss of Capital through depreciation. Temporarily, too, there will be the further drain involved in the expected heavy loss from Mortality caused by the war which is not likely to be met by the receipt of adequate extra premiums, since the Companies have decided not to charge such on previously existing policies on the lives of those who are joining H.M. Forces for the duration of the war.

The National Insurance Act 1911 has now been in force for two years, and calls for some reference. We have had the Report for 1913-14 on the Administration of National Health Insurance, and not the least valuable part of that Report is the portion to which is attached the signature of Mr. A. W. Watson, Chief Actuary to the Joint Committee, whose actuarial qualifications and practical acquaintance with the organization and working of Friendly Societies must be of inestimable benefit to those associated with him. We have also had the accounts in the newspapers of the Conferences of various Classes of Approved Societies, as well as the interesting Report of the Committee of Enquiry instituted by the Fabian Research Department, each dealing with the subject from a different point of view.

The Official Report consists of five separate Reports made respectively by the Joint Committee and the Four Commissions for England, Scotland, Ireland and Wales respectively. It is a mine of information as to the administration of Part I of the Act, but is too voluminous to be summarized—the index alone occupies eighteen pages, and eleven pages are devoted to the list of official publications. There are four separate Revenue Accounts, and if these are added together it will be found that on 11 January last the balances in the hands of the National Debt Commissioners for investments and at the Banks amounted to £15,564,992, and that a further sum of £937,689 had been paid to Approved Societies for investment.

It would be interesting to know the average rate of interest actually realized, and the classes of security chosen. According to the Report the total interest earned since the Act came into force amounts to £93, which on the face of it would appear to be most unsatisfactory, but I understand that the explanation lies in the complexity of the accounts and the consequent difficulty of preparing them in time for publication and not in any undue delay in investment, against which very stringent rules have been made. No doubt in future Reports, further financial information will be published. There was an amending Act last year, and another was in course of preparation for submission to Parliament this year, but has not reached maturity. In all probability there will be a series of other Acts in future years, for the Scheme cannot now be cancelled, and such a step as a reversion to any voluntary system is now impracticable.

That the Scheme will be more costly than was originally expected is already apparent. In his address to the National Conference of Industrial Assurance Approved Societies the President, Mr. A. C. Thompson, stated that the sickness among women appeared to be at least sixty per cent in excess of the expected rate, and that in some special classes of Women's Societies it was greatly in excess even of that high rate. He added that as a result of a very careful investigation which he had made he found that only a very insignificant portion of the total claims for sickness benefit was attributable either directly or indirectly to pregnancy and maternity. Mr. Thompson did not explain how he arrived at his percentage, neither did he state whether he had before him such sufficient and precise data as an Actuary would deem necessary before making an estimate, nor did he express any opinion whether any part of the excess was due to the mistakes of administrators inexperienced in the practical work of sickness supervision. Considerable weight, however, must be accorded to the conclusions of an administrator who has so great an opportunity of observing the working of the Act as Mr. Thompson possesses. In this connection a very valuable Report by the Departmental Committee on Sickness Benefit Claims has just been presented to Parliament, in which it is recommended that the general excess in Women's claims should be met by diverting to the funds of Societies a portion of the sums which now go in redemption of reserve values. The Committee, who probably had much fuller information before them than Mr. Thompson, state that they cannot

estimate the precise amount to be so applied, or the number of years which such a diversion would add to the time required to pay off reserve values, and they recommend that the Chief Actuary should make the necessary calculations. The Committee further recommend the grant of a new benefit payable to pregnant women in respect of the last four weeks of pregnancy, whether incapacitated or not, such grant to be provided by the Treasury, and allocated to the Societies so as to cover the actual cost of the benefit without increasing the amount available in their funds for sickness or any other benefit.

The Committee are unable to suggest any remedy for the serious excess which has arisen in the case of women workers with children who receive low wages, who gain a distinct pecuniary advantage in declaring on the Funds.

It is satisfactory to know that the Committee find that, taken as a whole, the experience as regards men's sickness benefit justifies the actuaries' estimates, but as was to be expected, the effect of segregation of persons exposed to special health risks or with special predisposition to sickness in particular societies and branches, has been to produce in certain cases excesses over the actuarial provision, while, on the other hand, in some cases especially where there has been a very strict selection of lives, an abnormally light rate of sickness has been experienced. It is further pointed out that the excessive sickness is in many cases the result of mismanagement, and in others arises partly from segregation and partly from mismanagement. The remedy proposed is the formation of a Special Risk Fund by a contribution from the Sinking Fund, supplemented by a Parliamentary grant, and that payments should be made thereout to Societies where deficiencies result from segregation, but not where they arise from maladministration. It is further proposed that future grants should be conditional on the Societies concerned having adopted such administrative changes as may meet the views of the Commissioners. It would seem that any such scheme will have to be very carefully watched in its operations, otherwise a premium will be put on maladministration. The Committee point out that over-insurance is a definite cause of excessive sickness claims, but make no recommendation beyond suggesting that insured persons who are willing and able to make provision on such a scale should be encouraged to secure alternative benefits.

I cannot pass from the consideration of the National Health

Insurance Act without expressing the opinion—an opinion which I am confident is supported by the experience of other Actuaries—that societies insuring sickness benefits, whether under the Act or voluntarily, must in the future prosper or otherwise in proportion to the interest which the members themselves take in the administration of their affairs, and the attitude which they adopt. Since the payment of benefits began there appears to have been a belief—held more generally perhaps by those who had not previously been insured—that the payment of normal benefits is secure, however defective the supervision of claims may be. This of course is an erroneous belief, and all of us, who have the opportunity, should never fail upon all occasions to impress upon those concerned the absolute necessity of the utmost vigilance in the supervision of sickness.

At our Annual Meeting in June my predecessor in this chair referred to the steady progress which has been made with the project of the Research Bureau, and expressed the hope that shortly it would be launched upon its career of usefulness, but it is my unfortunate duty to have to announce that the Council has decided that further progress so far as the collection of data is concerned must be arrested until after the war. It would be manifestly unfair to ask the various Companies who had promised their assistance to undertake the necessary work involved in the absence of so many members of their staffs, but there is good reason to think that some of our Fellows who are more particularly interested in the subject will not in the meantime be idle, as there is much preparatory work which can be usefully proceeded with. The proposed Bureau had two objects in view, each practically distinct from the other, the one being the preparation of new Tables of Assurance experience, on the lines of those of the “Institute of Actuaries” and the “British Offices”, the other the investigation of the effects of Family and Personal History on Life Assurance. The former object which is generally supported is of importance in view of the fact that the data for our latest published Tables are already over 21 years old, but as to the utility of the latter I have to confess that until recently I, at any rate, was doubtful. An eminent American Actuary, however, who was recently in this country has compelled me to alter somewhat my views, and to think that some practical benefit may ensue, provided that we do not expect too much. I am of opinion that any advantage which may arise will show itself, not

so much in the way of the exact financial assessment of the extra risk, as from the danger signals which the statistics will disclose, warning us of heavy mortality in directions hitherto unsuspected. At the Fifth International Congress in Berlin a suggestion was made that there should be an international investigation of hazardous risks, and a draft plan was prepared by the Permanent Committee for submission to Actuarial Societies throughout the world, but for obvious reasons it did not meet with any support here, and eventually it fell through. In America, however, two investigations in this direction have been made and published, and Mr. Arthur Hunter has been good enough to send to our *Journal* most valuable notes on the results obtained so far. In a paper which he and Dr. Oscar H. Rogers prepared for the Actuarial Society of America they summed up the conclusion at which they have arrived, a conclusion which in my opinion it is of the utmost importance to bear in mind, if we wish to form a true estimate of the value of the proposed investigation. Let me quote : “ Our studies of the result “ of this investigation, which we have summarized in this “ paper, have impressed us again and again with the great “ influence of the varying degrees of severity of selection upon “ the resulting mortality. The evidence is very strong that in “ any impairment a rigid selection will produce a low mortality, “ a lax selection will produce a high mortality, and that “ whenever we wish to reproduce in the future a given mortality “ in any impairment we must imitate closely the degree of “ selection which has produced that mortality in the past. We “ may not say of any impairment that the mortality in it will “ under all circumstances of selection be the same. On the con- “ trary, our action with respect to any impairment must rest “ not only upon the mortality which would occur in that “ impairment if unselected, but also upon the profound “ influence of the selection which we practise towards it.”

You will not be impatient, I trust, if I refer briefly to the Congress which was to have been held in St. Petersburg, or as it is now called, Petrograd, in June next. No official intimation has yet been conveyed to us that the Congress will not meet, but I have heard unofficially that our Russian friends consider that it will be impossible in any circumstances for them to receive us, and you will agree with me that it would be unwise to hold it in any other country, even should Peace have been made. As on all previous occasions, the number of papers by

Members of this Institute would probably have exceeded those of any other nation. The importance of the Congress, in the view of the Russian Government, was evidenced by the fact that six Ministers had accepted the office of Honorary President, and doubtless the reception which would have been given to us would have been most gracious. We all trust that with the speedy return of happier times, the Congress, though deferred, will meet with great success.

I should like to add, however, that it has for some time been the opinion of many of us that these Congresses are held too frequently, and at more than one meeting of the Permanent Committee we English have suggested that the interval between them should be five years instead of three, but we have always been outvoted by our foreign colleagues.

In this connection, may I be allowed, in my capacity of Honorary Correspondent of the Permanent Bureau, to express regret at the delay which has occurred in the publication of the Annual Bulletin, owing, no doubt, to the unhappy circumstances of our Belgian friends. Belgium is the home of International Actuarial Congresses, and we pay homage to our ally, her heroic King and Army in their hour of agony, and we pray sincerely that, when this war is over, there may be given to her a future of such prosperity and happiness that the memory of her present suffering will be recalled rather as a hideous nightmare than as an historical fact.

There is one matter in connection with the Institute to which I desire to call attention and that is the scarcity of papers for discussion at our Monthly Meetings. I appeal to some of the younger Fellows to come forward and support their Alma Mater! The Institute has given you far greater opportunities of acquiring a full knowledge of your professional work than we older Fellows had when we commenced our studies. I think sometimes that the desire to write a paper is checked by the fear that a subject although important is not large enough to fill up the number of pages in the *Journal* which so many of our contributions of recent years have occupied, but let me assure you that shorter papers are often of very great value, and may lead to most interesting discussions. Apart from purely technical papers, there would seem to be room in the near future for the consideration of the various emergency measures taken by the Government to preserve the Nation's credit in these times, reviewing fully their objects and

the results attained. I think also that papers on such subjects as the investment of our Funds in various foreign countries, and on Local Taxation and Finance, would be of great practical use.

I beg to thank you for the great patience you have shown in listening to me, and to express my regret that I have been unable to make my address more worthy of your consideration.

NOTE.

Comparison of National Income and Expenditure in the years 1887-8 and 1914-5.

Receipts into the Exchequer.

	1887-8	Per- cent	1914-5. As Estimated before the War	Per- cent
Customs and Excise ...	£45,250,000	50·4	£75,350,000	36·4
Death Duties ...	8,284,203	9·2	28,770,000	13·9
Income Tax ...	14,440,000	16·1	48,121,000	26·0
Super Tax	5,800,000	...
Stamps ...	4,715,797	5·2	9,900,000	4·8
Land Tax ...	1,030,000	1·1	700,000	·3
House Duty ...	1,940,000	2·2	2,000,000	1·0
Land Value Duties	725,000	·3
Taxes ...	£75,660,000	84·2	£171,366,000	82·7
Non-Tax Revenue ...	14,142,254	15·8	35,780,000	17·3
Total Receipts ...	£89,802,254	100·0	£207,146,000	100·0

Exchequer Issues exclusive of Expenditure not chargeable against Revenue.

	1887-8	Per- cent	1914-5. As Estimated before the War	Per- cent
National Debt ...	£26,213,911	30·0	£23,500,000	11·4
Army ...	18,433,330	21·1	28,885,000	13·9
Navy ...	12,325,357	14·1	51,550,000	24·9
Civil Service ...	14,779,185	16·9	18,772,000	9·1
Customs, Excise, Inland Revenue and Post Office	10,749,097	12·3	31,048,000	15·0
Public Education ...	4,922,765	5·6	19,564,000	9·5
Labour Exchanges and Insurance	9,312,000	4·5
Old Age Pensions	12,710,000	6·1
Public Health, &c.	250,000	·1
Roads Improvement	1,545,000	·7
Local Taxation	9,885,000	4·8
	£87,123,645	100·0	£207,021,000	100·0

Canadian Mortgages regarded as a field for the Investment of the Funds of British Life Assurance Companies, with some general notes on Canadian Indebtedness.
By ARTHUR DIGBY BESANT, B.A., F.I.A., General Manager and Actuary, Clerical, Medical and General Life Assurance Society.

[Read before the Institute, 21 December 1914.]

INTRODUCTION.

IN the course of the past summer I visited Canada, partly with a view to investigating upon the spot the opportunities which the Dominion might afford for mortgage investments, and partly in order to gain, at first hand, knowledge of the general financial conditions of the country.

The subject is one of great importance, and although the outbreak of war may have rendered all questions of new investments of purely academic interest for the moment, yet I hope that the following notes will be of interest to the Institute, and may be turned to some practical use in a not distant future.

In the course of my tour I travelled right across Canada from Quebec on the East to Vancouver City and Victoria on the West, visiting the principal towns of the various districts, interviewing bankers and leading citizens everywhere, and endeavouring to master the intricacies of the legal procedure involved in carrying out mortgage transactions in Canada. I received the utmost assistance from the officials of the Hudson's Bay Company in the various cities of Western Canada, and welcome this opportunity of acknowledging their valuable help and courtesy.

Let me briefly sketch the journey.

From Quebec, the most French of all Canadian cities, to Montreal and Ottawa the country is mainly agricultural, interspersed with extensive forests and possessing unsurpassed water powers of the utmost value for manufacturing and lighting purposes. Beyond Ottawa we travel for two weary days and nights—over twelve hundred miles—through the most desolate

and lonely country. There is one unending succession of barren rock, scrub and forest, with scarcely a sign of cultivation all the way, while the forest fires leave behind them long stretches of fallen and half-burnt trunks which add to the depression of the scene. This great stretch of country seems unlikely to be capable of much development, except possibly as regards minerals.

Just before we reach Winnipeg the scene changes, and from this point to the Rocky Mountains, nearly a thousand miles west, we pass through the rolling prairie country which has now become one of the largest and most famous wheat areas in the world. Coal, too, is found here to some extent, and at Medicine Hat and in its neighbourhood the natural gas forms a valuable and cheap source of power for manufacturing industries.

I need not dwell on the glorious scenery of the Rockies, or on the engineering triumphs involved in the construction of a railway across those mighty mountains, but there is one dramatic moment when the train, emerging from the last gorge on the Pacific side, plunges suddenly into a wide and peaceful valley, and in a moment we find ourselves in a fertile and agricultural district.

Thence to Vancouver we travel through a smiling country, with mountains, forests, rivers, and cultivated lands admirably adapted for fruit growing and mixed farming. The terminus of the Canadian Pacific Railway is at Vancouver, a beautiful city and destined to be one of the great ports of the world. Finally the steamer leads to Victoria, the most English-looking city of the entire Dominion. A great future lies before Vancouver Island, rich in timber, coal and minerals, but as yet its resources are almost undeveloped. How much has still to be done may be gathered from the fact that with an area about half that of Ireland it has a population considerably under 100,000, while the railways operate over a distance of 200 miles only.

It is difficult to convey any adequate idea of the vast size of the Dominion, or of the great distances which separate the chief towns from one another. But Quebec is as far from Vancouver as from London, and the journey by the quickest train takes five full days of continuous travel. Even such places as Calgary and Edmonton, the two leading cities of the

Province of Alberta, lie 200 miles apart—the distance which separates London from Liverpool; while Alberta itself, although by no means the largest Canadian province, covers an area four times as great as that of England and Wales.

THE FINANCIAL POSITION AT THE OUTBREAK OF WAR.

It should be borne in mind that my visit was made at a time when many circumstances combined to render the financial position of Canada extremely unsatisfactory. The gradually increasing land boom and Real Estate speculation, which culminated in 1912, were followed by a great collapse, the effect of which soon showed itself upon all the business interests of the Dominion. In particular, the absence of dealings in Real Estate caused a cessation of building operations in the prairie provinces, and this in its turn led to severe depression in the lumber trade of the West, so that wide-spread unemployment immediately followed.

An extremely interesting speech dealing with the conditions in Canada a few months before the outbreak of war was made by the Hon. Sidney Peel, President of the Trust and Loan Company of Canada, at their Annual General Meeting held in London last May. After mentioning that there was “a distinct check to the rising tide of Canadian prosperity” brought about primarily by the stringency of the financial conditions in all the world centres, which had decreased the demand for labour, and lessened the stream of migration to Canada, Mr. Peel said :—

“This restriction of development . . . has killed
 “the speculation in land which had run to such excessive
 “lengths, especially in building lands. The collapse of
 “this land boom has had the usual effects. Many people
 “find themselves landed with real estate at prices which
 “they can never recover and with instalments to pay
 “which they undertook with a light heart in the hope of
 “being able to pass their purchase on to someone even
 “more speculative than themselves.”

He went on to point out that the essential factor in Canadian prosperity is the farming industry, and that owing to the excellence of last year's harvest (*i.e.*, 1913), the farmer was never

in a more prosperous condition, although individuals had been hit through speculation. He concluded his speech as follows:—

“ I do not think we have much danger to apprehend
 “ from the altered conditions in Canada. Let me here
 “ interpose that the chairman of a mortgage company
 “ is always necessarily a pessimistic person. The collapse
 “ of a boom brings difficulties, but times of stringency
 “ are, as I have pointed out before now, by no means
 “ unfavourable to mortgage companies. They can choose
 “ their investments and make them at good rates and on
 “ reduced valuations.”

At the beginning of the summer it was thought that the worst stage had passed and that improvement was setting in, but all hope of this was destroyed by the war, and since then the depression has gradually increased. Under the circumstances it is inevitable that this paper must present a somewhat gloomy picture of the conditions existing to-day.

CANADIAN MORTGAGES.

Although many offices still confine their mortgages to the United Kingdom, this practice is now far from universal and many leading companies have long since extended their range of investment to cover mortgages on Colonial and Foreign property. I have extracted the exact figures for each office from the latest Blue Book containing the returns relating to Life Assurance Companies, and find that the total sum now lent on Mortgages within the United Kingdom amounts to £69,400,000, and on Mortgages out of the United Kingdom to £18,600,000, so that the latter figure represents no less than 21 per cent. of the total amount. Formerly Australia was the chief field for this class of investment, but Canada has now taken a leading place.

The Canadian Government Returns of 1913 show that British Offices transacting business in that country have invested over $3\frac{1}{2}$ millions of their Life Funds in Canadian mortgages, and in addition 3 millions have been so invested on the Fire account. Further large sums have been invested also in such mortgages by British Offices which do not transact Life business in Canada, and whose advances are consequently not included in the above Returns.

The chief attraction offered by these investments is undoubtedly the high rate of interest—6 per-cent to 8 per-cent

—which they yield. The main disadvantage is the difficulty of securing adequate supervision and control, but the amount of the sums now thus invested makes it clear that this difficulty can be successfully overcome.

Systems of Land Tenure.

It is necessary at the outset to call special attention to a fact of fundamental importance which governs all that follows and which causes innumerable difficulties and complications to the intending investor. Canada is not one country: it is a series of independent provinces, each with its own legislature, its own statutes, and its own machinery for carrying out its statutes. Thus the procedure for effecting a mortgage in the Province of Quebec differs widely from that employed in Manitoba, and this, in its turn, varies slightly from that in British Columbia. The whole of the land of Canada is divided into lots, numbered and lettered for exact identification, and on transfer by sale or mortgage the registered number forms the foundation of the transaction. The land is almost entirely freehold, even in the large towns.

Broadly speaking, Canadian land tenure divides itself into two main systems, the registration system adopted in Quebec and in the old settled provinces of the East: and the new "Torrens" system employed almost universally in the prairie provinces and in British Columbia.

Under the Quebec registration system, an official known as a Notary Public is appointed, to whom is entrusted the custody of all deeds relating to landed property. When a mortgage is required, the solicitor for the lender applies to the Notary Public, and is, for a small fee, furnished with an abstract of title, containing an epitome of all previous deeds affecting this particular "Lot." He may, if he think it necessary, check the abstract by inspecting the deeds themselves at the office of the Notary Public, or he may obtain copies of the deeds, but the actual title deeds never leave the custody of the Notary Public, and can, therefore, never be lost. If the solicitor be satisfied that the borrower has a good title he prepares the mortgage deed on a standard printed form, and this is then executed by both parties in the presence of the Notary Public. The mortgage deed is retained and filed, the Notary Public furnishing the lender with a certified copy, and this copy document is his evidence of title. It will be seen

that, apart from the question of the custody of the title deeds, the procedure is much the same as in this country. The Notary Public takes no responsibility; he merely produces deeds and furnishes official abstracts or copies: the solicitor for the lender has to satisfy himself that the chain of title is complete and that the advance can safely be made.

Under the "Torrens" system, on the other hand, the process is delightfully simple, and landed property changes hands exactly in the same way as stocks and shares. The Land Titles Office registers every fresh transaction, furnishes a certificate which is an *Absolute Guarantee of Title*, and cancels the previous certificate. Thus, if A wishes to sell a piece of land to B, all he has to do is to produce his certificate of ownership and to execute a simple printed deed in favour of B. The two documents are then handed in to the Land Titles Office, and in due course B receives his official certificate of ownership. There is no investigation of past title and the chain of previous owners is immaterial. The whole procedure is exactly that employed in England for Stock Exchange securities, and in addition the Province guarantees that the title is good. In the case of a mortgage, the certificate states that the land is owned by the borrower, but is charged in favour of the lender. The certificate is retained by the lender and is evidence of his title.

It will thus be seen that in Central and Western Canada a practical and simple method of tenure has been devised which permits the transfer of land at a minimum of cost and with a maximum of security, for when once the Land Titles Office has granted its certificate, the title of the owner becomes absolute and indisputable. In the Eastern provinces the old French traditions still largely govern the law, and the transaction is nearly as complicated as at home. From one point of view this is a little unfortunate, for a British Life Office which had decided to lend money on mortgage in Canada would probably prefer to begin by advancing upon the highest class of city property in the settled Eastern provinces, and more especially in Montreal and Toronto.

On Licensing and Registration.

Having thus sketched the two main systems of land transfer in Canada, I must deal next with the formalities with which a foreign lender has to comply before he can safely make advances

or obtain the protection of the Courts to secure his legal remedies in the event of default.

A British Life Company would be regarded in Canadian law as a "Foreign Corporation", and as such would require a licence authorising the transaction of mortgage business. In all probability a separate licence would be required for every province, involving, in each case, an initial payment of £30 to £40 and certain small annual registration fees. I interviewed the Chief Insurance Commissioner at Ottawa upon the question of obtaining a Dominion licence, but gathered from him that this would probably be ineffective, as the individual provinces collect their own taxation and would certainly exact the provincial licensing fees, to which I have already referred, before they would allow loan business to be transacted. The Commissioner suggested a special Act of Parliament as a way out of the difficulty, but this would be expensive and quite uncalled for. The procedure an office would adopt would be to take out a licence for any particular province as and when required; and in order to minimize expense it would probably concentrate its loans at the outset into one or two provinces.

And this question of licensing leads me to the heart of the greatest difficulty that has to be faced if an office decide to lend money in the Province of Quebec. Who is to represent the Company on the spot? As I have already explained, lender and borrower have to execute the mortgage deed in the presence of the Notary Public. So that there must be a local representative with power to sign in the name of the Company. I shall return to this point later on when dealing with methods of supervision.

This particular difficulty does not arise in the provinces governed by the "Torrens" system. There, as in England, the mortgagor alone need execute the mortgage and the certificate will recite the Company as mortgagee. On repayment, the receipt can be signed in London under the usual procedure, and I am advised that, provided the Land Titles Office be satisfied as to the repayment, they will grant the owner a new unencumbered certificate without paying much attention to the exact form of the discharge.

Thus it will be seen that both from a legal and practical standpoint it is much more easy to lend money in Winnipeg or Vancouver than in Montreal.

Types of Loans on Landed Property.

Canadian Loans on landed property fall into three main groups, secured respectively upon (a) Central City Property, (b) Residential City Property, and (c) Farm Lands.

In the course of my travels I have met enthusiastic advocates of each class, and I will mention briefly their respective advantages and disadvantages. Loans are generally granted for a fixed term of five years and are often repayable by instalments.

(a) *Central City Property.*—These loans are similar to many of our home mortgages and thus instinctively appeal to us more than the other types. While the basis of such loans is usually 50 per-cent of the Capital Valuation, it is possible to secure stringent valuers (they are called “Valuators”), and to obtain some check over their figures by means of the rentals received. The yield is 6 per-cent and upwards under normal conditions. In Montreal I had the opportunity of seeing the various properties on which advances had been made by one of our leading British Offices, and in every case interest and principal have been punctually paid. These loans are undoubtedly good, and with the local knowledge of their shrewd Canadian manager, and their capable local Board, they should be able to steer clear of difficulties and to maintain equal success in the future. But for success in such loans it is essential to have full local knowledge, a cautious “Valuator” and fairly settled permanent conditions such as are found in a relatively old city like Montreal.

On the other hand, such loans in the West are speculative in the extreme and valuations made up to a year or two ago have been based upon an inflated standard which has to-day utterly collapsed and has not yet reached any new level of stability. For example, in Vancouver City the outbreak of war following on the collapse of a land boom has so depressed matters that the Real Estate market has to-day practically ceased to exist. The fall in capital value from its previous inflated standard is difficult to measure, for there are no buyers, but the fall in rental values throughout the City is heavy and continuous. Until a readjustment of values has established itself, loans on even the best class of Vancouver City property should in my opinion be absolutely avoided. And this criticism applies generally to the cities of the prairie provinces, which have been largely overbuilt in anticipation of the continuation of a rapid growth of popula-

tion, whereas at the moment there is an exodus from the towns and the population is dwindling rather than increasing.

(b) *Residential City Property.*—The leasehold system is almost unknown in Canada, and the universal ambition of every citizen is to become the owner of the house he occupies. The general practice in buying a house is to arrange that part of the purchase money should remain on mortgage, and this especially applies amongst the well-to-do classes who find that they can employ their money more profitably in other ways. Even wealthy people are, therefore, quite content to let a mortgage on their house remain indefinitely, and such loans are regarded by many Canadian authorities as affording security of the highest class. From an English standpoint, mortgages secured upon individual houses would not be regarded as very attractive or suitable for the funds of an Assurance Office; but in the wealthy suburbs of the more important Canadian cities, where the population has hitherto been rapidly increasing, and where the demand for such houses has so far been greater than the supply, it seems reasonable to regard such residential property as likely to continue to appreciate steadily in value, and thus to form a satisfactory field for investment. The rate of interest yielded on this type of security varies from 6 per-cent to 7 per-cent under normal conditions at Montreal, and is a little higher at Winnipeg and Vancouver.

(c) *Farm Lands.*—This type of loan is largely granted in the prairie provinces, and, although very foreign to our preconceived ideas at home, has much to recommend it. It is strongly advocated by such an eminent financial authority as Mr. A. M. Nanton, a Director of the Canadian Pacific Railway and of the Hudson's Bay Company.

His firm for many years has invested very large sums upon the security of farm lands on behalf of British Offices and of important English and Scottish clients; and Mr. Nanton assures me that they have never lost a penny of capital in the process. Interest on such loans is often not punctually paid in years when the harvest is bad, and in that event it is customary to "carry the farmer," as it is called, until the ensuing harvest period. The chief advantage of this type of security is that there is not the same danger of inflated values as is met with in town plots, and, further, that as the loan is used to improve the land, the security for the mortgage is thereby automatically increased. Mr. Nanton was good enough to show me his books

and I gathered that most of the advances are of small amounts, and that, in general, not only is the interest promptly paid, but many of the loans are gradually reduced by repayment of the principal in instalments. But it is clear that local knowledge is essential for success in granting loans of this character, and that the only way in which an office could safely conduct them would be by placing entire confidence in a responsible local agent, and acting implicitly on his advice. Allowing for Agent's commission, the yield on loans of this character would be about 6 per-cent for the first year and 7 per-cent thereafter.

Modes of Investment.

There are two principal methods of transacting loan business in Canada. The most obvious course would be to have a direct mortgage in the name of the Company. But many practical difficulties at once present themselves. For example, as I have already mentioned, in the Province of Quebec the lenders must appear in person before a Notary Public, and must in his presence execute the mortgage deed, so that a Company must have a local representative to act on their behalf, or must grant a Power of Attorney to some one on the spot. I feel convinced that if a British Company decide to undertake mortgage business in Canada, practical considerations will enforce the appointment of local representatives into whose hands the entire machinery for dealing with mortgages must be placed. It would be simple to arrange within what limits such an Advisory Committee should have power to act, and the local Manager or Chairman could conveniently sign on behalf of the Company whenever a mortgage had to be completed. I may note, in passing, that in this respect the position of an Incorporated Company is comparatively simple as compared with that of an Office whose securities are held in the names of trustees. Canadian law is very adverse to any recognition of trusteeships, and a leading counsel of Montreal with whom I discussed this problem recognized the great difficulties that an office which is not incorporated would have in complying with this necessary formality, and still more in satisfying the Notary Public as to the authenticity of the discharge, and as to the authority of the person executing it when the time came to pay off the mortgage.

The other investment method is to act through the medium of a Trust Company. These Companies fulfil a very special

function in Canada seeing that the Banks are not allowed to make advances upon land. The Banks get over the difficulty by forming a subsidiary Trust Company. Thus the Royal Trust Company is to all intents and purposes the Bank of Montreal. A Trust Company merely acts as an agent for its clients. It undertakes the investment of money, investigates titles, arranges valuations and carries through all the preliminaries. It does not undertake liability for the loans in any way ; it merely uses its machinery in the interests of its clients and does its best for them. It is remunerated by a commission on the loan, and by an annual payment for collection of interest and for general supervision.

Undoubtedly the machinery of such a Company as the Royal Trust is at least as effective as that of any local Board—probably much more effective. The disadvantage of the system is that the mortgages must all be granted in the name of the Trust Company. In fact the name of the lender would not appear publicly at all. The borrower would deal only with the Trust Company, and while the Trust Company would be responsible they would have full powers to act without consulting the wishes of the lender. They would furnish a certificate, hypothecating to the lenders the various mortgages in which their money was invested, and that would be their sole security and their sole evidence of title. It will thus be seen how entirely the lending Company would depend upon the good faith of the Trust Company, and although such Companies of the highest class seem to be regarded much in the same way as we regard our "Public Trustee", I feel that most Boards of Directors would not be likely to take the responsibility of investing money in any other name than that of their own Company.

Provincial Income Tax.

An Income Tax is levied upon interest in parts of Canada. The rate varies in the different provinces, but at present is small in amount and rarely exceeds one-half per-cent, say 1*d.* in the £.

A licence to conduct loan business in Manitoba and certain other provinces involves, however, a curious and very unexpected feature. Under the Statutes, it is necessary to make an annual return of all interest received in the province. This return must include, not only mortgage interest, but interest received on all municipal bonds and on other securities of every

kind in the province, and taxation is then levied upon the whole.

Thus if a Company which already possessed large holdings of municipal stock of various cities in the Province of Manitoba were to grant a single mortgage in Winnipeg, it would have to disclose the fact of its municipal holdings, and income tax would be levied upon them, although at present they entirely escape Canadian taxation. In this way a penalty is exacted for transacting loan business in the Province, which seems not only unfair in principle, but which may be of serious importance should Canadian war taxation involve, as is possible, a large increase in the present rate of Provincial Income Tax.

Under Section 5 of the Finance Act of 1914, any taxation paid in this way would be deducted from the income liable to taxation in England, but of course the relief on this account would be very small in comparison with the loss of the amount deducted in Canada.

The General Investment Problem.

I have shown in detail that there are many practical difficulties in the way of undertaking mortgage business in Canada ; and that unexpected and unnecessary obstacles confront the would-be lender. The essential points to be borne in mind are :— (1) The danger of inflated valuations, especially in the Western cities ; and (2) the need of adequate local supervision. For the moment the second point is the more important, for in the present depression of the Real Estate market, values fixed a few years ago are crumbling and will continue to fall until they reach a natural level.

I regard the appointment of a local Board or of a Representative in whose judgment complete reliance can be placed, and must be placed, as the only means by which mortgages of a safe and high-class character can be secured in the Canadian cities. And this, in my judgment, is a permanent condition, for, as I was so often reminded during my tour, you cannot exercise much supervision in selecting the right or in avoiding the wrong mortgages if you are 3,000 or 5,000 miles away.

CANADIAN INDEBTEDNESS.

The growth of Canadian indebtedness during recent years has aroused much interest mingled with some anxiety, and a few general notes on the subject may therefore not be inopportune.

Before touching on financial questions, I should like to say how greatly I was impressed during my recent visit with the efforts which the Dominion is making for the Empire. Everywhere the feeling was the same; from East to West men were enlisting in their thousands, and provinces and cities were vying with each other in gifts of money and national produce. I would ask, therefore, that the sections of this Paper which deal with Canadian indebtedness should not be regarded as being merely coldly critical, but rather as having been written from a sympathetic standpoint, and with the feeling that it is to the mutual advantage of both Canada and England that the facts about the present financial position should be frankly recognized.

It is far from easy to ascertain the full extent of Canadian borrowings, for apart from the large loans which have been floated on the London Market amounting now to five hundred and fifteen millions sterling, it is estimated by Sir George Paish that upwards of one hundred millions sterling of American capital has been invested in the country, and that not far short of another one hundred millions has been lent, much of it privately on mortgages of real estate and by way of loans to farmers. The total amount of foreign capital supplied to Canada thus amounted at the end of last year (1913) to nearly seven hundred millions sterling, carrying with it an interest burden of about thirty-two millions per annum. It is startling to find that one half of this great sum has been lent within the past seven years. Reckoning dollars at five to the £, these loans may, roughly speaking, be analysed as follows:—

(a) Dominion Government (net)	£70,000,000
(b) Provincial Governments ...	26,000,000
(c) Municipalities ... (say)	70,000,000
(d) Railways ...	300,000,000
(e) Industrial Companies and Real Estate Mortgages ... (say)	234,000,000
Total ...	£700,000,000

As the population is under 8,000,000—little more than that of London—the gross debt is thus seen to be nearly £90 per head, or, putting the same facts in another way, every Canadian family of five persons has to pay £20 annually to England or the United States for interest on borrowed money.

It is not, however, sufficient to regard merely the amount of

the debt ; we must also take into account the assets which it represents, and endeavour to trace whether the heavy borrowings necessitated by the growth and development of a new country have been justified. That there has been extravagance in some quarters is undoubted : and certain industrial and other loans which have involved the lenders in heavy loss have been open to severe criticism.

But taking a broad view, there can, I think, be no question that the bulk of the immense amount of capital poured into Canada in recent years has been wisely expended. Much of the borrowings by the Dominion Government has been spent upon remunerative enterprise and public works represented by tangible assets. This applies also to the greater part of the debt incurred by the Provinces. The portion of the Dominion and Provincial Government expenditure most open to criticism is that made in respect of certain railways where there seems little prospect of an adequate return on the capital expended. There will be much more divergence of opinion in this country as to the wisdom of much of the municipal expenditure, and in a subsequent section of this paper I deal with this subject in more detail.

The expenditure upon the railways has had the most far-reaching effects. As Sir George Paish writes in an interesting article published in the *Statist* of 3 January 1914 :—

“ In proportion to population no country has ever
 “ built railways as rapidly as Canada has done in recent
 “ years. The effect of this railway construction on the
 “ prosperity of Canada has already been amazing. The
 “ new railways have brought into existence a great many
 “ new towns, have caused the old cities to grow in size,
 “ have wonderfully stimulated the growth of population.
 “ The great extension of the railway system has
 “ caused vast areas of new land to be placed under
 “ cultivation and has opened up new mineral districts.
 “ Moreover, industries have sprung up in many
 “ directions, and one cannot pass through the great
 “ cities without becoming conscious of the fact that
 “ Canada is not merely a country with unlimited agri-
 “ cultural possibilities, but that it has already become
 “ an important manufacturing and industrial State.”

A notable feature of the past few years has been the increased attention which British capitalists have devoted to industrial development in the Dominion. Large sums have also been

lent to farmers in the prairie provinces, and this expenditure should prove remunerative to the lenders as well as advantageous to Canada.

The question as to whether Canada has overborrowed is not an easy one to answer. Judged from the standpoint of her enormous natural resources and the possibilities which lie ahead, the expenditure up to date has produced such striking results in the growth of the productive power of the country that the employment of further capital may be expected to bring about even greater financial and commercial developments in days to come. Against this argument, however, must be set the fact that easy borrowing leads inevitably to extravagant expenditure ; and, long before the outbreak of war, many of the highest financial authorities had recognized that the interests of Canada would be better served by checking rather than by forcing the pace in the near future. Canadians, it has been said, have been taught to think only of what may be obtained by the expenditure of money, and to think not at all of the day of paying back. I have heard it urged, too, in Western Canada, that the immediate need is for more scientific farming of the land already under cultivation, rather than for further loans to extend the present wheat area. Farmers, not mere "land miners," are needed, and consolidation rather than expansion.

The next few months will go far to settle the wisdom or lack of wisdom of the way in which the money has been expended. The settled provinces of the East will face the storm with fair confidence, but, unless the Provinces or the Dominion come to the rescue of the newer cities of the West, it is difficult to see how the latter can meet their current obligations in the immediate future.

From this brief general survey I now pass on to a more detailed analysis of the expenditure under its various headings.

Dominion Debt.

From the figures presented by the Hon. W. T. White, the Minister of Finance, it would appear that the Dominion debt has shown very little growth during the past few years, and that against the one hundred millions of liabilities there is a set off of over thirty millions of assets consisting of Sinking Fund, Specie Reserve and miscellaneous investments, so that the net debt is really under seventy millions sterling.

Most of this debt has been incurred for developing and

opening up the country, and in view of the enormous size of the Dominion its amount may certainly be considered at the present time as not only legitimate but moderate.

The Dominion Government supplies large subsidies for capital outlays upon the larger public works and national undertakings, a policy which Mr. White justified in his Budget speech last April as follows :—

“ It must be obvious to all that the taxpayer of the day is not to be expected to defray each year the entire capital cost of works such as railways, canals, harbour improvements, dry docks, and other expensive undertakings which once constructed become national assets of great productive value to the Dominion for generations to come. The propriety of constructing these works, partially at least, with funds raised upon the credit of the country does not appear to me to be open to challenge or controversy.”

He then continued :—

“ Last year it was considered advisable to render material assistance to railways in order that their work of construction might not be interrupted by reason of the prevailing adverse monetary conditions and the consequent stoppage of funds which previously had been obtainable in the London money market by the railway companies themselves through sales of securities.”

His estimate of the capital expenditure for the year then ending (31 March 1914) in respect of railway subsidies and other special accounts amounted to 11½ millions sterling, of which about seven millions was expected to be found out of surplus revenue, so that the contemplated addition to the debt was only four millions.

A good deal of criticism has been raised as to the policy of the Dominion Government in undertaking the construction of railways which offer little prospect of becoming revenue producers. The Intercolonial Railway, extending from the ocean ports of Nova Scotia and New Brunswick to Montreal, is owned and operated by the Dominion Government. This line does not pay its working expenses, but its construction was necessary in order to provide railway facilities to Halifax and other ports which are free from ice in the winter season. The Eastern portion of the National Trans-continental Line from Moncton (New Brunswick) to Winnipeg, and the Hudson Bay line, both

of which are now under construction with Government money, seem to have much less commercial justification, and this applies also to the large cash subsidies granted to the companies embraced in the Canadian Northern system. In addition to this actual expenditure, the Dominion Government has guaranteed the principal and interest on certain Canadian Northern and Grand Trunk Pacific railway securities to the extent of nearly twenty millions sterling.

As the Dominion Government does not levy direct taxation, but obtains nearly the whole of its revenue from Customs and Excise, the falling off in Imports arising from the outbreak of hostilities will cause a serious loss of income, while concurrently the expenditure for war purposes will be very heavy. So far, ten millions sterling has been voted on this account, but a much larger sum will certainly be needed. It seems inevitable, therefore, that even if some of the expenditure on new public works can be postponed the near future must show a large increase in the Dominion debt.

Provincial Debt.

The Provinces in Canada are independent countries so far as their internal government is concerned.

They, like the Dominion, practically levy no direct taxation and their chief sources of revenue are the Dominion subsidy, the proceeds of the Sales of Land, Timber and Mining royalties, Succession Duties, Fees and Licenses. The chief items of expenditure are Public Works (Roads, Bridges and Ferries), Legislation, Administration of Justice, provision of Hospitals and Asylums, and large subsidies for Education. In most of the Provinces expenditure slightly exceeds income.

A valuable volume showing in detail the Canadian Provincial and Municipal indebtedness as at the end of last year has recently been published by Messrs. Wood, Gundy & Co., of Toronto and London. In almost all cases the figures were supplied by the municipal officials, and they may, therefore, be taken as reliable. This work is especially interesting because for the first time information is now available of the total amount of the liabilities incurred, whereas all previous statistics have dealt only with the loans floated on the London market.

The total Provincial funded debt in December 1913 was twenty-six millions sterling, of which eighteen millions had been raised in London. Two years previously the London debt was fourteen

millions, and in 1902 under ten millions. The increase in the indebtedness is spread over the whole country, though it is most marked in the newer provinces of the West. The eight millions raised outside London have gone chiefly to the Provinces of Ontario and Alberta.

In addition to these direct obligations, the Provinces have guaranteed the principal and interest of thirty-six millions of railway securities, chiefly to facilitate the construction of branch lines of the Canadian Northern and Grand Trunk Pacific. This policy has only been adopted during quite recent years, but it has now become an important and popular form of aid. Time alone will show whether it is a wise development, but in the meanwhile this contingent liability cannot be ignored in considering the finances of the Provinces. Nor should it be overlooked that while the expenditure may rapidly increase, the present sources of income do not seem equally capable of expansion.

The Provinces have expended money lavishly in the erection of palatial parliamentary buildings, and the public offices generally are built on an extravagant scale. For example, at Regina and Edmonton, the capitals of the new Provinces of Saskatchewan and Alberta, the imposing parliamentary buildings just erected would excite admiration in London itself, and Winnipeg, not to be outdone, is replacing its old building by one of increased magnificence. The rivalry between the Provinces and the universal love of display have undoubtedly led to extravagance in many directions.

Municipal Debt.

The municipal indebtedness of Canada, according to Messrs. Wood, Gundy & Co.'s statistics, had in December 1913 reached the total of over 102 millions sterling. Twelve millions of this is termed "Local Improvement" debt and is charged primarily upon the owners of the property benefited by the expenditure, though, of course, in the last resort the municipality would be liable. Of the 102 millions, only 40 millions has been publicly raised in the London market, and I think that it will come as an unpleasant surprise to investors in this country to find that this huge additional liability exists.

The following table gives a complete list of the Canadian cities whose loans are quoted in the London Stock Exchange List. In Column (1) is given the amount of the loans raised

in London. in Column (2) the loans raised elsewhere (chiefly in Canada and the United States of America), and in Column (3) the total funded debt. The figures have been arranged in the order of the indebtedness of the cities.

Municipal indebtedness, as at December 1913, of Canadian cities which have raised loans in London.

City			Loans quoted on London Stock Exchange	Loans raised elsewhere (chiefly in Canada and the U.S.A.)	Total funded debt
			(1)	(2)	(3)
			£	£	£
Montreal	9,215,000	5,649,000	14,864,000
Toronto	4,286,000	7,165,000	11,451,000
Winnipeg	6,223,000	1,791,000	8,017,000
Vancouver	4,621,000	1,014,000	5,635,000
Edmonton	3,777,000	1,704,000	5,481,000
Calgary	2,016,000	2,200,000	4,246,000
Victoria	1,107,000	2,022,000	3,129,000
Quebec	1,617,000	768,000	2,385,000
Ottawa	443,000	1,760,000	2,203,000
Regina	1,080,000	827,000	1,907,000
Hamilton	687,000	1,136,000	1,823,000
Saskatoon	818,000	969,000	1,787,000
Maisonneuve	419,000	997,000	1,416,000
Port Arthur	639,000	748,000	1,387,000
Fort William	198,000	1,022,000	1,220,000
Moose Jaw	448,000	735,000	1,183,000
New Westminster	478,000	683,000	1,161,000
Point Grey	241,000	909,000	1,150,000
South Vancouver	310,000	779,000	1,089,000
St. John...	308,000	519,000	827,000
Lethbridge	142,000	618,000	760,000
Westmount	109,000	656,000	766,000
Prince Albert	103,000	588,000	691,000
Burnaby...	118,000	483,000	601,000
North Vancouver	257,000	271,000	528,000
Sherbrooke	103,000	239,000	342,000
Moncton...	52,000	189,000	241,000
...			39,836,000	36,444,000	76,280,000

The total of the London loans two years previously, *i.e.*, at the end of 1911, was 22½ millions, while in 1906 and in 1901 it was under nine millions. It is very significant to notice that during the five years 1902–1906 inclusive, the amount of the indebtedness was virtually stationary so far as the

London market was concerned. The explanation seems to be that such loans as were required in that period were absorbed by Canada herself. The era of expansion of municipal borrowing in London began about 1907 and, as will be observed, developed with extraordinary rapidity during the years 1912 and 1913.

The 36 millions which have been raised by these cities either publicly or privately outside London were issued chiefly in Canada or in the United States, and while the latter country is a large permanent holder of these bonds, there is no doubt that a considerable proportion of the remainder ultimately found purchasers in England.

The remainder of Canada's municipal debt, amounting to about 26 millions, is spread over 338 cities, towns and villages, embracing practically every place with a population of 1,000 and upwards.

The following table shows the indebtedness of the more important cities which have raised their loans without application to the London market.

London (Ontario)	£1,169,000
Halifax (Nova Scotia)	1,012,000
Medicine Hat (Alberta)	734,000
Brandon (Manitoba)	668,000
St. Boniface (Manitoba)	659,000
Prince Rupert (British Columbia)	488,000
Brantford (Ontario)	481,000
St. Catharines (Ontario)	480,000
Guelph (Ontario)	405,000
Outremont (Quebec)	370,000
Peterborough (Ontario)	354,000
Three Rivers (Quebec)	353,000
Berlin (Ontario)	349,000
	<hr/>
	£7,522,000
325 small municipalities, aggregate loans about	18,000,000
	<hr/>
Total	£25,522,000

In addition to the funded debt, Treasury Bills are issued by the cities to cover current liabilities. I have no means of ascertaining how much is now outstanding on this account; but as Sir F. Williams Taylor stated that in 1912 they amounted to nearly five millions in respect of the cities which had issued loans in London, the total additional liability to-day may reach a considerable amount.

Much of this expenditure of a hundred millions has been incurred for remunerative enterprises, for, whether rightly or wrongly, municipal ownership of all "Public Utilities" is regarded, especially in the West, as being advantageous to the community. Many new towns have, therefore, been provided with municipal water, electric light, electric tramcars, telephones, &c., indeed with all the comforts enjoyed by cities of much older foundation. Up to a certain point this policy may be justified, but in many instances it has been carried far beyond the limits of prudence.

The municipalities impose direct taxation by means of what is known as the Corporation Tax and out of this all payments for police, sewage, lighting, improvements and general administrative expenses are met. This tax is based upon the assessed capital value of all land and buildings within the areas under the municipal control. A special tax, based upon the same principle, is levied for education. At Montreal the Corporation Tax is 1 per-cent, and the Education Tax four-tenths or five-tenths per-cent of the assessed values. This corresponds roughly to an English rate of about 6s. in the £. In many places, especially in the West, the taxation is far higher. For example, at Lethbridge it is 3·3 per-cent, or over 13s. in the £. And in reality the scale of taxation is often higher than it looks, for the levy is made upon an inflated assessment. Generally speaking, Canadian municipalities are allowed to borrow up to something like 20 per-cent of the assessed capital value. It will thus be seen that the temptation to raise the assessed value in order to increase their borrowing powers, is very great; and this, no doubt, has been a powerful incentive in bringing about the inflation to which reference has already been made.

In view of the large holdings of Canadian municipal bonds by British Assurance Companies, I made special enquiries to try to learn how this type of investment was regarded locally, more especially in the West where the money has been spent so lavishly.

Everywhere I was met with the question, "Why does England run after these municipal loans when mortgages on the land paying a higher rate of interest are available?"

The average Canadian, who knew that there was no adequate supervision over the issue and expenditure of such loans, has not of late been attracted by the security offered or the rate of

interest, and has willingly left them to the English and American market. Indeed I found that there was a consensus of opinion that in the last few years money has been lent far too freely by England, and on far too cheap terms, especially to the small new cities. In England, of course, all such loans have to receive the sanction of the Local Government Board before they can be issued and the security is thus held in high repute by the public. Mr. John Coles, F.I.A., was so struck by the pressing need of some similar central control in Canada that during his visit last year he took up the question seriously; and mainly on his initiative a discussion was started in the Press which has already had important results. In the Province of Saskatchewan a Supervising Committee has just been appointed and it is expected that other provinces will follow the lead so given.

On several occasions I pointed out that the flotation of municipal loans, each at a higher rate of interest than its predecessor, naturally causes a continuous fall in the value of the older issues, but I had great difficulty in convincing the Canadian that the matter was of much moment. He invariably replied that, as all the loans were repayable at par at some fixed date, and there was no necessity to sell, current quotations need not be considered of the first importance. The argument that English Life Offices were accustomed to regard the market price as the basis for Balance Sheet purposes, and that all such falls in value involved the constant writing off of large sums, did not seem to appeal to him. Possibly the American habit of regarding the subject from the amortisation point of view has some effect in determining the attitude of the Canadian business man in this matter.

As regards the way in which loans have been expended, I found that it was difficult to secure any admission of extravagance. Every Canadian spends freely and wishes his city to have finer public buildings than its neighbour. He recognises that much of the work is for the benefit of the future population and is content that the next generation should pay. So streets are laid out, sewers and tramways constructed and lighting facilities provided, far in excess of the needs of the present population and on the assumption that the numbers will increase in the future as rapidly as in the immediate past. As long as all went well and fresh money could be obtained easily this course continued unchecked.

The rapid growth, however, of Canadian municipal indebted-

ness during recent years has excited a good deal of uneasiness in this country, and many months before the outbreak of war the terms on which such loans were taken up by the London market had grown so onerous that a slackening in the stream of new issues had become inevitable. The war, coming without any warning, has suddenly caused a complete stoppage in the importation of money and the effect upon the municipalities has been immediate and severe. They were caught unprepared and works of "Public Utilities" of every description are either suspended or are being completed by means of short-dated loans at ruinously high rates of interest. Unemployment in the towns is already so rife that every effort is being made by the municipalities to continue whatever work is in progress, but there can be no doubt that some of the more heavily indebted towns have an anxious winter before them. Municipal income is now falling off owing to difficulty in collecting the taxes, and for the first time expenditure has to be paid for out of income, since fresh loans are no longer available. So far there has been no default but the full effect upon Canadian municipal finance of the absence of new loans has yet to be seen.

It is interesting to note that, contrary to general impression, the population of Canada tends to gravitate more and more to the large towns. Thus, in 1871, one in thirty-two of the population of Canada resided in Montreal: in 1891 the proportion had risen to one in twenty-two, in 1901 to one in twenty, and in 1911 to one in fifteen. Toronto shows a similar tendency. As regards the towns in the West, the majority did not exist thirty years ago: many of them, indeed, not twenty years ago; and their phenomenal growth has unfortunately diverted settlers from the land.

I append a few details of certain cities where I can speak of the condition of affairs from personal enquiries.

Victoria (Capital of British Columbia).—Population, 50,000. Gross debt per head, £63. A flourishing port and a residential city with English characteristics. It is likely to gain from the opening of the Panama Canal and the general development of the mineral wealth of Vancouver Island, but at present shows signs of the general stagnation of trade. A large sum is being expended in improving the harbour.

Vancouver City (British Columbia).—Population, 122,000. Gross debt per head, £46. One of the world's natural ports, and bound to have a great future. Terminal of the Canadian

Pacific Railway. Chief industries are shipping, lumber and salmon (canned). At present suffering from extreme land depression. Building is at a standstill. Many offices are unlet and the huge new blocks secure tenants at the expense of the older buildings. Rents are steadily falling. Much unemployment, due to the collapse of lumber and shipping industries, all the large vessels having been requisitioned for the war. Local rates are very heavy and are likely to rise further. The assessed values, on which rating is based, are inflated and are much in excess of market values to-day. The prospects for the coming winter are serious, and the local view is that some years will elapse before a return of prosperity.

North Vancouver is a dreary and scattered town of 8,000 inhabitants on the slopes across the harbour and connected with Vancouver by a steam ferry. Docks, &c., are projected, but the work on them was not proceeding at the time of my visit. A Dominion Government subsidy of over £1,000,000 has since been arranged. Many wide streets have been laid out, but the houses are few. The municipal debt is very high (£66 per head) and as the town is unlikely to be incorporated with Vancouver in the near future, the security does not appear attractive.

New Westminster (British Columbia).—Population, 17,000. Gross debt per head, £68. Consists of one long main street, and has good wharves on the Fraser River. There are a few modern blocks, but the town appears to be suffering from the general depression. Formerly the capital of the Province, but now overshadowed by the neighbouring City of Vancouver.

Calgary (Alberta).—Population, 85,000. Gross debt per head, £50. A typical prairie city, whose population in 1901 was only 5,000. It is now a great receiving and forwarding centre for grain and mixed farm produce. There are hardly any manufactures, but large railway works of the C.P.R. are at Ogden a few miles away. The collapse in the land boom has caused much unemployment. Rents have fallen heavily. All building is at a standstill, and there is a desperate need for ready money to complete municipal work already undertaken. The local newspapers in August even advised that the city should accept a loan for a year offered by an American syndicate at $8\frac{1}{2}$ per-cent, rather than incur the heavy capital loss involved in postponing the work. A small loan has since been raised in London at over $5\frac{1}{2}$ per-cent. The coming winter will be a time of great anxiety on account of labour and economic troubles. If new loans

cannot be obtained, it is difficult to see how municipal expenditure can be met out of income.

Edmonton (Capital of Alberta).—Population, 70,000. Gross debt per head, £78, being the highest of any Canadian city. It is the centre of a rich mixed farming district and of a great coal area. Fairly extensive manufactures. In all general respects the position at Edmonton resembles that at Calgary, but the municipal extravagance is even more marked. The tramcar system extends miles beyond the houses and the needs of the present population. The assessed values, which form the basis for local taxation, are much in excess of market values, and are even now being increased. Money has been spent on a most lavish scale and as further loans have been cut off, it has recently been found necessary to pay contractors partly in Bills. The prospects for the winter are very gloomy.

Lethbridge (Alberta).—Population, 13,000. Gross debt per head, £59. Principal industries, coal and farming. The *Monetary Times of Canada* on 9 October stated:—"Borrowing its own sinking fund of \$100,000, the construction work will be carried forward as contemplated. . . The City of Lethbridge has no payments on debentures to meet until 1937, and bonds will be sold later to replace the sinking fund."

Regina (Capital of Saskatchewan).—Population, 45,000. Gross debt per head, £42. A new town of the prairies in the heart of the wheat growing district and one of the chief distributing centres for agricultural machinery. The Canadian Pacific and the Grand Trunk Railways both have stations here, and the town seems likely to increase in importance. There are a few large modern blocks and a palatial group of parliamentary buildings has been erected on what five years ago was virgin prairie. Signs of depression are less marked than at Edmonton or Calgary. One interesting feature of Regina is that a daylight-saving scheme is in successful operation. In the summer months the city time is one hour in advance of the standard railway time.

Winnipeg (Capital of Manitoba).—Population, 185,000. Gross debt per head, £43. A large and busy city, a great railroad centre and the metropolis of the grain trade of the West. There are many blocks of modern buildings and a large residential suburb of fine houses. Office accommodation does not seriously exceed the demand, and rents, though falling, are fairly stable. There is considerable trade activity and less unemployment than

further west. Winnipeg is the Headquarters of the Hudson's Bay Company in Canada.

Fort William and Port Arthur (Ontario).—Combined population, 42,500. Gross debt per head of Fort William, £50, and of Port Arthur, £77. These two cities are close together, and are the chief Canadian ports for the shipping of wheat on the great Lakes. Apart from the grain elevators, these towns do not present much sign of activity and there are few modern buildings. Even in the centre of Port Arthur there is much waste land and primitive wooden houses ("shacks"), so that the general effect is depressing and both towns present a very unfinished appearance. The municipal tramcar system runs for eleven miles, much of it through undeveloped land, far beyond any sign of houses, and it must be many years before the population can need this costly service.

Ottawa (Capital of the Dominion of Canada).—Population, 95,000. Gross debt per head, £23. A prosperous-looking, well-finished city, with fine buildings. There is a large lumber trade, and abundant water power. The current depression is not so marked as in the West. The city recently tried to raise a loan at $4\frac{1}{2}$ per-cent, but did not obtain a single bid locally in spite of wide advertisement in the newspapers.

Toronto (Capital of Ontario).—Population, 425,000. Gross debt per head, £27. A busy manufacturing city with accumulated wealth, stable conditions and an increasing population. There are several modern office "skyscrapers," but the supply of offices does not exceed current demands and rents are maintained. The war has dislocated industry and has caused considerable unemployment, but not to anything like the extent found in the West. The loss of German trade has been a serious blow, and during my visit a firm employing 5,000 hands had to close on this account. In the Province of Ontario the farmers are prosperous and generally own their land free from mortgage incumbrances.

Montreal (Quebec). Population, 550,000. Gross debt per head, £27. The chief city and commercial capital of Canada; a port and a manufacturing centre, with many fine office buildings, and every sign of business activity. Rents and capital values seem well maintained, and, as in Toronto, the commercial depression is not unduly severe. There is a large residential suburb round Mount Royal, and, as the population is steadily increasing, these houses are regarded as an excellent security

for mortgages. The municipal control seems good and not extravagant, but there is chronic friction between the French and the English representatives.

Quebec (Capital of Quebec).—Population, 90,000. Gross debt per head, £27. The oldest city of Canada. French influence is predominant. The population is stationary and, in spite of its importance as a port, the city does not present the signs of the modern spirit so generally prevalent in Canada. A prosperous, sleepy place.

Canadian Railways.

The latest official figures published deal with the railway statistics for the year ending 30 June 1913. At that date the mileage operated was over 29,000, and a further 18,600 was either projected or under construction. It may be noted for comparison that the railway mileage in the United Kingdom is 23,400. The total mileage, owned and leased, of the Canadian Pacific Railway is 11,500, of the Grand Trunk and Grand Trunk Pacific 4,500, of the Canadian Northern 4,000, and the Dominion Government owns and operates about 2,000 miles.

The capital liabilities of the Canadian Railways amounted to three hundred and ten millions sterling, made up as follows :—

Funded Debt (Bonds, &c.)	...	£125,000,000
Debenture Stock (C.P.R.)	...	33,000,000
Stocks	152,000 000
Total		£310,000,000

Of the above, the Bonds and Stocks outstanding on account of lines under construction amounted to £36,000,000, so that the capital account of operating railways was £274,000,000.

The average rate of interest paid on the funded debt was just over 4 per-cent and it is satisfactory to note the official statement that the full interest was paid on all outstanding bonds.

The debenture stock of the Canadian Pacific is separately classified because of its unique character. It differs from the bonds in being a perpetual stock. The Company engages to pay 4 per-cent interest on the principal, and this interest is secured by a lien on all the railway property in perpetuity.

Of the stocks, only about 85 millions, representing under 60 per-cent of the whole, is dividend paying. The dividends amounted to about 5½ millions, which gives an average yield of

$6\frac{1}{2}$ per-cent on the dividend paying stocks, and of about $3\frac{2}{3}$ per-cent on the whole nominal capital. The rate earned was somewhat less than in the years 1911 and 1912, and this reflects the prevailing depression. Among the non-dividend paying stocks may be mentioned those of the Grand Trunk ($22\frac{1}{2}$ millions) and of the Canadian Northern ($15\frac{1}{2}$ millions).

In considering the above figures it is important to bear in mind that while the capital account is swollen by expenditure on railways which have been recently constructed, there has not yet been time for all these lines to earn an equivalent revenue. As the proportion of new construction lessens, the earnings seem likely to grow more rapidly than the capital account, and increased dividends should result.

As has often been pointed out, the natural channel of trade between the western portion of Canada and the United States runs north and south, but in order to enable Canada to preserve her political entity, and to develop her resources under the Imperial flag she has been compelled, in defiance of nature, to construct her main lines of railway from east to west.

From a commercial point of view, the thousand miles of barren country which separates the manufacturing East from the agricultural West is a great handicap. Throughout this long stretch, there are practically no towns of any importance and but few industries, so that the railways cannot earn much local revenue either from passengers or from merchandise to help to pay their operating expenses.

The three great trans-continental railways are the Canadian Pacific, with its terminal at Vancouver, opened in 1886, the Grand Trunk and Grand Trunk Pacific, which has its terminal at Prince Rupert, 500 miles north of Vancouver, opened last summer, and the Canadian Northern, which will also terminate at Vancouver, but of which large sections still remain to be constructed.

So far as British Columbia is concerned the Canadian Pacific has had hitherto almost a monopoly of traffic, though this will in future be shared by the Grand Trunk Pacific. On the eastern side of the Rocky Mountains throughout the great wheat belt all three railways are in active operation, and gravitate by many lines to Winnipeg, the great distributing centre of the West. Thence each line leads to the twin cities of Fort William and Port Arthur where the grain is shipped into the

Great Lake steamers on its journey to Chicago or Eastern Canada.

It has sometimes been stated that with the completion of the trans-continental lines the great era of railway construction in Canada has come to an end. I venture to think this is far from being the case.

In considering the prospects of the railways throughout Canada the question of the harvest in the prairie provinces is a factor of the utmost importance. It must be remembered that, great as is the output of wheat at the present time, the area of cultivated land does not represent more than a fraction of that suited for the production of wheat, and there can be little doubt that, as the population increases and a larger area comes under cultivation, there must be a great corresponding development of the railway system. The construction of branch lines in every direction will need much further capital in the future. Possibly at the outset these lines may not be remunerative, but they will lead to the more rapid development of the districts through which they pass, and ultimately should justify the capital expended on them.

Apart from this construction of new branches, long stretches of important main lines are still only single-track, and in the Rocky Mountains much work still remains to be done in remodelling the permanent way and in providing tunnelling to reduce the steepness of the gradients.

The opening of the Panama Canal obviously cannot fail to affect the Canadian railway system, but it is yet far too early to hazard an opinion as to whether the changes will be favourable or adverse. From different quarters I heard the most diverse views on this subject. At present, as has been noted, the wheat all travels eastwards, and the moving of the crops in the autumn months is the chief activity of the railways throughout central Canada. It is claimed by some that it will be cheaper to move the wheat westwards to the Pacific ports and thence by sea to England through the Panama Canal. On the other hand, the cost of haulage over the steep gradients of the Rocky Mountains would go far to neutralize this saving so that there does not seem any great likelihood of any large change in the present system. It appears to be generally agreed, however, that so far as British Columbia is concerned the opening of the Canal will be advantageous and will lead to a great development of the fishing, fruit and mineral industries,

and that the railways operating in those districts should be helped thereby.

Industrial Companies.

While England has taken the chief share in financing the Railroads and in purchasing Government and Municipal securities, the capital provided by the United States has been more largely employed in promoting industrial enterprise. This is perfectly natural, for their means of obtaining local information as to industrial opportunities have, of course, been far more numerous than those open to English investors.

During the course of my tour this point was more than once brought home to me closely, for the Canadian bankers spoke bitterly of the discouragement from which legitimate industrial enterprise suffered owing to the readiness with which many unsound and speculative ventures had been taken up by the English market. They complained that an agent with a glib tongue seemed to be able to float any reckless undertaking in London, and that the English habit was to invest first and investigate afterwards. It has thus come about that legitimate enterprise has been handicapped and that even the soundest industrial securities have been prejudiced by the carelessness of our investing public. I need not pursue the subject in any further detail, for it is not in the least likely that British Life Assurance Companies will embark largely upon this type of investment.

The magnitude, however, of the monetary interests involved is very large, for upwards of sixty to seventy millions of American capital, seventy millions of British, and ten millions of European capital have been lent for the development of hydro-electric works, coal mines, iron and steel industries, lumber, pulp and paper industries, fisheries, milling industries, various lines of manufacture and other more or less local and domestic enterprises.

Real Estate Mortgages.

In the first section of this Paper I have dealt in detail with the various types of mortgages on Real Estate. So far as I can trace there are no reliable statistics available which afford a trustworthy guide to the amount so invested, and the Land Titles Offices do not publish any returns of their transactions. It appears, however, that over eight millions have been lent by

Insurance Companies operating in Canada (other than native), and that the Loan and Trust Companies have advanced nearly fifty millions sterling, though there seem no means of ascertaining what proportion of the latter sum is imported capital.

But in the Prairie provinces additional capital is still urgently needed for farming development, for the construction of buildings, for the purchase of agricultural machinery, and generally for increasing quickly the productive power of the country.

According to official figures only one quarter of the land capable of cultivation is at present devoted to agriculture, and in this estimate no account is taken of land in the Northern areas beyond the limits of the present wheat belt. Experiment is now stated to have developed a wheat which will ripen a fortnight earlier than any hitherto grown in the district, and it is confidently claimed that, as soon as this scientific discovery can be put to commercial use, the present wheat belt will be extended 100 miles northward. The supply of farm lands is thus practically unlimited, and consequently they do not stand at inflated prices.

As the rate of interest on these loans is high, there seems likely, when the present depression has passed away, and immigration on a large scale again sets in, to be here a sound opportunity for the investment of large further amounts of British capital.

SUMMARY.

I will close this paper with some general observations on the Canadian financial outlook. For much information set out in the notes which follow I am indebted to Sir Frederick Williams Taylor, the General Manager of the Bank of Montreal.

- (1) Canada is at present passing through a very adverse time, and the forthcoming winter will be a period of grave anxiety.
- (2) So far the Eastern cities are standing the strain much better than those of the Middle and the West, partly because of their manufacturing industries and partly because they have had time to accumulate some reserve of wealth.
- (3) In spite of the development of the farming industry in the Prairie provinces, the population in the cities has been increasing much more rapidly than in the country. It is hoped that the present depression

will force men on to the land, so that they may become producers instead of merely distributors of wealth.

- (4) The total amount of English capital publicly invested in Canada amounted, in December 1913, to over five hundred millions sterling, many millions of which have been wasted in municipal and industrial enterprises.
- (5) In 1913 English capital poured into the country at the rate of six millions sterling a month. This has suddenly stopped, and few Canadians recognize how enormously the industries of the country have depended upon this constant influx of new capital.
- (6) The indebtedness of Canada to England involves a monthly payment for interest of two millions sterling.
- (7) The payments for war expenses have now to be met in addition. Every gift of Canada to the Empire will have to be met ultimately by a loan in London.
- (8) Canada will now go through a time of severe trial in attempting to meet expenditure out of income instead of out of loan, and the intensity of the strain will increase with every day's duration of the war.
- (9) It has already been found necessary to adopt a Moratorium in the Prairie provinces to suspend the due payment of interest and instalments of purchase money on land loans, and even in the East there is a strong agitation in favour of some similar legislation for the relief of distressed borrowers.
- (10) "Unfortunately economy is not a popular text in a country that has been borrowing so freely for so long."—(*The Economist*, 24 October 1914).

These are the adverse facts and it is well to face them. But I want you to bear in mind that these difficulties are temporary and will pass away. The natural wealth of Canada is enormous and she will emerge a healthier, stronger and more self-reliant nation. The greatest asset of Canada is her annual harvest and, in comparison with the value of this, municipal extravagance can hardly be said to be of fundamental importance. The harvest this season is inferior in quantity to that of last year.

but prices are so much higher on account of the war that the farmer is prospering.

Moreover, in considering the future, it must be remembered that so far mineral and even agricultural development is in its infancy. But the spirit of confidence in the future of the country is universal, and shrewd observers welcome the check to speculation which the present set-back has caused and the impetus it has given to the movement for forcing men on to the land.

In summing up impressions of Canada we have to remember, as Sir F. Williams Taylor reminded me, that we are dealing with a vast country and that we must avoid thinking parochially. There have been periods of depression before and they have passed. In the same way every Canadian believes that this one will pass in its turn and that a new era of development and prosperity lies ahead.

In conclusion, I must express my warm thanks to Mr. George H. Lawton, A.I.A., who accompanied me to Canada, for the careful notes he made of the various interviews throughout the tour and for much valuable help in the preparation of this paper. I am also indebted to Mr. W. N. McIlwraith, of Messrs. Wood, Gundy & Co., for his assistance in preparing the statistics embodied in the section dealing with municipal indebtedness, and to Mr. G. W. Parfett for many useful suggestions.

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ABSTRACT OF THE DISCUSSION.

Mr. OWEN KENTISH, in opening the discussion, said that the paper dealt with so large a subject that he had a very difficult task before him in attempting to deal in any adequate way with the many important questions which it raised. He was most in sympathy with the concluding paragraphs, in which it was pointed out that the mineral and agricultural development of the country was in its infancy, that the spirit of confidence in the future was universal, and that Canada would emerge from the present difficult times a healthier, stronger and more self-reliant nation. He wished that this aspect of the subject had been more prominently brought out in the paper. It seemed to him that Canadian matters were judged by the author too much from the standpoint of an ancient country like Great Britain, and that sufficient allowance had not been made for the difficulties which must confront the men in power, in Government, Province and Municipality, in dealing with the problems of a rapidly-growing country. The balance sheet of a life office which had been founded within the last five years would not be looked at in the same way as that of an office that had been in existence over half-a-century, and the same allowance, or even greater allowance, should be made in criticising Canada's balance sheet.

He had little to say with regard to the first part of the paper, which dealt with matters of fact. It was a valuable record of the points that must be borne in mind by those who might at the present time, or in the future, contemplate mortgage investments in Canada, and would no doubt be referred to by insurance managers in the days to come, when people had money to invest outside their own country. But the economic crisis which had reduced values to their proper level should be welcomed by everyone, because they knew that recent values were inflated. They had never accepted, except at a liberal discount, the opinions of the best valuers on the spot, and that attitude was now justified. He wondered, however, if, in regard to Canadian investments generally, the British life offices would make the mistake that they had so often made in the past, of waiting until prices had risen, and thus losing their opportunity of investing on the most favourable terms. Personally, he believed that the next few years would see exceptionally good opportunities for investment in Canada.

In connection with the part of the paper which dealt with the question of Canadian indebtedness, there was one point on which he was disposed to differ from the author, namely, in regard to the assessment of debt at so much per head of the population. They were told that the total debt of Canada was £90 per head. But over 40 per cent of the total was for railways. Did the author contend that Canada paid the total interest on her railway debt? When the sources of, say, the Canadian Pacific Railway revenue, were analysed, it was found that much of it was provided by the European consumers of Canadian wheat. But even taking the figures as they stood,

and omitting the last item (industrial and mortgage indebtedness), for which the British figures were not available, the British Isles gave rather surprising results. The National, Railway, County and Municipal debts of the British Isles amounted to £2,070,000,000 and, taking the population at 48,000,000, the amount per head was £43 against £58 for Canada. The population of the British Isles, however, included, in a much larger proportion than that of Canada, persons on the border line of penury, females, old people and little children. Even if the basis of comparison were correct, he contended that Canada was just as able to bear £58 per head as the British Isles were to bear £43. Again, taking the municipalities, it was not enough to say that one town had a gross debt of £50 per head and another £25, leaving it to be inferred that the second was twice as safe as the first. It was necessary to know what proportion of the population of each was actually producing wealth, whether the town consisted mainly of men in the prime of life, or whether there were old people and children there in large numbers. For instance, Manchester had a net debt of £32 per head, and Hastings only £8 10s., but, if asked which stock he would rather hold, he would reply without hesitation, Manchester's. A comparison of this kind was of value only when two towns stood on approximately the same footing.

There were two points on which he desired information from the author. Reference was made in the paper to the municipal indebtedness on Treasury Bills, and he gathered the author believed that the sum now owing was largely in excess of the amount there mentioned. He thought, however, the author would find on enquiry that the indebtedness on temporary securities at the present time, excluding sums raised directly as the result of the War, was under three millions—a satisfactory reduction. And he was not quite clear as to the author's comparison of Canadian rating with rating at home. It seemed to be based upon a rental of 5 per-cent upon the capital value of property, but he was certain that nowhere in Canada could premises be rented on those terms. He thought the rentals varied from $7\frac{1}{2}$ per-cent to 10 per-cent of capital values, which would reduce the rate, which the author put at 6s. in the £, to from 4s. to 3s. in the £. On the question of assessments, it might not be out of place to point out that Vancouver and Edmonton were the classic examples quoted by the advocates of the single tax. In those cities improvements were not rated at all, in order, probably, to stimulate development. Taking two adjoining sites of equal area, the rating would be identical, notwithstanding the fact that the building on one was twice as valuable as that on the other. It would seem that site assessments would almost certainly tend to become inflated in such circumstances, and possibly the very high site values in Vancouver might be attributed to that cause.

He would like to say a few words on the question of extravagance, so often referred to in the paper. The first reference was to the public buildings at Regina and Edmonton. It might be remarked that those were examples of Provincial and not Municipal expen-

diture, and that the fine buildings at Regina, which cost $1\frac{1}{2}$ million dollars, were built entirely out of the proceeds of the sale of surplus city lands, not from loaned capital. Probably, however, the Provinces were looking ahead. A Government must be housed somewhere. In 1898 Victoria put up a building which no doubt the author would have called palatial, and at first it was certainly larger than was necessary. Five years later it was just large enough, and in 1913 it was necessary to erect a further building of the same size as that originally erected. With such an example before them, he thought that many of the municipalities were justified in putting up larger buildings than immediate requirements warranted. He believed all Canadians were agreed that from its position the City of Edmonton and Strathcona was absolutely certain to become one of the most important cities in Western Canada. On the general question, it must not be forgotten that municipal ownership of "public utilities" was regarded differently in Canada and Great Britain. In this country the opinion seemed to prevail that all municipalities were more or less incompetent. Canada, perhaps from painful experience, thought that municipal ownership was for the good of the community, and it was the almost universal practice for the water and electric light supplies to be operated by the municipality. It was the custom of the country, and had to be accepted. Personally, he could not blame those in authority if, in laying down plant for the supply of water or the provision of sewers, they took long views and spent far more than the size of the town warranted. It was a most costly business to scrap such systems and replace them by others when the town had outgrown the supply.

The author referred to electric trams and telephones as examples of municipal extravagance, but he (the speaker) believed there were only eight tram systems in the whole of Canada, and only two telephones, owned by the municipalities. The electric light systems were almost invariably profitable, and most of the tram systems were also profitable. The miles of unremunerative line at Edmonton were explained by the geographical situation of that city in relation to Strathcona, and the putting of the two cities, now one, in communication with each other was a public-spirited act of the municipality for which it could hardly be blamed. The other tram system which was referred to, at Port Arthur and Fort William, connected those two cities, and to do so it had to run through "undeveloped land"; and although the author might say that "it must be years before the population can need such a costly service" he found on enquiry that the line was worked at a profit. He admitted that more had been spent on many of these undertakings than had been absolutely necessary, and that to provide local employment streets had been constructed and buildings erected before they were wanted. But when a country was filled with optimism for the future it was most difficult to impress the importance of strict economy upon the people, and if Canada had been able to borrow too readily and too cheaply in the past, he did not think those who were ready and anxious to lend her money were quite playing the game when

they turned round and abused her for taking advantage of the opportunities that were held out to her. There were many cases on record of financiers going from this country, cap in hand, to beg those same municipalities to place loans on the London market through them. The present crisis would teach men on both sides of the Atlantic caution, and principles of sound economics were more likely to be observed both there and here in the days to come.

Canada to-day was in the position of a great business that needed capital to develop it, and to starve a growing business on the capital side was a most serious thing. There were certain favourable signs, however, which should be considered. By her great natural wealth Canada came successfully through the financial crisis of 1907 and, if she could adequately develop her resources, she would come as successfully through the present crisis. The two years which followed the Franco-Prussian War were the years which showed the greatest recorded immigration into the United States. European peoples, escaping from the burdens of militarism and from the heavy taxation which followed that great war, sought a home in the new world, and very probably the same thing would happen at the close of the present war. There was also the possibility that industrial unemployment in Canada would send men back to the cultivation of the land. One immediate effect of the war would almost certainly be a large increase in the area prepared for the 1915 wheat crop, to help to fill the gaps caused by the devastation of vast agricultural districts in Europe. There were also gaps caused by the industrial paralysis of countries engaged in the European war. In the year 1912-13 Germany and Austria exported to Canada over £16,000,000 worth of goods and a proportion of those goods Canada might be expected to make for herself, while the enforced economy resulting from the war would help to make up for the inflation caused by an undue optimism. As far as Canadian investments were concerned, the author had done very well to point out the need for the exercise of great caution, but that caution should not be exercised to the extent of turning a deaf ear to every legitimate demand for capital. With a wise discrimination and full investigation of the securities offered there would be in the not distant future many opportunities for safe and remunerative investments.

Sir T. P. WHITTAKER, M.P., said that he visited Canada in the boom, and therefore under conditions somewhat different from those which the author had experienced. At that time optimism prevailed everywhere. But there was bound to be a serious re-action. The present trouble in Canada was not the result of the War, although it had undoubtedly been accentuated by it. The trouble had come before the War and was inevitable. It was quite true there was such a thing as a parochial view, but the view of an old country was, in his judgment, not necessarily parochial nor was it to be despised. It was the view of experience. The people in this country had had the accumulated experience of lending money to every new country that had been developed during the last century or so, and knew what had happened in those countries. Therefore, their view of what

was likely to happen in another new country, although it was the view of an old country, was very valuable indeed. It was not a question, as the opener had stated, of starving a new country, but of due proportion, of wise lending and wise expenditure of capital; expenditure might develop too rapidly and bring about a financial crisis. This country went through the same crisis seventy years or more ago in connection with the development of its railways, and an awful crash was the result of the unwise expenditure that was made at that time. It was necessary to assimilate and digest as one went along, and to spend capital as it was possible successfully to employ it.

There were one or two points he desired to mention in connection with Canadian mortgages. The first was that in new and rapidly-growing towns very quick changes took place in localities. A road might be a good residential road for a first-class style of house, but in a very few years it might be altogether unsuitable for a good residential district, and the houses were then a drug on the market; they would be unlet or unused till they could be developed for business purposes. Those changes in the rapidly-growing towns of Canada occurred very quickly indeed, and required to be most carefully watched. It also had to be borne in mind that probably four-fifths of the houses in Canada were built of wood and were not intended to be permanent. Values of property in Canada were ludicrously inflated during the boom. He knew of one property which was bought for \$40,000; it then changed hands successively at \$60,000, \$80,000, and \$110,000, only six weeks elapsing between the purchase at \$60,000 and the last sale at \$110,000. The gentleman who gave him the particulars and showed him the property informed him that its legitimate value was not more than \$40,000. The value had been rushed up; but what was the good of a valuation of that particular property or of the property adjoining it, based on that series of sales?

One point had not been referred to in the paper, but it seemed to him the main cause of all the difficulty. So far as the prosperity of Canada arose from its agricultural industry, its timber industry, its mining and minerals, its fisheries, and its other natural resources, it was absolutely sound and bound to grow. But the boom had not arisen from that; it was the result of the large expenditure of money as capital in the country in the building of railways, the making of roads, the putting down of trams, the building of electric light and power stations. That expenditure naturally necessitated the employment of a large number of men who wanted houses and goods, and therefore a large number of houses were built to accommodate the men engaged in those enterprises; large numbers of stores were opened to supply their wants and that again increased employment. But railways, roads, trams, gas works, and electric light works would not have always to be built at that rate; once they were built the only thing that remained afterwards was the annual income, a very different thing. It was obvious that, as soon as those great undertakings had been constructed, there would be fewer of

them built in the future. The boom which was the result of the expenditure of capital in the country was bound to come to an end more or less, and it was necessary to come back again to that portion of the prosperity which was due to the natural resources of the country. That was the experience of all new countries; it had occurred in Australia, in New Zealand, and in Argentina. There was first a great rush of capital; the expenditure created prosperity, but when that expenditure ceased there was a slump and trouble ensued. The same thing had occurred in the United States, again and again. After he visited Canada he made a tour down the west coast of the United States, with the object of going through a country that had passed through the same experience a quarter of a century ago; and he found that the values of property in Seattle, Portland and other towns on the Pacific Coast of the United States were not yet quite back to the level of the boom in 1890. Those towns had their boom just as Canada had it; then values fell, and at the time he visited them they had not recovered to the original figures. He thought Canada would recover more rapidly; but, nevertheless, the difficulty had to be faced.

The author had given a very useful warning in his paper in regard to the varying laws that existed as to investments in the various Provinces of Canada, and it was very necessary that investors should acquaint themselves with those laws before lending money on mortgage, otherwise they would find that in some of the Provinces taxation on the investments would have to be paid. Farm loans were very strongly recommended in Canada, but difficulties occurred in connection with them in the West. The farm land might be twenty or thirty miles away from the railway, and the railway might only just touch a station where there were two or three shanties away out in the Prairie. The lender had to rely on travelling inspectors, who probably depended for their commission upon the loan "going through." There was also very great difficulty in collecting interest from the farmers in Canada. They did not get their own money very quickly for the wheat, and therefore were not always punctual in payment. It was those difficulties which made the rate of interest on farm loans higher than on town loans. The rate of interest would not be higher unless there were greater risks and difficulties. The same thing applied to school loans in outlying agricultural districts, which were very much recommended in Canada. A point to be borne in mind was the position the lender would be in if the necessity arose for enforcing the security. There would be a power, he imagined, to do so, but what was it and how was it going to be exercised? If there was a defaulting local authority in Canada, and the power, such as it was, to recover was exercised, he thought it would do more than anything else to teach a very useful lesson. He quite agreed with the suggestion Mr. John Coles made some time ago, that the loans ought to be supervised by the Government, and ought to receive the sanction at least of the Provincial Government, as loans in this country required the sanction of the Local Government Board. One difficulty in connection with the

expenditure of municipal money by public authorities which had been seriously felt in some parts of Australia and in particular in New Zealand was that the resulting employment of a large number of people created a labour vote which was exercised in favour of the continuance of work after the necessity for it had ceased. In that way the community might be driven into expenditure, not on its merits, but on political grounds.

It also had to be borne in mind that the land of Canada was virgin soil, but the time would come when that natural advantage would cease, and the farmers would have to go in for mixed farming, which was a very different thing. It might be that some day in some parts of the Eastern States of Canada derelict and vacant farms would be found, as had happened in the eastern parts of the United States in consequence of new virgin soil becoming available in the western States. So far as railways were concerned, the Canadian Pacific had been enormously successful, but how would it get on in the future in the face of the great competition of the new lines that were being built? The railway expenditure in Canada had been tremendous, and the competition for traffic would be very great. The author would probably remember, when he went by the Canadian Pacific to Vancouver, passing for a great distance along one of the rivers, and seeing on the other side of the river a new railway being constructed at enormous cost through a very difficult country. That meant competition and reduction in rates, and the two railways would have to live practically out of the same traffic that existed at present. Another question which had to be borne in mind was that the natural market for the Western States of Canada was in the United States, and Canada had been driven to a great deal of expenditure on its railways in order to retain the traffic from West to East, when, according to ordinary natural conditions, it ought to have gone South. These conditions might lead to a great reduction in railway rates, and, if so, the position of the Canadian railways might be seriously affected.

Mr. F. W. HIRST (Editor of the "Economist") said his friend Sir Thomas Whittaker had made some very sage and astute observations upon Canada which had evidently met with a good deal of approbation from those present. Although Mr. Kentish's speech was a very interesting one, he was glad that Sir Thomas had spoken in a cautious rather than in an optimistic vein. New countries were optimistic; and it was natural when things were very much depressed that those feelings of excessive optimism should disappear and exaggerated pessimism take their place. The time when people were apt to invest in new countries was when exaggerated optimism prevailed; the time when no one would invest in them, when prices were at the bottom, was the time when anybody who wanted to make money should invest. But one of the objections to investing in Canada at the present time was that, although it was possible to obtain a very good rate of interest, it might prove very difficult to withdraw one's capital. The author of the paper had, in his opinion, done a service to the country by giving a fair and reasonable view of things

in Canada. He would have paid a very poor compliment to the Institute if he had tried to look at everything through rose-coloured spectacles. For one thing he would not have been believed. It was impossible to make people believe that things were very good in Canada, in Argentina, or in Brazil; and when Mr. Kentish said it would not be fair to judge new countries by old countries he could not help asking what was the condition of the old countries? Were they so very much better? Were Turkey, Greece and Portugal so very much better than some of the new countries? He was perfectly certain the best thing they could do for Canada, Australia and this country was, as Lord Morley once said, to dwell as far as possible in the Palace of Truth.

He agreed with Sir Thomas Whittaker and also with Mr. Coles that the Canadians needed very much a severe system of Local Government supervision. The difficulty was to know how it should be carried out, because there was the Federal Government and there were the Provincial Governments. What was really wanted was a proper Federal system of local control. It might happen that when one or two of the small places defaulted, as he was afraid they might do, the Canadian Government would see the necessity of coming to the rescue in some form or another. When they did so, if leading people in the City of London were wise they would advise the Canadian Government very strongly to establish a system of supervision, which would improve Canadian credit in this country.

The capitalist institutions of London had a great many interests in Canada, and they ought to try to maintain Canada's credit and prevent things from becoming very bad. The country should not be let down too much when it was in difficulties; it must be kept fed at any rate a little in order to keep existing investments alive. He desired to thank the author for his concise and sober paper—a model for journalists. He sometimes thought that the difficulties new countries suffered from, especially Canada, were very largely due to the absurdly optimistic articles, puffing articles of various kinds, which appeared in a certain class of newspaper. He believed such articles had done a very great deal of harm, but he was afraid the reasons which prompted them would probably last as long as human nature.

Mr. G. H. LAWTON desired, in the first place, to express his conviction that the paper not only gave the true facts, but conveyed a correct impression of the financial condition of Canada at the present time. Mr. Kentish had mentioned the question of the Edmonton trams. It was a parochial point, but he could not resist the temptation to read the following paragraph which appeared in the "Monetary Times of Canada", in the issue of the 13th ultimo, which arrived a few days ago. It was headed "Running Civic Railways", and was as follows—"Seldom does municipal ownership confess the error of its ways. . . . Commissioner Harrison, who is in charge of the public utilities of Edmonton, and who is one of the most business-like of its citizens, has given some facts and figures which should cause local authorities in other cities to ponder. . . .

Before the Edmonton street railway begins to show a profit it must carry at least 47,000 passengers per day. At the present time it is carrying on the average 32,000 passengers per day, and is losing at the rate of 500 dollars to 800 dollars per day (say, £140 a day, or £50,000 a year.) After submitting these figures, Mr Harrison adds : 'I find that our system is over-capitalized; that we have twice as many miles of track per thousand of population as any city in the Dominion of Canada, and in many instances four times as many miles; in other words, we have extended our system far more rapidly than good business practice would warrant.' One of the suggestions he makes seems to suggest danger—that the establishment of skating rinks and amusement parks at the end of the line may increase the traffic, and in time overcome the deficit. Investors who hold Edmonton's bonds may view civic skating rinks with suspicion." At all events, the reference in the paper to the Edmonton tram system was certainly not an over-statement of facts.

Sir Thomas Whittaker had mentioned Canadian School Bonds, but he understood him to refer chiefly to those in the West and in outlying districts. He would like to make a few remarks in regard to school bonds, especially those issued in the City of Montreal, which formed a suitable investment for the funds of a life office. In Montreal elementary education was free, as it was practically throughout Canada, and the cost, apart from large subsidies by the Provinces, was met by tax. There they had the interesting problem occasioned by the prevalence of two religions. At the outset a taxpayer had to elect whether he would contribute to the Protestant or Roman Catholic schools; if he decided to remain neutral his taxes were apportioned *pro rata* between the two. He had before him particulars of an issue by the Protestant School Board Commissioners, which took the usual form of 30-year bonds with half-yearly interest. The security consisted of a first charge on the Protestant City School Taxes which were stated to approximate to five times the total amount required annually for interest and sinking fund. On the bonds themselves was printed the following clause—"The City of Montreal is bound by law to retain in its hands out of the yearly proceeds of the school tax in the city a sufficient sum to pay the interest and form a sinking fund for the repayment of these bonds at maturity, and the bearer of this bond shall have the right to claim from the City of Montreal payment of capital and interest when due." In effect, therefore, the security was the double obligation of the Protestant Board of School Commissioners and the City of Montreal. There was also a special lien on the assets of the School Commissioners, and in that way those school bonds would appear to be a stronger security than the bonds of the City of Montreal itself. The issue in January last consisted of $4\frac{1}{2}$ per-cent bonds offered at just over $97\frac{1}{2}$, so that the immediate yield was about £4 13s., a rate slightly higher than that obtained from the City's own bonds at that time, but not being quoted on the Stock Exchange they were not, of course, quite so marketable. The total amount thus issued was something over £750,000; and it might be added

that it required an Act of the Legislature before an issue of School Bonds could be put on the market.

Perhaps the section of the paper which was of the greatest interest to the majority of those present was that dealing with municipal indebtedness. The growth of the total debt in recent years was illustrated by the following figures which he had extracted from the annual review of the Bond Market in Canada, published by the President of the Dominion Security Corporation, Mr. Wood, in January of each year. During the year 1911, the total issue of municipal bonds was nearly $9\frac{1}{2}$ millions sterling, in 1912 it was just over $9\frac{1}{2}$ millions sterling, while in 1913 it was 23 millions sterling, considerably more than double the amount issued in either of the previous years. A significant feature, indicating the trend of the market in 1913, was that the later offerings of similar securities realized substantially lower prices. For instance, Winnipeg $4\frac{1}{2}$ percents were offered in February, 1913, at 100, and in July at 97; Edmonton 5 percents were offered in April at $101\frac{1}{2}$, and in September at 96; and Montreal $4\frac{1}{2}$ percents were offered in March and May at 100, and in November at $98\frac{1}{2}$. When the total municipal figures for each year were analysed as to the sources from which the money was obtained, a noticeable feature was the increase in American investments. In 1911 the United States absorbed only 5 per-cent of the total new issues; in 1912, 8 per-cent; and in 1913 over 19 per-cent. The reason was, no doubt, that the American investor interested in that class of security found that the yield was greater from Canadian municipal debentures than from similar securities in his own country. Possibly, too, the Canadian securities were much more actively pushed in the States than hitherto. As Great Britain had taken an average of about 60 per-cent of the total issues, it followed that Canada herself had each year taken up a decreasing percentage of such bonds; in fact, it had fallen steadily from $34\frac{1}{2}$ per-cent in 1911 to $28\frac{1}{2}$ per-cent in 1912, and to 22 per-cent in 1913. That characteristic in regard to Canada's share, which showed itself so clearly in the matter of municipal bonds, persisted throughout each class of security—Government, railway, industrial—with the one exception of public corporations, *i.e.*, tramway, light, heat, power and telephone companies; and when the total of all the classes of new issues was considered it was found that whereas Great Britain had provided approximately 76 per-cent to 74 per-cent of the new money, and the States' share had risen from $6\frac{1}{2}$ per-cent to $13\frac{1}{2}$ per-cent, Canada's proportion had fallen from nearly 17 per-cent to just over 12 per-cent. He thought, therefore, it was clearly shown that in recent years Canada had become more and more dependent on capital raised outside her own country.

That Canada had needed and could use all the capital she could get was emphatically brought home by the fact that in a stringent period like the year 1913, when money, owing to the Balkan War and other conditions, was by no means easy to obtain, the total of all the bonds issued reached the record amount of £75,000,000, whereas in 1912 it was £54,500,000 and in 1911 £53,000,000. Before

leaving the subject of new issues, he would like to mention two further points. The first was that towards the £56,000,000 which had been raised by the railway companies in the three years in question, Canada had contributed the insignificant sum of £140,000, Great Britain being responsible for £50,000,000 (nearly 90 per-cent of the total) and the United States for the balance. The second point was that he found that of the new industrial issues in 1911 to 1913, inclusive, Great Britain provided £18,000,000 while the States found less than £3,000,000, and it would therefore appear that, although at the present time the total amount lent by the two countries was nearly equal, the relative position was undergoing some change, Great Britain providing a larger proportion than was formerly the case.

Turning to the last section of the paper—real estate mortgages—it was stated on page 63: “In the Prairie provinces additional capital is still urgently needed for farming development.” Great as had been the industrial development of the Dominion, he still thought that for many years to come, as well as at the present time, the out-standing and essential factor in Canada’s welfare would be her annual harvest. In the immediate future, with Russia and France—two of the five countries that produced the largest amount of wheat—involved in war, the value of a good harvest in Canada next year was all important; whatever quantity of wheat she could put on the world’s markets would find a ready sale and a good price. In Canadian papers one read of efforts, and successful efforts, to increase the acreage of cultivation, which brought him to his last point, *i.e.*, the question of farm mortgages. Sir Thomas Whittaker had briefly referred to them, but he desired to present them in a somewhat different aspect. Apart from the importance attaching to that question, due to present conditions, the matter was one well worth consideration. It could not be denied that there were great practical difficulties inherent in granting mortgages on farms. At the present time almost the only way to overcome them was to employ a suitable agency with a widespread organization—widespread because, of course, it was advisable not to have all such loans in one section of the country, but to scatter them throughout as large an area as possible. If advantage were taken of the agency method, the procedure was roughly as follows—The farmer filled up a form of application stating how much he wanted and what he wanted it for. The agency instructed its local inspector, who should be a salaried official with no axe of his own to grind, to call on the man and see if he was at the time, or was likely to become, a successful farmer, for, of course, it was no good advancing money for farming to a man who had no aptitude in that direction. The inspector also valued the property, and, if his headquarters approved, a sum not exceeding 50 per-cent of such valuation was then agreed upon as a maximum advance, but that sum was by no means always lent out of hand. In many cases instalments were advanced as and when specific work, such as breaking a certain portion of the land, was done. In that way the security grew with the amount of the loan. The advance was repayable at the end of a term of years, usually five, and repay-

ment by instalments, though not usually obligatory, was permitted. The object of not insisting on periodical repayments of capital was to enable the farmer to tide over bad harvest years, when he could often contrive to pay the interest though not an instalment of the capital. He considered the chief drawback to that class of mortgage was that implicit confidence had to be placed in the agency employed, any adequate control from this side being out of the question. He would not repeat the advantages and disadvantages mentioned in the paper, and had only ventured to enlarge upon farm mortgages because, if what he might call the moral side of the investment problem appealed to the members, and if they felt it any part of their duty to help in the development of the Colonies by providing money for their securities, then he thought that farm mortgages seemed to offer the most direct means of advancing the prosperity of the Dominion of Canada.

MR. S. G. WARNER said that two Canadian investment experiences had come under his observation. The first was an experience of nearly 20 years' lending operations in the district known as "The New Northwest", and comprising the provinces of Manitoba, Saskatchewan and Alberta. The amount of capital involved had been substantial, and there had not been any loss. He agreed with the author of the paper that the essential thing for a proposed lender on such security to do was to find a first-class representative. An English company, far distant from its investment field, might arrange for brief periodic visits thereto, but these alone would not secure the chief end in view—the safety of its capital. It was possible, however, in Canada, "new country" though it might be, to find men of experience and influence to whom they could delegate the supervision of the business with every confidence that it would be safely and profitably conducted. The loans, as a rule, were for comparatively small amounts, and (as Mr. Lawton had said) for terms of years. Often the difficulty was to keep them as long as was desired, for the borrowers were able and willing to repay them, in whole or in part, sooner than the contract provided. It was perfectly possible under such auspices as he had described to get strictly conservative valuations, and lending with an ample margin on the figures so given would involve little if any risk.

Sir Thomas Whittaker had referred to the importance of the proximity of mortgaged property to good transport facilities, but to that and all kindred points an efficient representative on the spot was fully alive. One rather interesting point was the difference between wheat farming and mixed farming. Of the two probably the mixed farm presented rather the more desirable security, because there was stock as well as grain, and if one were less successful in any year the other might counterbalance the deficiency. Over large parts of the area, however, mixed farming did not exist and a lender must be content with the pure wheat land. That, however, was excellent. It had been described as virgin soil, and doubtless there must be possibilities of exhaustion in the future; but that future, so far as one could now judge, was indefinitely remote. The

paper referred to the process of "carrying the farmer." In a bad year it was unquestionably necessary to allow interest to remain unpaid—but the position would not fail to right itself in time.

The other experience was in British Columbia, and was of rather longer duration than that just mentioned. It began in a period of prosperity, and had to pass through one of adversity. The loans in this case were not on agricultural but on urban properties. It became necessary in some cases to enter into possession, but values vindicated themselves as years went by and the history of the operations, viewed as a whole, was one of success.

It seemed natural, at first, to think that a time of depression was a time when investment should be avoided, but further reflection might suggest a different view. The essential question was, whether the security was inherently sound. Had Canada resources, agricultural and mineral, which in all human probability must make for ultimate greatness? If so, then surely, in accordance with the old maxim that it is a wise man who can buy on a falling market, it would be better to invest when values were depressed than when they were inflated. The investor must, however, place his capital in the right class of security, having regard to the true direction of the country's progress, and that suggested some interesting reflections, in the light of what had been said about the Canadian farm lands. Half-a-century ago in England, farm land was looked on as a gilt-edged investment. Our forefathers said there was "nothing like land." Then followed a time of adversity for farmers and land-owners, and English investment sought other channels. But beneath the old idea lay a profound truth. It was obscured here by two main causes; our feudal system of land tenure and the competition of foreign wheat. But now we went to the new region whence the competition came, and where the other adverse influence was absent, and lent our money there. In an historic phrase, we called in the new world to redress the balance of the old.

The most obvious advantage in Canadian investment was of course the higher rate of interest it brought—and so far as economic conditions could be foreseen that advantage was likely to continue for a long time to come. He was not sure, however, that it was a permanent or the most profound advantage. If and when economic development reached a stage at which over the whole civilized world the rate of interest was practically equalized, life assurance companies would still have no reason to regret that prudently and judiciously they had extended their investment operations far and wide; for by that means they would have broadened the base of their finance, so strengthening its fabric; minimized the risk of loss from local and temporary fluctuations; and applied effectively to their investments the great principle of average on which their whole business was founded.

Mr. GEOFFREY MARKS, in closing the discussion, congratulated the author on a very useful paper, the value of which was much enhanced by the index and the list of references appended to it.

He thought that they ought, as investors, and some of them as representing very large investing bodies—to take a long view of the securities which were open to them in Canada, and in doing so, to disregard to some extent the position in which Canada now found itself. It would be blinking the truth to say that the present position was anything but bad—very bad,—but he had faith in the ultimate destiny of Canada. He believed that the present position was a phase of its development which would pass away. Skill in the management of investments depended entirely on the power to distinguish between what was permanent and what was temporary, or to disentangle what was essential from what was unimportant. Canada was only following the destiny of all young countries. Extravagance to them seemed necessary and natural. Any country which was rich in undeveloped resources went through the same course. In Canada when land was cheap farms were paid for out of their own profits in a few years; the railways which then existed were choked with traffic; industries flourished; trade improved; large fortunes were quickly and easily made, either in legitimate trade or in speculation, the cost of living was high, but it was easily met, and the people in their private lives became accustomed to a scale of living which seemed equally necessary and natural when they applied it in public affairs and in business.

Young countries were mostly protective; the large credits which they were enabled by borrowing to set up abroad were represented by large imports; their national revenues, and therefore their prosperity, seemed to be increasing quickly, until a time arrived, as it had done with Canada, when universal financial stringency, accentuated by the present terrible war, put an end to borrowing and brought about a sudden collapse. Sir Thomas Whittaker and Mr. Hirst had pointed out that that had been markedly the history of the United States; it was the present history of Brazil; it had been that of Argentina, and it was also the present situation in regard to Canada. Those crises came at intervals. They were the result of the unequal development of the country's natural resources and of its power to exploit them. If the two movements could keep pace these crises would not happen. But that could not be, and he submitted that in the nature of things crises such as that through which Canada was now passing were unavoidable. They did not permanently impair the country's possibilities, and they might afford, as one of the speakers had suggested, a very useful and advantageous lesson to those who were responsible for its management. What had to be considered was, what real foundation the country had for what might appear to some people to be extravagance. He took it that the essentials to prosperity in any country were, first, that it should be rich in natural resources; secondly, that it should have a sound system of banking and currency; thirdly, that its facilities for transportation should be adequate; and, fourthly, that it should have a proper labour supply. As briefly as possible, he would mention a few reasons why he thought Canada possessed all those essentials. He did not think anybody who had visited both Canada and the

States would deny that Canada had at least all the possibilities that the United States possessed. The agricultural produce of the United States, excluding cotton, was of the annual value of some £1,800,000,000. Canada had developed only about 10 per-cent of its present available wheat area, and the annual value of Canada's agricultural products was about £180,000,000, just one-tenth that of the United States. It seemed to him very significant that the period of development at which Canada appeared to have arrived had enabled it to attain to a position where, if its resources were compared with its liabilities, there was not a great deal of difference between Canada and the United States. He certainly thought that Canada was far more developed in every respect than the States were at a corresponding period of their expansion.

As regards banking and currency the Canadian system was better, even at the present time, than that of the United States, although it was not equal to that of this country. The number of banks in Canada was under thirty; they worked in the closest co-operation, and were guided by men, both as directors and executive officers, who were not only informed as to all the forms of finance but also interested personally in every kind of industry, commerce, and manufacture and in the great transportation systems. They were therefore in a position to keep their hand on the pulse which beat in every artery throughout the country, and that, combined with their general level-headedness, due to the fact that most of them were either Scottish or of Scottish extraction, seemed to him to afford a very fine safeguard for the future of the country.

The question of transportation had been adequately dealt with. It always presented in a new country great difficulties. It was impossible to avoid over-development of railways if the country was to be properly developed. There must be stages when railways had to run through long miles of country which produced nothing at all into districts that at times produced too much, or at any rate more than the railways could handle. But that was a situation which in time remedied itself, and to his thinking it was far better that the railways should be developed and built in advance of the natural resources of the country than that the natural resources should have to wait for the railways.

In regard to labour, he thought again Canada compared favourably with the United States. The annual immigration into the United States was about 1 per-cent of the total population, and from his own observation and from conversations he had in Washington with members of the staff of the American Government Department, he had come to the conclusion that the immigration question was one of very great difficulty to the United States: indeed, he thought he might go so far as to say that, except that it supplied a large amount of unskilled labour, immigration at the present time was by no means an advantage to the United States. On the other hand, Canada obtained about 5 per-cent of its population as annual immigrants; about 75 per-cent of them were English-speaking, and the rest were Scandinavians and Germans of a very substantial

and satisfactory kind. A Canadian had told him in this connection that the war was unfavourable to the development of Canada, inasmuch as the German immigrants into Canada were of a character which was of the highest value to the country, and that he viewed with some apprehension the stoppage of that source of supply.

He would close by saying that he agreed with the optimistic view of Canada's future. He believed in being an optimist. Nothing had ever been done in this world except by optimists. The only danger of optimism was when it became exaggerated, and he thought it might be taken for granted, as the tone of the debate showed, that that was not a danger which was likely to affect this country in its relations with Canada.

The PRESIDENT having proposed a cordial vote of thanks, which was carried with acclamation, Mr. A. D. BESANT, in reply, said that in view of the lateness of the hour, he would not attempt to go in detail into the many points that had been raised. With regard to Municipal Taxation, Mr. Kentish had questioned the accuracy of the English equivalent given in the paper of the Montreal Corporation and Education Tax. He had arrived at his figures by exactly the opposite course to that suggested by Mr. Kentish. On a 5 per-cent basis an English rate of 6s. in the £ would be equivalent to a rate on a capital assessment of 3·6*d.* in the £, *i.e.*, 1·5 per-cent, which was the exact scale of taxation in force at Montreal. In connection with the total municipal indebtedness of Canada the paper suggested that the outstanding Treasury Bills might to-day reach a considerable amount, though there were no means of ascertaining the figure. That, however, had kindly been furnished by Mr. G. A. Cassels—the London Manager of the Bank of Montreal—who stated that the total amount of outstanding Municipal Treasury Bills at the present time was only £1,668,000. The figure compared very favourably with the £5,000,000 existing in 1912.

With regard to the question of general expenditure, it seemed to him the point that had to be decided was exactly where the line should be drawn. Some, among whom he would include Mr. Kentish, were a little more optimistic than others, and some, among whom he should be inclined to include himself, were a little less optimistic. It was all a question of degree. As stated in the paper, the great bulk of the expenditure had been wise and legitimate, but he still thought cases had occurred in which there had been extravagance, and in some cases marked extravagance. He instanced as one of those features the wonderful Parliamentary buildings which had been put up at Regina, which from the English point of view was quite a small town of 40,000 people. Although, as a matter of fact, that building had been paid for out of the sale of surplus lands, he still thought that the money provided in that way might have been used for a better object, and that it was an instance of extravagance. It was necessary, of course, to take long views, but he did not think it wise to take too long views. If unduly long views were taken they led to undue competition, as for instance, in the

case Sir Thomas Whittaker had mentioned where two railway lines ran for many miles close to one another. Another instance of that was the existence, between Fort William and Winnipeg, of three great lines which entered into competition with one another for taking down the wheat. There was certainly no need for those different tracks at the present time.

As to the present condition of Canada as a field for investment, he cordially agreed with Mr. Warner that times like the present afforded an excellent opportunity for investing on the most favourable terms. In conclusion he desired to repeat that the set-back in Canada was purely temporary. Nothing could affect the two great natural resources of the Dominion, her annual harvest and enormous mineral wealth. Development in the future was certain, and with that development prosperity must lie ahead.

[Mr. J. R. Hart sends us the following notes on some legal points arising on Mr. Besant's Paper—Eds., *J.I.A.*]

The main part of the procedure in carrying out a mortgage is, of course, directed to protecting the lender in those cases—few in number—where the personal covenant has failed. So long as principal and interest are met by the borrower, the mortgaged property is not resorted to, nor are many of the provisions in the deed put into force. It is, however, of great importance to an intending lender to consider not only the procedure in carrying out the mortgage, but his position in case of default, and his rights and remedies under the deed. The author does not appear to have dealt with this aspect of the subject of mortgages. This is easily to be understood, as limitations must necessarily be placed on the various sections of a paper of such a vast scope as that in question. It should, however, I think, be mentioned, that in the province of Quebec, where French law prevails, the lender does not get a mortgage, as we understand a mortgage in England. He gets only a charge, or what is known as a "hypothèque." This does not carry with it such full rights as a mortgagee in this country possesses. There is, for example, no power to foreclose, and the property not being vested in the lender, but remaining in the borrower, the former has probably no power of leasing, and has restrictions on other rights possessed by a mortgagee over here.

Passing on to the Torrens system which is in force in the west, I think that the statement that the whole procedure is exactly that employed in England for Stock Exchange securities needs some qualification. The Torrens system possesses undoubted advantages, inasmuch as title prior to the dealing in question is dispensed with. It provides that a mortgage does not operate as a conveyance of the legal estate, but merely as a charge, and that on satisfaction of the mortgage a formal receipt only is necessary. But, although transactions are probably carried out in this way, it is considered necessary by the legal advisers of some mortgagees that for the protection of their clients the latter should have a mortgage by a deed in the form employed in England under our old conveyancing system. The legal

estate is conveyed by such deed from the borrower to the lender, and the latter apparently possesses all the rights enjoyed by a mortgagee here, subject to this point, which must be borne in mind. The borrower remains the registered owner, and it has yet to be seen, when a good deal more experience of the exercise of mortgage powers is available, whether any complications will arise from the two titles, the title on the register and that off the register. This procedure for mortgaging land, is, of course, quite different from that adopted in a mortgage of stocks and shares. In the case of the latter, the register would have no concern with the relationship of mortgagor and mortgagee.

There are a few other points arising out of the author's interesting notes. In the case of the legal system in Quebec it would be well, I think, to make it clear that the registration there is registration of deed only and not registration of title as under the Torrens system : and in the paragraph describing the procedure in Quebec, it might, I think, be mentioned that the lender has the protection of such deed registry, although it does not dispense with prior investigation of title.

In dealing with Canadian land tenure generally, the author has divided the systems into two main systems, that in Quebec and that in the west. I think, however, it should be mentioned that there are really three systems, namely, the two mentioned by the author, and in addition the ordinary English law which prevails in the province of Ontario, and into which has apparently been adopted a deed registry in some parts and a voluntary title registry in other parts.

A small point of detail as regards the Torrens system should, I think, be mentioned. The land certificate is not deposited with the lender, as mentioned by the author, but is held by the registrar while the mortgage is in force. There have been a good many difficulties in this country under our Registration of Title system arising on the custody of the certificate, and the provision in Canada for its retention by the Registrar would appear in all the circumstances to be the most satisfactory.

It is very difficult over here to obtain full information as to the exact legal position of a mortgagee under the various systems, but I think the above notes may be useful as supplementing to some extent the valuable information given by the author.

[We add the following extract from a communication we have received from Mr. A. McDougald—EDS., *J.I.A.*]

The author specifies as the essential points to be borne in mind in undertaking mortgage business in Canada :—

1. The danger of inflated valuations, especially in the Western cities ; and
2. The need of adequate local supervision.

I would suggest that the need of adequate supervision is always the most important point, but supervision would require to be more than local. It would have to be co-extensive with the location of

the securities. Later in the paper the author refers to farm lands as likely to prove a sound investment for British capital. If he means insurance capital I should be disposed to agree, conditionally on the setting up of a regular system of short date inspection and audit of the manner in which the mortgagors farm their lands and of the use to which they put the borrowed monies. Here the character and status of the borrower are of prime importance. The additional cost of this machinery to safeguard the capital would in respect to a given volume be more than met out of the superior interest yield.

The author speaks of Canadian mortgages. In the Province of Quebec, however, the lender is not a mortgagee but is the holder of a *hypothèque* or *lien*. Two features of a *hypothèque* are that (a) the legal estate in the immoveables does not pass, but only a real right thereupon is held by the lender; and (b) a definite maximum sum, or sums, named in the deed limits the amount recoverable thereunder by the creditor. It is interesting in passing to note that this form of security is said to be an evolution of the Roman law much modified under the old French law and ultimately embodied in the Quebec Code of 1774, with subsequent amendments.

There are various kinds of *hypothèques*, the borrower and lender kind being that known as a conventional *hypothèque*. In the management of these loans it is important to keep down accumulations of arrears, for the reason that, after a specified time-limit, interest arrears cannot legally be recovered. In the event of default the sheriff is moved to bring the property to sale, and in the meantime the borrower remains in possession of the rents. Out of the proceeds of sale the creditors are paid according to the legal priorities. The position of the hypothecary in the queue of creditors is not without interest. The law gives prior claim to some ten other classes of creditor, including conditionally those for the expenses of the last illness of the borrower and the cost of his funeral. These and others are generally of no practical moment, but possible claims by the Roman Catholic Church and by the builders of the structure on the land, as also by the vendor from whom the borrower purchased, are matters which require to be most carefully guarded against. These considerations are closely allied to the question "Who can hypothecate"? For the purpose of this discussion the answer is "An adult solvent male of the Protestant persuasion, who holds his property free from actual or potential claims by the Roman Catholic Church." The land tenure met with in Quebec Province is that known as "*franc aleu roturier*" or free and common soccage—say freehold. In that portion of the Province which was settled by the English there is no necessary appearance before a notary in executing the loan deed, though in the French settled parts the intervention of the notary is a legal requirement. The whole system of conventional *hypothèques*, though it may sound strange to unaccustomed ears, is in practice quite simple. All documents are complete and so readily available for reference that a title can often be investigated and the loan put through in a few hours.

Turning to another phase of the subject in the Province of Quebec, it is to be noted that from the point of view of security, and extreme punctuality of payment, the Roman Catholic Church offers the most desirable "mortgage" securities in Canada. The credit of that Church being so high she borrows at an average rate under 4 per-cent in normal times. This low rate offers no attraction to the Canadian insurance companies, and but small inducement to the few British companies lending in Canada. Loans are sometimes granted out of British insurance monies to Roman Catholic bishops in corporate capacity or to a "fabrique." The latter is a legally recognized corporation governing the church parish and administering its property, which usually comprises the schools, presbytery, church and cemetery, all, or nearly all of which, it has power to hypothecate.

These hypothèques are held in high regard by Protestant and Catholic funds alike. No instance of the default of a fabrique is on record, nor have I ever heard of borrowers finding it necessary to exercise their constitutional powers of commandeering the private lands of the Roman Catholic holders within the parish for the service or support of the loan. It is generally expected that short of the moral and legal conversion of the province from Roman Catholicism, these ecclesiastical securities will retain the high degree of favour they have so long enjoyed.

I pass now to the author's remarks on licensing and registration. It is interesting to observe that a recent decision of the Privy Council in re John Deere Plow Co. and British Columbia Province regarding companies incorporated under a Dominion charter cuts out the fees referred to by the author. As bearing on this point it is well to remember that a revision of the Dominion Companies' Law has for some time been on the *tapis*, and cannot under normal conditions be much longer delayed. Again, the procedure which an office would adopt in starting loan operations in the Dominion would be influenced by its estimate of the issues in the pending suit between the Provincial and the Federal Governments. Lastly, it is to be observed that the Canadian Life Offices' Association has recently decided upon steps to test the constitutionality of powers claimed, and hitherto somewhat vigorously exercised, by the Provinces in their taxation of federally licensed as well as federally incorporated companies.

Whilst advocating the investment of pooled funds through the medium of companies federally incorporated (a Charter for which would be comparatively inexpensive) I incline to the view that in the case of an individual British Life company much can be said for the acquisition of lending facilities under cover of a federal licence to transact business in terms of the Dominion Insurance Act.

On the subject of mortgage security margins the Insurance Act, I think, stipulates for a minimum of 40 per-cent. Good enough in normal times on sound valuations of central city properties of a marketable type, such a margin would generally be too small in respect to properties at the other end on the lending scale.

It may serve a purpose to note the attitude of the Canadian insur-

ance companies to Canadian mortgage securities. In the Dominion Blue Book assets are shown in considerable detail and disclose some rather remarkable proportions of the several companies' mortgages to invested assets. In the case of one of the largest companies such proportion is so small as to be negligible. On the other hand the proportion in the case of some of the smaller companies runs as high as 75 per-cent and upwards. Over the whole, the average at the opening of the century was well over 30 per-cent and is now closer to 40 per-cent. It is noticeable too that companies within easy reach of the North West show in several cases a substantially higher interest yield than their rivals, thus pointing to the natural conclusion that the higher rated classes of mortgage referred to by the author are most favoured by the companies geographically nearest thereto.

LEGAL NOTES.

By WILLIAM CHARLES SHARMAN, F.I.A., *Barrister-at-Law*.

THE case of *In re Blackburn Philanthropic Assurance Company (Limited)* (1914), 2 Ch. 430, deals with an interesting point as to the power of a Friendly Society which by special resolution has converted itself into a company, to extend and alter the objects of its memorandum afterwards by special resolution, there being at the time no register of shareholders, and none of the shares having been allotted. The case came before the Court as a petition for the confirmation by the Court of the special resolution altering the memorandum of association. It was held that the conversion of the Society into a limited company did not involve the simultaneous conversion of the members of the Society into members of the company, and there was no special resolution properly passed which the Court could confirm. The facts of the case are as follows:—

Conversion of
Friendly Society
into Company.
Power to alter
the objects of
Memorandum.

The *Blackburn Philanthropic Friendly Society* was established in the year 1839 under the name of the *Blackburn Philanthropic Burial Society*. It first commenced to transact life assurance business in 1863, the sums payable at death varying from £8 to £100. The limit of assurances was raised in 1888 to £200, and in 1912 to £300. Between these two dates the Society had changed its name by the substitution of the words "Friendly Collecting" for the word "Burial" in its original title, and had on the coming into operation of the Friendly Societies Act, 1896, and the Collecting and Industrial Assurance Companies Act,

1896, become subject to the provisions of both Acts. The Society operated in the counties of Lancaster, York, Chester, and Leicester exclusively, and had attained large dimensions; the members numbered over 373,000, and the average amount of funds per member was about 35s. It was considered advisable to extend the objects and increase the powers of the Society, and the board of management were advised that the proper procedure to attain these ends was to convert the Society under Section 71 of the Friendly Societies Act, 1896, into a Company limited by shares, and then alter the provisions of the memorandum of association with respect to the objects of the Company under Section 9 of the Companies (Consolidation) Act, 1908.

On 6 and 27 August 1913, a special resolution was passed converting the Society into a Company limited by shares, and on 2 September 1913, the Company was registered as the *Blackburn Philanthropic Assurance Company, Limited*, with a capital of £1,000,000 in £1 shares. The objects of the company were restricted to the objects of the society, and the memorandum was registered without the names of any subscribers being appended thereto.

The articles of association registered with the memorandum contained *inter alia* the following provisions:—None of the original shares shall be offered to the public for subscription; the shares shall be allotted and issued as fully paid to all members of the society in proportion to their total contributions paid in respect of existing assurances. Application forms for shares were distributed amongst the members, but owing to incorrect statements, and the necessity of verification, it was estimated that two years might elapse before the allotment of shares could be made.

A special resolution altering and extending the objects clause of the memorandum was passed, and confirmed at extraordinary general meetings of the company held on 17 February and 5 March 1914. This resolution came before the Court for confirmation. Eve, J., in the course of his judgment, said:—

“There is nothing, in my opinion, in s. 71 of the Friendly Societies Act, 1896, to support the view that the conversion of the society into a company limited by shares involves the simultaneous conversion of members of the society into members of the company. Apart from the practical difficulty amounting in the present case to the impossibility of determining within any reasonable time after the conversion the

“pecuniary interest of each member in the company’s share
“capital, and of complying with the statutory requirements as
“to opening and posting up the register of members, and making
“the statutory returns, such an interpretation would introduce
“a remarkable modification into the law of joint stock com-
“panies. By s. 24 of the Companies (Consolidation) Act, 1908,
“the two classes of persons who are to be deemed members of
“any company under that Act are (1) the subscribers to the
“memorandum, and (2) every other person who has agreed
“to become a member of the company ; but if the registration
“of a society as a company operates to convert the members
“of the society into members of the company, there is a third
“class of persons who are to be deemed members of ‘ a company
“under the Act’ ; that is to say, persons who have not signed
“the memorandum of association, and who have never agreed
“to become members of the company ; persons, indeed, who
“may have actively opposed the conversion and who would never
“voluntarily agree to become members of the company. I
“cannot bring myself to believe that a conversion under s. 71
“was ever intended to have such consequences, and I hold that
“no results follow therefrom.

“It is said that this conclusion leaves the members in a very
“unenviable position, as the society of which they were formerly
“members no longer exists, and they have no interests in the
“company which has superseded it. I do not agree, for I think
“that under the concluding part of s. 71 the members retain the
“same rights against the assets of the society in the hands of the
“company as they would have had against them had the society
“continued to exist as a registered friendly society. The
“release embodied in the form of application for shares adopted
“in the present case shews that this view is shared by the
“company’s advisers.

“The point, therefore, which has been raised falls to be
“decided under the Companies Act and not under the Friendly
“Societies Act, and there being no persons who have sub-
“scribed the company’s memorandum of association, the matter
“is resolved into the inquiry whether there are any persons who
“have agreed to become members of the company. An agree-
“ment to become a member of or, in other words, to take shares
“in a company, may be come to in many different ways ; in
“this, as in most cases, it is contemplated that the contract
“will be founded on an application for shares by the intending

“member. In a contract so founded there are two other
 “essentials beyond the application for shares; that is to say,
 “an allotment and a communication to the allottee of the fact
 “that the allotment has been made. In the absence of any
 “one of these essentials there is no contract. Here, although
 “there have been applications made for shares, neither of the
 “other essentials is present in a single case, and in my opinion
 “it is impossible on these facts to hold that any person has as
 “yet agreed to become a member of the company. Some
 “have undoubtedly offered to do so, but their offers have not
 “ripened into contracts, and can still be withdrawn. In my
 “opinion, no effective resolution was passed or capable of being
 “passed at the Meetings of 17 February and 5 March, to both
 “of which were summoned not only the members of the company,
 “but all persons entitled to attend and vote thereat—that is to
 “say, every person who had been a member of the society for the
 “full period of six calendar months immediately preceding the
 “incorporation of the company.

“On the short ground, therefore, there is no resolution
 “which the Court can confirm.”

The decision of Mr. Justice Lawrence in the case of
Harrington v. Pearl Life Assurance Company (Limited), (1913),
 30 T.L.R. 24 (*J.I.A.*, vol. xlviii, p. 105), was affirmed
 by the Court of Appeal, 30 T.L.R. 631. The case
 deals with the position of an assurance company
 which has accepted the first premium on an assurance
 after the commencement of the fatal illness of the life proposed,
 and in ignorance of that illness. The facts are fully reported
 in the previous note on the case.

In expressing agreement with Mr. Justice Lawrence that the
 plaintiff had no case, Buckley, L.J., said there was no question
 that the policy which defendants in their letter of 18 October
 said would be duly forwarded on receipt of premium provided
 that it was received within 30 days was a policy in the ordinary
 form as issued by this particular company, one of the conditions
 of which was that there was to be no insurance until the premium
 was paid. There was therefore no policy of insurance in force
 until the premium was paid. On 18 October there was in
 existence a contract between Bentley and the defendants, and
 the bargain was that if Bentley complied with certain stipulations
 he should have a policy. The premium was paid on 8 November.

Payment of first
 premium after
 commencement
 of fatal illness.
 Position of assur-
 ance company.

At that time who was the person who could pay it to the defendants? In his opinion, it was Bentley alone. The facts then existing were these, that Bentley was lying ill in a hospital, and that he died the same evening. The premium had in fact been paid by plaintiff at midday, but it was clear that to the knowledge of Bentley at the moment when the money was paid, the facts had so altered, and the circumstances had so changed, that the defendants would be entitled to say that they would not receive the premium, and that the doctrine laid down in *Canning v. Farquhar* (2 T.L.R. 3 and 6, 16 Q.B.D. 727) was applicable. The case thus stated had, in his opinion, no substance in it, but something else had happened to which he must refer. It appeared from the evidence that Bentley, who had from time to time borrowed money from plaintiff, executed an assignment on 4 November to the plaintiff of all his interest in the policy for £300. The rota number mentioned was the number shewn on the proposal form. In point of fact, no policy was then in existence and nothing was assigned, and, although the plaintiff had himself paid the premium, in law it was Bentley who made the payment, and the plaintiff was in no contractual relation with the defendants. The policy, if issued, would have been Bentley's policy, and the circumstances were such that Bentley could not have taken it out because if the defendants had known the facts they would never have issued the policy.

In his judgment the appeal failed, and must be dismissed.

Lord Justices Kennedy and Phillimore agreed.

The question as to whether section 36 of the Assurance Companies Act, 1909, operated to validate policies obtained by fraud was considered by the Court of Appeal in the case of *Tofts v. Pearl Life Assurance Company (Limited)* 31 T.L.R. 29.

The facts of the case are as follows:—In 1902 the plaintiff was informed by an agent of the defendants that policies covering the mourning expenses to which he would be put on the death of his father and mother would be valid, and thereupon he agreed to take them out. The policies in fact provided for funeral expenses and not mourning expenses. Section 36 (2) of the Assurance Companies Act, 1909, provides that “No policy effected before the passing of this Act . . . shall be deemed to be void by reason only that the person effecting the policy had not at the time the policy was effected an insurable interest in the life of the

Assurance Companies Act, 1909.
Validity of policies.

“person assured . . . if the policy was effected by or on account of a person who had at the time a *bonâ fide* expectation that he would incur expenses in connexion with the death or funeral of the assured.”

In a County Court action by the plaintiff to recover the premiums, the Judge found that there had been fraud on the part of the defendants' agent, and he gave judgment for plaintiff. On the question being brought before the Divisional Court the Court differed. Mr. Justice Lush held that the policies were made valid by the above section, and the plaintiff could not recover. Mr. Justice Bray held that the section did not validate policies obtained by fraud and the plaintiff was entitled to succeed. As the Court differed, the appeal was dismissed.

The defendants appealed, and the Court of Appeal dismissed the appeal without calling upon counsel for the respondents.

Lord Justice Buckley in giving judgment said that in his opinion the judgment of Mr. Justice Bray in the Divisional Court was quite right, and he would have been quite content to adopt it as his own, but he ought undoubtedly to give his judgment in his own words. In the first place the defendants said that there was no evidence to support the County Court Judge's finding of fraud. It was quite plain that there was an issue between the parties whether the insurances were intended to cover mourning or whether they were meant to provide a fund for funeral expenses. The case set up by the plaintiff at the trial was that the policies were intended to provide for mourning and there was ample evidence to show that the defendants' agent knew that that was the plaintiff's intention. In his (the Lord Justice's) opinion, the County Court Judge was entitled to draw the inference that the agent, well knowing what the intention of the plaintiff was, represented to his employers that the policies were intended to cover funeral expenses which were an insurable interest. In his opinion, there was ample evidence to justify the County Court Judge's finding as to fraud.

It had been argued in the second place, on behalf of the defendants, that even if there were evidence of fraud, provision had been made by section 30 of the Act of 1909 to validate such policies. His Lordship then read the section, and said that it had been argued that these policies would not be void by reason only that the person effecting them had not at the time the policies were effected an insurable interest in the lives of

the persons assured, if they were effected on account of a person who had at the time a *bonâ fide* expectation that he would incur expenses in connexion with the death or funeral of the assured. In his opinion, the effect of the section was to remove the objection arising from the fact that the person effecting the assurance had not an insurable interest, but if the policy were capable of attack on the ground of fraud, such right of action was not taken away. In an unreported case of *Johnson v. The Refuge Assurance Company*, the Court of Appeal had expressed an opinion in consonance with the view he had just expressed, namely, that the Act had no effect on a plaintiff's right to sue in fraud.

The third point that had been taken was that after the plaintiff had been paying premiums for some time he thought the policy might be illegal, but after some correspondence with the company he continued to pay the premiums on the footing that the policies were valid and legal, and that that constituted a waiver of his rights. In his (the Lord Justice's) judgment there was no waiver on the part of the plaintiff, and the action brought by him was maintainable. On the facts as found by the learned County Court Judge the plaintiff had established fraud, and was entitled to rescind his contract. The appeal must therefore be dismissed.

L.JJ. Phillimore and Pickford delivered judgment to the same effect.

It is satisfactory to be able to note that the moratorium has come to an end. The final Proclamation was dated 30 September 1914, and authorized a limited extension of the
The Moratorium. period by which certain payments could be postponed. By the terms of this Proclamation the operation of the Proclamations dated 6 August and 12 August (*see J.I.A.*, vol. xlviii, page 418), was applied to payments which became due and payable on or after 4 October 1914, and before 4 November 1914, with the following exceptions :—

(a) Any payment in respect of rent.

(b) Any payment due and payable to and by a retail trader in respect of his business as such trader.

It was further provided that in respect of payments which had already been postponed under the previous Proclamations, the person from whom the payment was due should not be

entitled to claim the benefit of the further postponement unless interest was paid up to date.

Bills of Exchange (other than cheques and bills on demand) accepted before 4 August 1914, the original date of maturity of which was after October 3, if not paid on presentation, and provided the acceptor had not expressly refused reacceptance thereof, were postponed for one calendar month.

If, however, the bill had already been postponed under the previous Proclamation, the period of further postponement was to be 14 days.

The New English Life Tables.

THE Supplement to the Seventy-fifth Annual Report of the Registrar-General of Births, Deaths and Marriages in England and Wales, Part I (Cd. 7512), is of special interest to Actuaries as being the first of this long series of official reviews of the vital statistics of the nation in the preparation of which the professional services of the Actuary have been requisitioned. It will be agreed that in making a departure from established practice and in seeking expert assistance in an essentially technical matter, the Registrar-General has exercised a sound judgment; and there will be a general disposition to congratulate Mr. George King upon the notable service to the State which, in consequence, he has been enabled to render.

Mr. King's report consists of an interesting examination of the Statistics of the Censuses of 1901 and 1911 and of the deaths of the ten years 1901-10 (these being the data for the Life Tables which, conventionally, were the principal purpose of the investigation), a description of the method of construction adopted, a discussion of some of the more important deductions derivable from the Life Tables and an account of the author's short method of constructing abridged Life Tables which was first made known to the profession in *J.I.A.*, vol. xlviii, page 294. Following the report are fifteen complete Life Tables, the first seven of these being based, respectively, upon the Populations enumerated at the two Censuses 1901 and 1911 and the deaths in the intervening ten years (English Life No. 7), the population at 1911 and the deaths of the three years 1910-12 (English Life No. 8), Males and Females being in each case dealt with separately, and the enumerated spinsters, married women and widows at the Census of 1911 with the corresponding deaths in the three years 1910-12. The remaining eight tables, described as Sectional Life Tables, are based upon the estimated population at risk in the years 1911 and 1912, and the deaths in these two years (Males and Females being dealt with separately) in (a) the Administrative County of London, (b), the aggregate of County Boroughs, (c) the aggregate of Urban Districts and (d) the aggregate of Rural Districts.

The general result of the investigation is to show a marked decline in the national rate of mortality at all ages, and in respect of both sexes. as compared with that disclosed by previous investigations. This decline is shown to have been continuous during the ten years, for not only are the death rates of English Life Table No. 7 lower than those of No. 6, but No. 8 Table shows a pronounced improvement over No. 7, which latter Table, it may be suggested, is consequently already rendered obsolete, for most practical purposes. Mr. King finds further evidence of continuity in the decline of the death rate in the fact that the rates of mortality indicated by the Tables prepared for the purposes of the National Insurance Act (*J.I.A.*, vol. xlvii, pp. 549-552) and which were based upon the estimated population at 30 June 1909, and the deaths of the three years 1908-9-10 occupy an intermediate position between those exhibited respectively by English Life Tables No. 7 and No. 8.

The scientific interest of Actuaries in the new Tables will naturally tend to concentrate itself upon the method of construction. This is very fully described in Mr. King's paper "On a New Method of Constructing and of Graduating Mortality and other Tables" (*J.I.A.*, vol. xliii, p. 109), and its characteristics are analysed, in relation to its present use, in the review of Mr. King's graduation of the ages recorded in the Census of 1911 appearing on pages 207-214 of volume xlviii of the *Journal*.

It is probable that the subject of graduation will always excite controversy, but even those whose preference is for mathematical, as opposed to what have been called mechanical, methods of adjustment, will be prepared to concede, doubtless, that where the data from which a Life Table is to be constructed are presented in the form in which, alone, Census particulars have on previous occasions been available, the facts being recorded in groups combining the figures relative to five or ten ages, the method devised by Mr. King possesses the substantial merit of supplying an easy and effective method of transition from group values to individual values of the function under treatment. Mr. King's statement in regard to the curve given by his method that "it would be difficult to imagine a curve of smoother graduation, unless it were a curve really of the third order with a constant third difference", seems, however, open to question since it is easy to find mortality curves, not of the third order, which on the criterion chosen by Mr. King, namely, the progression of $\Delta^3 q_x$, produce decidedly smoother results than those brought out in the new Tables. This may be seen by comparing the values of $\Delta^3 q$ for his English Life Table No. 8 (Males) with the values for the National Insurance Table referred to above. In the English Life Table the sum of the third differences from ages 20 to 59 (ignoring + or — signs) is 57; and there are 17 changes of sign; between ages 60 and 79 the sum of the third difference is 129, and changes of sign occur six times. In the National Insurance Table (which also was constructed from Census data) the sum of the corresponding differences at ages 20 to 59 is 36 and there are 14 changes of sign; in the group of ages 60 to 79 the sum of the differences is 62 and there

are no changes of sign. The point is not of great importance, but it seems desirable that it should not pass unnoticed.

Again, lest any one method of treating Census data should be vested with an authority to which it can only become entitled by comparison with other methods and elimination of the less fit, it seems advisable to point out that since 1909 some important contributions to this branch of actuarial science have been made and that, consequently, the matter hardly stands where it did when Mr. King produced his method. The lectures of the late Sir G. F. Hardy, K.C.B., on the Theory of the Construction of Tables of Mortality, &c., have been published by the Institute; and the principles developed in the course of his lectures by Sir George Hardy, than whom, certainly, there was no greater authority on this subject, have been adopted, by the Actuarial Advisory Committee, for the preparation of the important National Insurance Life Tables above described. The method of construction employed in this case, or any similar method of the same class, namely, curve-fitting on modern statistical principles, differs fundamentally of course from that adopted by Mr. King. While this is not the place in which to attempt to pass judgment it may be suggested that the subject should still be regarded as open to discussion. Mr. King's method obviously affords a ready and attractive instrument for the construction of life tables where the data are suitable, both in form and in volume, and it will always be sure of its place, therefore, in the equipment of the practising actuary. But when the purpose in view is the graduation of a great National Life Table it may be suggested that facility and rapidity of operation are minor points, and that no labour is in vain which is directed to the ascertainment of a method of graduation that will command general assent as the best obtainable in all the circumstances.

Underlying, however, the question of graduation, if not, indeed involving it, is the larger subject of the proper treatment of population statistics for the effective dispersion of errors (the use of the word "elimination" in this connection seems out of place) as a necessary preliminary to the preparation of Life Tables. Errors of age of what may be called a local character, such as those resulting from the partiality for even numbers which has been brought to light by the fulness of detail in which the last enumeration of the population has been treated, are no doubt distributed very successfully by Mr. King's method. But the method cannot be said to touch the large systematic errors which, it has always been believed, have been introduced into the Census statistics by the deliberate and conscious misstatement of age in which particular sections of the population, notably women, have indulged. Apparently it is Mr. King's opinion, based upon the age-distribution shown by the last Census, that systematic errors of this kind are not present in sufficient volume to exercise any material influence upon the figures. Indeed, the supposed tendency among women to understate the age at certain periods of life is not even mentioned in either of Mr. King's reports dealing with the present Census, an obvious excess

of the women enumerated in England and Wales at early adult ages being attributed to immigration from Ireland and Scotland or from abroad.

Migration is of course an element to be taken into consideration in the study of all population statistics. Where the respective numbers of emigrants and immigrants differ considerably, or where there are marked differences between the age distributions of the two classes, some effect upon the censal distribution of ages is to be expected, and it may happen that disturbances of this kind, especially if concentrated in certain age-groups, will exercise such an influence as to make it impossible legitimately to replace the enumerated population by a distribution following a mathematical law. If this be the position in any particular case, and if it can be established that nothing but real movements of the population account for apparent abnormalities in the distribution, it would appear that in these circumstances a mechanical process can with entire propriety be applied to the figures as they leave the hands of the enumerator. But the premises here enunciated are surely fanciful; it is, at least, difficult to believe that no deliberate understatements of age lie concealed in the schedules when the number of women enumerated in England and Wales in 1911, at the ages 20-24 exceeds by some 2,300 the number of girls enumerated at the ages 10-14 in 1901. [The actual figures are: in the 1901 Census, females of ages 10-14, 1,670,770; in the 1911 Census, females of ages 20-24, 1,673,066.] Official statistics appear to show that the number of emigrants from the British Isles generally exceeds the number of immigrants and since the latter appear to include a large proportion of British subjects, many of whom may be taken to be returning to the mother country, the excess of emigrants over immigrants at the young adult ages is probably very great. So far, therefore, the relevant statistics appear to tell against the theory on which the assumption of accuracy in the recorded figures is based. Nor is the theory much helped, apparently, by the statistics of migration from Ireland to England. The total number of emigrants from Ireland (male and female) in the year 1911 is stated by an official return (Cd. 6131) to have been 31,058. The Irish-born in this total were 30,573, and of these only 1,604 went to England. The year 1911 was apparently in no way abnormal, and so far as emigration from Ireland is concerned the effect upon the English figures must, on these statistics, be regarded as altogether insignificant. While no parallel figures relating to Scotland are available for comparison there is no reason to think that emigration in this case has, to any pronounced extent, taken a merely southern direction. Many observers will be disposed, therefore, to conclude that the explanation of the curious run of the figures which is suggested in the previous report, dealing with the graduation of the ages returned at the Census of 1911, is somewhat inadequate. Some light is thrown upon this interesting problem by the following table, which has been prepared to show the numbers of persons expected to have survived the ten years 1901-11, on the assumptions (*a*) that the ages of the

population were correctly recorded in 1901, (b) that the subsequent mortality was that shown by Mr. King's English Life Table No. 7, and (c) that the population was undisturbed by migration, inwards or outwards, during the ten years separating the two Censuses.

Ages at 1911	MALES				FEMALES			
	No. (in thousands) expected to have survived from 1901	No. (in thousands) enumerated	Difference		No. (in thousands) expected to have survived from 1901	No. (in thousands) enumerated	Difference	
			+	-			+	-
15-19	1,698	1,655	..	43	1,706	1,682	...	24
20-24	1,620	1,503	...	117	1,624	1,673	49	...
25-29	1,543	1,456	...	87	1,582	1,623	41	...
30-34	1,100	1,376	...	24	1,579	1,501	...	78
35-39	1,246	1,261	15	...	1,417	1,352	...	65
40-44	1,067	1,075	8	...	1,190	1,158	...	32
45-49	930	926	...	4	1,020	999	...	21
50-54	779	768	...	11	853	834	...	19
55-59	625	608	...	17	699	670	...	29
60-64	483	477	...	6	560	543	...	17
65-69	337	366	29	...	410	441	31	...
70-74	234	237	3	...	305	317	12	...
75-79	122	127	5	...	171	182	11	...
80-84	56.9	56.4	...	0.5	86.5	87.8	1.3	...
85 and over	22.9	22.8	...	0.1	40.0	41.1	1.1	...

Looking at this Table in detail, it is seen that the male population appears to have suffered a heavy drain at ages under 35; and it is presumably correct to ascribe this, in the main, to emigration, there being no particular reason to suppose that in these groups or in the corresponding groups enumerated at 1901, the ages given were inaccurate to any substantial extent. Among the women, while a loss in the youngest group in the Table possibly points to emigration, there is apparently a large accession to the population aged 20-29, and this notwithstanding the probability that at these ages the excess of emigration over immigration was responsible for a considerable loss of population. (See Board of Trade Return on Emigration and Immigration, June, 1914, wherein it is shown—page 11—that 46,530 women of ages between 18 and 30 emigrated to the British Dominions, overseas, or to non-European countries during the year 1913. The corresponding number of immigrants was only 7,619; and the total figures of immigration from European countries are not such as to suggest that the influx from this source would redress the balance.)

This apparent accession to the population of women at ages between 20 and 30 is evidently connected with the apparent loss of population at ages over 30 and seems to point to the transference of large numbers from one group to another as the result of deliberate understatements of age.

Among the men there is an apparent accession to the population at ages 35 to 44 and this probably is real, the result perhaps of the return home of men (then aged between 25 and 35) who were engaged in the South African War at the time when the Census of 1901 was taken and the number of whom was probably sufficient to convert the deficiency of immigrants, as compared with emigrants, at this age group, into a surplus. It may be thought perhaps that the "mean population" as obtained from the enumerations of the population at 1901 and 1911 should have been corrected at the age groups 30-44 to allow for the disturbance of the theoretical conditions resulting from the sudden increment to the population of the forces which returned to the United Kingdom in the early part of the decennium, as shown by Table 34 of the 75th Annual Report of the Registrar-General (Cd. 7028). A possible weakness in the "mean population" method of obtaining the "exposed to risk" under these exceptional conditions is, of course, emphasized by existing circumstances. With a greatly increased Navy and Army serving away from the Country, the present war may result in a disturbance of such magnitude (especially if, unhappily, the conflict should be greatly prolonged) as to make it necessary to introduce a correcting adjustment into the process of calculating the "exposed to risk" by the "mean population" formula when the next occasion arrives for an investigation of the national life experience.

At the ages between 45 and 65, there is, in the case of both men and women, an apparent loss of population followed by a curious accession at all the higher ages. It is impossible to dissociate this feature from the introduction of Old Age Pensions in 1908. Whether the Census of 1901 or that of 1911 is the more correct in regard to these instances of apparent discrepancy, it is of course impossible to suggest without prolonged investigation, but it is worthy of observation that in noting a similar feature in the Report of the Census of Scotland for 1911 the Registrar-General for that country states that certain irregularities in the groups 65-70 and 70-75 indicate "that the returned ages of these periods require explanation, or at least that there has been some influence tending to correct misstatement in the returns of 1901 or to introduce it in the returns of this Census." With the further suggestion of the Registrar-General for Scotland that the passing of the Old Age Pensions Act, which took place between the two Censuses, is not unconnected with the feature, the case for some preliminary adjustment of the enumerated figures, when the production of National Life Tables is about to be undertaken, may be held to have received some degree of official recognition.

Mr. King's observations as to the Census figures at the infantile ages lend further support to the proposition. In an interesting series of comparisons between the numbers enumerated and the expected number of survivors (as obtained from the births taken over a series of years and the corresponding deaths at infantile ages) he shows that while from the age of 2 onwards, the number of children enumerated accords fairly with the expectation, the number returned as living at the ages 0 and 1 is seriously short of that expected—the

difference adjusted to 1 July 1911 being, for both sexes, over 85,000, or between 5 and 6 per-cent of the number recorded. It is better to refer students to the report itself, in which an admirably clear statement of the problem and its treatment is set forth, than to explain the method by which this difficulty was dealt with, but the fact that a difficulty arose, and not for the first time, or one confined to the Census of the "predominant partner" in the United Kingdom, is sufficient to indicate the possibility of the figures containing other large systematic errors. In view of the evident general inaccuracy of the Census figures as to infants it is an interesting point that an apparent inconsistency in the relative numbers of boys and girls enumerated at the Censuses of 1901 and 1911 can be satisfactorily explained. At the ages under 5, the boys enumerated at 1901 were fewer than the girls by some 6,000; while in 1911 at the same ages, the boys enumerated exceeded the girls by about 18,000. Scrutiny of the relative numbers of births and deaths, treated on a plan similar to that adopted by Mr. King, suggests that in 1901 the respective numbers of boys and girls should have been almost exactly equal, while in 1911 the boys should have outnumbered the girls by about 21,000. The relative increase in the number of boys by about 24,000 is thus, in the main, substantiated, and may be regarded as confirming the excess of the decline in the rate of mortality among males at the infantile ages as compared with that among females, which is otherwise shown by a comparison of q_x in the English Life Tables, Nos. 6 and 8 respectively.

Before leaving this subject it seems worth while to press the advantage of the use of such a Table as the foregoing for the purpose of studying the distribution of the population with regard to age, as against the plan which in his previous report Mr. King appears to have adopted, namely, that of comparing the differences between the graduated numbers living at successive ages at the same Census. Even if full reliance could be placed on graduation, as a means by which accidental errors may be removed without introducing other errors, it would seem difficult to draw any dependable inference as to the effect of migration, from a scrutiny of the changes in the numbers living between one age and another, at the same date; for migratory movements, while differing widely in respect of ages separated by considerable intervals, will affect contiguous ages in very much the same degree. But apart from this point it may be remarked that, in addition to dispersing errors, graduation, in the case under discussion, disposes of such natural differences in the numbers enumerated as would be expected to result from the differences in the numbers born from year to year. The extent of these differences over a long series of years is shown by Table 5 contained in the Annual Report of the Registrar-General for England and Wales for 1912. By the aid of this Table an interesting comparison can be made between the number of births in any year ending on 1st April with the numbers enumerated at the corresponding age last birthday on 1st April, 1911, and as illustrating the point here raised the following table has been prepared to show the estimated number of births

of males in each of five years ending 1st April, 1892-6, respectively with the numbers of males living at ages 15-19 at the Census of 1911, both as enumerated and as graduated by Mr. King.

Males (in thousands.)

Year of Birth ending 1 April	No. of Births (as estimated)	No. living at 1 April 1911		Age last Birthday at 1 April 1911
		As Enumerated	As Graduated	
1892	466	323	316	19
1893	457	333	323	18
1894	465	330	330	17
1895	454	335	335	16
1896	470	334	340	15

It will be seen that in the graduated figures of the numbers living at 1911 all trace of the yearly variations in the number of births has been lost. While this may be relatively unimportant so far as the rate of mortality at particular ages is concerned, where, as in the case of English Life Table No. 8, the numbers of both the living and the deaths have been adjusted by the same formula, it limits materially the statistical uses to which the differences between the graduated figures can safely be put.

The fact that in the table here given the enumerated figures show minima at those ages at which the corresponding births show maxima is not easily explained. It is possible that we have here some indication of the extent to which age *next* birthday is returned on the Census schedules instead of age *last* birthday, an error to which attention is directed by the Registrar-General in his general Report upon the Census. [Cd. 6610.]

Passing from the enumeration figures to the deaths, it is worthy of note that, for the first time in the history of the subject, the operator on censal statistics has been enabled to make use of returns giving the number of deaths registered at each year of age. These data are available for the years 1910-11-12, and Mr. King has made use of them for the purpose of sub-dividing the figures given in decennial groups for the years 1901-10 in order to obtain the deaths in quinary groups corresponding with the enumerated populations. For the English Life Table No. 8 Mr. King was enabled, in view of the completeness of the data, both as to "exposed to risk" and deaths, to choose such quinary groupings of the ages as enabled his formula of distribution to be employed to the greatest advantage, having regard to the apparent character of the errors of misstatement by which the figures were affected. The particular groups chosen are those commencing with "4" and "9" as the second digit in the age, and these have been employed also to exhibit the results of the comparison between "actual" and "expected" deaths which is adduced as one criterion of the success of the graduation. It is perhaps somewhat disturbing to find that the age groups

in which these results are shown correspond with the points selected for the "pivotal values" by which the graduation was effected. There seems to be no conclusive reason for adhering, in a mere summary of results, to the particular grouping which was used for the graduation; unless, indeed, it is to be asserted that the "actual deaths" are given with a close approach to accuracy in these particular groups, while in any other groupings, they would be given very inaccurately. In view of past discussion upon the method, it may, perhaps, be suggested that more definite evidence as to the success of the graduation would have been afforded had Mr. King first graduated the numbers of deaths (which present a most irregular appearance), for the removal of the "local" errors that, not improbably, constitute their chief defect, then presenting the aggregate of the deviations summarized in other age groups than those lying between the points for which the "pivotal values" were derived.

The series of Life Tables for Females deduced from the population (as estimated) in 1911 and 1912 and the deaths of those years is of particular interest as showing separately the rates of mortality among spinsters, wives and widows. The experience of spinsters reveals a lighter mortality than that of wives (though the difference is not very marked) at practically all periods of life up to age 70 excepting that comprised within the ages 45 to 55, but throughout the scale until about age 80, the mortality among widows is apparently much greater than that prevailing among either spinsters or wives. These differences are striking, and, especially in the case of widows, much may be expected to be made of them. Actuaries, however, will view this feature with caution. It is evident that the spinster class is being continuously depleted by marriage and the same is true of the widows, among whom, indeed, as Messrs. Hardy and Wyatt have shown in the Tables appended to their report upon the National Insurance Bill, the rate of re-marriage is very considerably higher than the rate of marriage prevailing among spinsters. Here, therefore, there is a continuous exercise of selection, which in the case of widows must exert a profound influence as withdrawing from the group its healthiest and most vigorous members. While the results brought out are, of course, true so far as they go, the explanation of them, approached from this standpoint, may be distinctly less subject to emotional colour than the bare statistical conclusions, as presented in Mr. King's report, suggest. It would have been an advantage therefore if the report had contained an explanatory note such as might serve to put the uninformed upon their guard. In particular, the presentation of the function e_x , deduced from these aggregate statistics, appears open to criticism. Whatever may be the real "expectation of life", on the general average, in the case of a large group of young spinsters, the majority of whom will pass into the "married" group, some into the "widowed" group, and some back again into the "married" group, with further alternations between these last two groups in the case of a very small proportion, it certainly cannot be presented with regard to scientific accuracy by a calculation made from "spinster"

statistics at all periods of life. At most, such a calculation gives the "expectation of life" of the minority, wholly indeterminate at the date of origin, who will remain spinsters, and its value as so limited is, at least, problematical. The same consideration applies *mutatis mutandis* to the corresponding function tabulated in respect of wives and widows.

The sectional life tables mark a distinct advance upon the former plan under which the one general life table applicable to each sex was supplemented by a table drawn from the experience of selected "Healthy Districts." Under that method of compilation, the death rate prevalent over the whole area was the criterion by which was determined the inclusion or otherwise of the experience of a particular district in the supplementary tables, and the latter proved nothing, therefore, except the relative incidence of mortality at different periods of life among a body of lives which, taken as a whole, was already known to be subject to specially light death rates. The method was open, therefore, to the rather serious criticism, from which the new plan is wholly free, that it settled beforehand what the main features of the sectional table were to be.

Setting apart the "London" tables, the sectional tables in their apportionment into Rural Districts, Urban Districts and County Boroughs may be regarded as corresponding with the Rural, Town and City Districts into which the older friendly society experiences were divided; and they exhibit, as would be expected, the same relative features, the rates of mortality being decidedly lower in the Rural Districts than in either of the other sections, while the Urban Districts show a lighter mortality than that prevailing in the County Boroughs. It is not, however, evident that the differences between the three tables can safely be regarded as affording a direct measure of the relative effects of existence under rural and urban conditions. Many factors, of course, are operating upon the death rates, each tending to produce its own result upon the particular section of the community which it affects; and where these factors are not operating with equal force over each of the four sections of the new classification their influence cannot be regarded as negligible in a comparison of the respective sectional rates of mortality. Occupation is one such influence, but materials by which the effect of occupation, as entering into the sectional experiences, could be disentangled from other particular influences are not available. Geographical differences, on the other hand, can be investigated to some extent by the aid of the Manchester Unity Tables, published a few years ago, in which considerable prominence was given to this element. It is true that the presence of occupational influences was indicated by this experience, but the distribution of occupations between the three areas was, on the whole, so general, as evidenced by the table on page 33 of the report, that differences of mortality due to this cause can almost be disregarded as an element affecting the comparison between the general average experiences of the different areas.

The effect of the geographical element in its operation upon aggregate tables is at least interesting. From the abstract of the

values of e_x contained on page 60 of the report on the Manchester Unity Experience it appears that while the differences, at the age of 25, between the "rural" and "urban" groups are, in area 1, 1.19, in area 2, 1.63, and in area 3, 1.37, the difference between "rural" and "urban" for the whole society is 2.14, being thus considerably greater than the corresponding difference in the case of any one of the three divisions of the data. The reason is of course that the "exposed to risk" contributed by the several areas entered respectively into the combined (whole society) "rural" and "urban" groups in wholly different proportions, as may readily be seen by Table C, pp. 160-175 of the report. Thus area 1, which showed much the lightest mortality in both sections contributed 63 per-cent of all the "rural" exposures, but only 25 per-cent of the "urban" exposures; while area 2, which showed, again in both sections, the heaviest mortality, contributed but little over 9 per-cent to the "rural" data, against 25 per-cent to that of the "urban" group. Area 3, which in regard to mortality occupied a midway position between the other areas, contributed to the "rural" and "urban" sections respectively 28 per-cent and 50 per-cent of the total exposures in those sections. In view of these differences it has seemed that the results of a distribution of the population enumerated at the Census of 1911 into areas corresponding (in the absence of other criteria) as nearly as may be with the Manchester Unity areas, as indicated by the map on page 27 of the report, might be of some interest, and such a distribution has accordingly been made. The following Table is the outcome:—

Geographical area adopted in the Manchester Unity Experience	SECTION FOR CENSAL LIFE TABLE		
	Rural Districts	Urban Districts	County Boroughs
	Percentage of the Population in the Section which was resident in the geographical area named in the first column		
1. Non-manufacturing	60.8	38.8	17.7
2. Textile	8.1	23.7	36.3
3. Manufacturing, (excluding Textile)	31.1	37.5	46.0
	100.0	100.0	100.0

If the conclusions drawn from the Manchester Unity experience as to geographical distribution of mortality be well founded (and so far as the writer knows they have not been questioned), the figures here set forth may be held to be of some significance in relation to the differences between the death rates represented by the new "rural," "urban" and "county borough" life tables. Density of population, environment, occupation and habits of life, acting in combina-

tion, produce certain variations in the death rate and these are found to satisfy the popular belief as to the relative effects of country and town life. The operation of these factors is obviously reflected in the new tables, but there are clearly other causes operating in the same direction. It is important that these should be kept in view lest, under the influence of the bare statistics, an exaggerated impression should be formed as to the possible effect, upon the vitality of the population, of the tendency towards urbanization which has for so long a period been manifest in the Census returns of this and other countries.

REVIEW.

Vital Statistics Explained. By JOSEPH BURN, F.I.A., F.S.I.
London: Constable & Co., Ltd.

MR. BURN'S book consists of a series of lectures delivered in connection with the Chadwick Trust, and it follows that his subject is treated largely from the point of view of the student of sanitary science. As the author says, the subject falls naturally into three main divisions:—

1. The census as an index to the development of national life, with a criticism of the methods adopted in its collection and tabulation.
2. The annual reports of the Registrar-General, with special references to the number and causes of death. Indications of the progress of sanitary reform.
3. The preparation of mortality tables :
 - (a) national,
 - (b) municipal,
 - (c) occupational.

Practical uses of mortality tables by medical officers and others.

In dealing with the first heading, a general historical account of the processes of census taking is given, which calls for little comment. We note, however, that Mr. Burn expresses the opinion that in the census the date of birth should be asked for rather than the age last birthday.

In chapter II a comparison is made of the populations of Great Britain, France and Germany. In instituting this comparison a variety of circumstances are brought under review and the effect of the birth and death rates on the growth of population is closely examined. Thus it is shown that between the years of 1881 and 1911, the population of France has increased from 37 millions to 39 millions only, whereas that of Germany has increased from 45 millions to 65 millions, notwithstanding that throughout that period the death rate in France was less than that in Germany. The explanation is to be found in the high German birth rate which is over 50 per-cent in excess of the French.

These figures lead to a further consideration, namely, the proportion of what Mr. Burn terms the "effective" population. In this connection the following table taken from page 29 is of interest :

	Germany	England	France
One-quarter of the population is under age	10·75	12·00	14·25
One-half of the population is under age	23·50	25·75	30·00
Three-quarters of the population is under age	40·75	42·00	48·75

Thus Mr. Burn arrives at the conclusion that "as far as youth and strength are concerned, the order of merit is Germany, England and France."

In considering the Annual Reports of the Registrar-General it is pointed out that it is impossible to discuss the changes in the birth rate without considering the corresponding anterior changes in the marriage rate. The latter function is therefore first dealt with and the general tendency to defer marriage is clearly shown.

In dealing with the birth rate, three methods are adopted for purposes of illustration :

- (a) Birth rate calculated on total population at all ages,
- (b) Fertility calculated on the Female Population aged 15-45 years,
- (c) Legitimate Fertility calculated on the Married Female Population aged 15-45 years.

These three bases afford, of course, an increasing degree of precision in estimating the fertility of the population and we could have wished that Mr. Burn had proceeded a stage further to the natural conclusion that the only true index of fertility is to be found in tables showing the rate of issue at each age.

No attempt is made to measure the effect on the birth rate of the deferment of the age at marriage, although this could have been readily ascertained from the Rates of Issue given in the first Annual Report on the National Insurance Act. Using these figures we find that the above feature would account for a reduction in the birth rate of about 10 per-cent since 1871, whereas the actual reduction was over 25 per-cent.

Incidentally, as illustrating the danger of using vague standards for measuring these statistical quantities, we may refer to Mr. Burn's quotation from the third volume of the report on the last census of Scotland, given on page 42 :

"There is a level chance of a young woman, marrying at 17, being the mother of at least ten children . . . this chance in the case of a woman marrying at 20 is that she will be the mother of at least nine children"

This is a somewhat startling statement, but on referring to the report in question we are relieved to find that it bears the important qualification "provided the marriage continues to the end of the fertile period of the woman's life."

The vital question, however, as Mr. Burn says, is not "How many children are born?" but "What proportion of children born survive the perils of infancy?" In this connection an analysis is made of the causes of death and the opinion is expressed that much remains to be done in reducing this "slaughter of the innocents."

Chapter IV is devoted to an analysis of the death rates, and diagrams are given illustrating the changes in the mortality from certain epidemic diseases, from Tuberculosis and Phthisis and from Cancer. The consideration of these facts leads naturally to Mr. Burn's main recommendation, which is of sufficient importance to justify our quoting his words in full:

"I hope and believe that in the future mortality tables will be formed at comparatively short intervals both for the whole of the country and for many separate districts, so that the rates may be compared and the progress towards improved mortality rates indicated clearly and indisputably. I think that, to make such tables as useful as possible, they should show, not only the mortality rates for each year of age, but also for each age the rates of mortality due to various diseases."

The author concludes with a simple demonstration of the construction of such a table by osculatory interpolation, and students who experience difficulty in this part of their work will find the subject very lucidly dealt with, and a graduation of the figures for England and Wales is worked out.

We must confess to some feelings of alarm when Mr. Burn adds the weight of his authority to the suggestion that Life Tables should be constructed for numerous districts, without a corresponding warning against the use of mortality tables constructed from limited data by persons unskilled in actuarial science. Apart from this the object is an eminently desirable one, and we would suggest that there is an opportunity for some of the younger members of the Institute to offer their services to local bodies for the purpose.

Mr. Burn's book is embellished by numerous diagrams, and our gratitude to him in this respect would be enhanced if some of them—notably those on pages 33 and 47—were a little less troublesome to decipher. It is necessary to remember also that the lectures were addressed to a non-actuarial body and that in some places, therefore, Mr. Burn has contented himself with a less degree of precision or qualification than might otherwise have been the case.

These, however, are small criticisms of a thoroughly interesting work which gives rise to many novel and interesting lines of thought and cannot fail to bring home to the reader the desirability of a closer co-ordination between actuarial and sanitary science.

New Year Honour.

THE announcement on 1 January 1915 that the honour of Knighthood had been conferred on Mr. A. W. WATSON will have been received with very great pleasure by the actuarial profession.

At a meeting of the Council of the Institute of Actuaries, held at Staple Inn Hall on 12 January 1915, the following resolution was unanimously passed:

"The Council of the Institute of Actuaries offer their most cordial congratulations to Sir Alfred Watson. They welcome the honour conferred on him not only as a recognition of his official services to the State, but also on account of his exceptional knowledge of the Friendly Society system and of the constant exercise of his great influence and abilities, throughout his professional career, in favour of a high standard of Friendly Society finance based on sound actuarial principles."

CORRESPONDENCE.

MR. NATHAN'S PAPER ON SECTION 72 OF THE NATIONAL INSURANCE ACT.

To the Editors of the Journal of the Institute of Actuaries.

SIRS,—I trust you will allow me the privilege of removing any misconception to which a statement appearing in the October number of the *Journal* may give rise.

In his Note on p. 388, it would seem that Mr. Nathan gathered from my remarks during the discussion of his recent paper that I advocated the treatment of sickness in excess of the normal (non-hazardous) rates as entirely due to accident risk, and consequently the use of unloaded values in estimating the release under a scheme. This is not a correct representation of my views, as I think Mr. Nathan will find on referring to the report of the discussion.

I am, Sirs,

Your obedient servant,

R. G. MAUDLING.

St. Stephen's House,

Victoria Embankment,

Westminster, London, S.W.

16 December 1914.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

The Analysis of Life Office Expenses. By CHARLES HUGH MALTBY, F.I.A., of the North British and Mercantile Insurance Company.

[Read before the Institute, 25 January 1915.]

I.—INTRODUCTION.

IN turning over the pages of the *Journal*, one is struck by the number of papers which would have been rendered far more valuable if more detailed data of the actual expenses of management had been available. Probably less is known about expenses than about any other factor involved in our daily actuarial work; and this comparative ignorance is the more remarkable when we remember the immense amount of thought and research which has been expended on the subjects of Mortality and Interest.

Perhaps good reasons existed in the past for this comparative neglect; but it appears to me that the time is approaching—if it has not already arrived—when it will be absolutely necessary to investigate the subject of expenses to the fullest possible extent. To mention only one aspect of the subject, there is a general impression that expenses are tending to increase, and hence it becomes important for each office to ascertain whether this impression is correct in its own case, and if so, the causes of the increase. These questions can only be answered by a full analysis; but if this is possible any increase will be robbed of much of its force, even if its cause cannot be removed. Certainly expenses are at least as worthy of investigation as mortality, and will probably yield more valuable results, since they, at least, are susceptible of regulation.

Practically the only treatment of the subject of analysis of expenditure in the *Journal* is the paper read in 1910 by

Mr. H. J. Rietschel—vol. xlv, p. 415—which is doubly valuable, since in the discussion the late Sir George F. Hardy presented us with what appears to be one of the main keys to the solution of the problem.

The present paper is but little more than a development of the suggestion then thrown out, but I think that the further consideration of the subject will be of value, since any steps towards a more accurate, and practical, method of analysis can only be made as the result of the public discussion of tentative attempts at a solution.

I do not advocate the analysis of expenses with any idea of the publication of results, or of their comparison with those of other Offices. There are certain details of its operations which every Company, in justice to itself, is bound to keep private; and of these, the results of an analysis of its expenses would certainly be one. Moreover, any comparison of results would only be misleading, since the details of any method adopted would probably vary in every case.

II.—IS AN ANALYSIS POSSIBLE ?

The first question which arises when the problem is considered, is whether a full analysis is possible at all, in view of the nature of the expenses involved, and the character of our business. In Life Assurance we cannot always trace a direct relationship between a class of expense and any "unit"* in terms of which it can be expressed; such as exists in a manufacturing concern between a product, and the labour, or the materials, involved. Thus, certain broad assumptions may be necessary—as, for example, that made in Section X, in a comparison of the general expenses with the Mean Life Fund—but if this is admissible, no insurmountable obstacle appears to exist. Of course, a certain amount of labour will be involved; but, as I hope to show, an amount not incommensurate with the importance of the subject.

III.—USES OF AN ANALYSIS.

The uses of the results of a full analysis would be manifold, and there is hardly a detail of our daily work in which a more complete knowledge of the expenses involved would not be of the greatest assistance.

* For want of a better word, I use—somewhat incorrectly—the term "unit" to denote a quantity as a percentage of which a given expense may be expressed.

Perhaps the most obvious use is in connection with the loading of office premiums, and the calculation of special rates ; where the possession of such knowledge should do much to reduce, if not to prevent, the evil of "rate cutting", and the payment of excessive commissions. At present we may be led to follow precedent, and the practice of other offices, far too blindly, without any real knowledge of one of the most important factors involved.

Again, economy of management is one of the most powerful weapons an office can wield in the struggle for business, and a comparatively full knowledge of the incidence of expenses is almost essential, if economy is not to degenerate into a mere process of cutting down the most obvious items of expense.

If an annual analysis is made, the results will gain in value as experience is accumulated, and the changes from year to year are noted, reflecting as they must almost every detail of an office's operations.

If it is admitted that our present knowledge of expenses is defective, that no insurmountable obstacles bar the way to a remedy, and that probably the results will be worth the labour involved, the next step is to consider how an investigation can best be made ; and I suggest the following method of procedure as an approximate solution of the problem, and a basis for discussion and further work.

IV.—REQUIREMENTS WHICH ANY SYSTEM MUST SATISFY.

The following requirements have been kept in view in devising the method :

- I. Any method suggested must involve the minimum amount of work consistent with reasonable accuracy.
- II. The particulars required must be easily obtained from existing records ; and no radical changes must be required in methods of book-keeping at present in use.
- III. Any method suggested must give effect to the varying expenses involved under different classes of policies ; and, as far as possible, allot to each class its full and proper share.
- IV. The results must be presented in such a manner that they can be readily combined with the factors of Mortality, and Interest, for use in actuarial calculations.

- V. The method must be elastic, and easily adapted to the varying requirements of different offices ; since, in matters of detail, the scheme which will suit Office A may be totally unsuitable for Office B.

It will be gathered, therefore, that when any method of analysis is first put into operation it may only be possible to proceed on broad lines, and refinements in minor matters of detail must be left until the results of practical experience have accumulated, and the books are perhaps better adapted to the requirements of the system. In any event, attempts at the extreme subdivision of minor items of expense are to be avoided since the trouble involved is likely to be out of all proportion to any additional accuracy obtained.

V.—EXPENSES DEALT WITH.

The expenses dealt with in the paper are those charged against the Life Department in the Annual Accounts, and I have assumed that in the case of “Composite” Companies the division of any general expenses of the whole Office—such as “Directors’ Fees”—will be made on the basis usually adopted by the Company whose Life expenses are under examination, since probably a different method is used in every case.

In passing, however, attention may be called to the great complexity of the problem involved, since no theoretical basis appears to exist for the equitable apportionment of general expenses between two or more Departments transacting widely differing forms of Insurance. I am afraid that from its very nature the problem is only capable of an empirical solution ; although, under normal conditions, a division based in one way or another on the Funds belonging to each section is probably as free from objection as any other system which can be suggested.

In any event, these joint expenses should be kept as small as possible, in order that the actual cost of each department may be accurately known.

From this point onward, therefore, I deal solely with the analysis of the expenses of a Company transacting Life business only, since the adjustments necessary in other cases will readily suggest themselves ; a typical example being dealt with in Section XIX.

VI.—MAIN GROUPS INTO WHICH EXPENSES MAY BE DIVIDED.

The expenses of a Life Assurance Company may be divided into the following six main groups; of which the first five deal with “expenses of management”, and the sixth with Commission.

Group 1.— As pointed out by Mr. Duncan C. Fraser, in **Initial Expenses.** the discussion on Mr. Rietschel’s paper—vol. xlv, p. 448—a certain amount of new business is absolutely necessary to the well-being of an Office. Over and above the necessity for replacing the wastage due to claims, a certain amount of expense is also incurred in maintaining and extending the reputation of the Office, and in enabling it to hold its place amongst its competitors; so that the expenses incurred before a policy is issued may be divided into:

- (a) The expenses incurred in searching for business, as for example, Advertising,
- (b) The expenses incurred in actually placing a policy on the books after a proposal has been obtained, as for example, Medical Fees.

In my opinion, only those expenses falling into section (b) should be designated “Initial Expenses”, and can be regarded as a debt due to the Office by the new entrant. Those expenses falling into section (a) are general expenses incurred for the good of the Office as a whole, and as such are allocated to Group V below.

I regard the salaries and expenses of Inspectors, Special Agents, and Branch Managers, as akin to Initial Commission, since payment depends largely on the amount of business produced; and these items are accordingly treated as Initial Expenses. No doubt, however, in many cases much might be said for the view that part at least of these particular items really belongs to section (a), and is incurred in searching for business. Although this Group thus comprises comparatively few items, I do not think its range is too restricted, since the treatment of Group V allots to each new policy its proper share of such expenses as Advertising, Rent, &c.

Group 2.—
Renewal
Expenses.

Incurred in connection with the collection of the premiums after a policy has been placed on the books.

Group 3.—
Claim Expenses. Incurred in connection with the payment of the sum assured upon the maturity of a policy by death, or survival. Although these expenses are very small, they differ in incidence, and in nature, from those in any other Group, and they are accordingly treated separately for the purposes of this paper. In practice, no doubt, other methods will suggest themselves.

Group 4.—
Investment Expenses. Arising out of the Investment and management of the Funds of the Life Department.

As will be seen later, no reason exists why these expenses should not be combined with those of the next Group, since they are both divided in proportion to the same “unit.” In many cases this may be the more convenient course to adopt; although as a matter of interest, I think it preferable to treat them in separate groups when this is possible.

Group 5.—
The general expenses of Office Management. Since there are objections to the use of such terms as “general” or “indirect” expenses, or “fixed charges”, I propose using the brief, but expressive, technical term “Oncost” to indicate those expenses falling into this fifth group.

In the case of a manufacturing concern, this term is used to distinguish all those items in the cost of a product which do not come under the heads of productive labour, materials, or profit, such as upkeep of premises, administration, &c. Extending the same general principle to Life Assurance, I include in this group, not only such expenses as Directors’ Fees, Rent, &c., but also expenses, such as Advertising, incurred in searching for business, and expenses incurred for the benefit of the Office as a whole. I also propose including all items of expense which cannot be accurately, and conveniently, divided up amongst the groups to which they properly belong, as for example, the expenses of the Actuarial Department.

This will be, in most offices, the largest of the five groups, but I think that its size is an essential feature of our business, and is not without advantages. Since the group can easily be split into its component parts year by year, its comprehensiveness does not mean that a large portion of the expenses is lumped together into a crude undigested mass; while, as it is to be compared with that “unit” which is the least liable to accidental fluctuations, its size will to some extent cause it to act as a fly-wheel on the machinery of the analysis.

Group 6.— As already mentioned, the sixth group is made up
Commission. of Commission—Initial and Renewal.

The above apportionment ignores the fact that, strictly speaking, certain minor items of expense should be set off against certain miscellaneous sources of profit, *e.g.*, expenses incurred in connection with Surrenders. In practice, however, it is impossible to give effect to such an adjustment, which is fortunately of infinitesimal importance.

VII.—APPORTIONMENT OF INDIVIDUAL ITEMS OF EXPENSE AMONGST THE MAIN GROUPS.

Having dealt with the main groups into which expenses may be divided, the next step is to consider the proper group to which each individual item of expense should be allotted, the method suggested being shown in the annexed schedule.

The basis adopted has been made as broad, and as simple, as possible ; and probably in every case, the expenses of management can easily be divided into the main items shown. It is, however, intended to be only a very general indication of the procedure to be adopted in any particular case ; since in practice, the exact details will vary in different Companies, and can only be determined by one familiar with the peculiar circumstances of the office in question. For example, in a small Company it will be impossible to divide salaries to the full extent suggested, and some approximate basis of division must be arrived at.

Elaborations of the method shown will readily suggest themselves, and may be well worth the extra trouble involved, especially when it is desired to trace the causes of any changes in the results of the analysis, from year to year.

Some small miscellaneous items of expense may cause a little trouble at first, but these are really of very minor importance, since their effect on the final ratios is negligible.

APPORTIONMENT OF INDIVIDUAL ITEMS OF EXPENSE AMONGST THE FIVE MAIN GROUPS.

Item of Expense	New Business Expenses	Renewal Expenses	Claim Expenses	Invest- ment Expenses	Oncost
1. Rent, Rates, Taxes, and maintenance of Head Office	*
2. Salaries, &c., of Head Office Staff:—					
Principal Officers and their private assistants	*
Secretary's Department	*
New Business Department	*
Policy Department	*
Inspectors and Special Agents... ..	*
Agency Department	*
Renewal Department	*
Claims Department *...	*
Loan and Investment Department	*	...
Actuarial Department	}	}	}	}	*
Accountant's Department					
Cashier's Department					
Typists					
Post and Counter Clerks					
Messengers	*
3. Directors' Fees and Expenses	*
4. Auditors' Fees	*
5. Advertising	*
6. Medical Fees... ..	*
7. Printing, Stationery, Books, &c.—					
(a) Prospectuses and New Business Literature	*
8. (b) General Office Stationery, &c.	<i>p</i>	<i>p</i>	<i>p</i>	<i>p</i>	<i>p</i>
9. Travelling and similar Expenses	<i>p</i>	<i>p</i>	<i>p</i>	<i>p</i>	<i>p</i>
10. Stamps on:—					
Policies and First Premium Receipts	*
Premium Renewal Receipts and Postage of Notices	*
Interest Receipts and Postage of Notices	*	...
Circulars to Public and Agents	*
General Postages	*
11. Legal Expenses	<i>p</i>	<i>p</i>	<i>p</i>
12. Valuation Expenses } are subjects of					
13. Superannuation } special notes.					
14. Other General Expenses, Petty Cash, &c., which cannot be divided	*
15. Branch Expenses:—					
Rents, &c.	*
Salaries, &c., of Managers	*
" " " Inspectors and Special Agents	*
Salaries, &c., of Branch Clerical Staffs	*
(Stamps in a large Branch can be subdivided as per Item 10 above.)					

* Indicates the group to which a particular expense is allocated.

p Indicates that the item is divided amongst several groups.

SUMMARY OF THE ITEMS ALLOTTED TO EACH GROUP.

The various items allotted to each of the main groups are accordingly as follow :

- Group I. Initial Expenses.* Salaries, &c., of New Business Dept.
Salaries, &c., of Policy Dept.
Salaries, &c., of Inspectors, Special Agents and Branch Managers.
Medical Fees.
Policy Stamps and Interim Receipt Stamps.
- Group II. Renewal Expenses.* Salaries, &c., of Renewal Dept.
Stamps on Premium Receipts and Notices.
Salaries of Branch Clerical Staffs.
- Group III. Claim Expenses.* Salaries, &c., of Claim Dept.
Share of Legal Expenses.
- Group IV. Investment Expenses.* Salaries, &c., of Loan and Investment Dept.
Stamps and Postage on Interest Receipts and Notices.
Share of Legal Expenses.
- Group V. Oncost.*
- (a) *Expenses which are undoubtedly Oncost.*
Rent, Rates, and Maintenance of Head Office and Branches.
Salaries, &c., of Principal Officers and their private assistants.
Salaries of Secretary's Dept.
Salaries of Typists.
Salaries of Post and Counter Clerks.
Salaries of Messengers.
Fees and Expenses of Directors and Auditors.
General Legal Expenses.
General Postages.
- (b) *Expenses incurred in searching for business.*
Advertising.
Circulars, Prospectuses, and Publicity literature generally.

Agents' Postages and Expenses.

Salaries, &c., of Agency Dept.

Postage on Circulars to Agents and Public.

(c) *Expenses treated as Oncost, since they cannot otherwise be conveniently divided.*

Salaries and expenses of Actuarial Dept.

Salaries and expenses of Cashier's Dept.

Salaries and expenses of Accountant's Dept.

Minor general expenses. Petty Cash, &c.

NOTE.—General Stationery, and Travelling Expenses, are sub-divided according to the Department by which they are incurred, and so allotted to the proper Group.

VIII.—SOME QUESTIONS WHICH MAY ARISE IN PRACTICE.

Item 1.— This item includes such expenses as Fuel, Lighting, Rent, Rates, Taxes, &c., of Lift, &c.
Head Office.

When a Company only occupies a part of its building, and lets the remainder, a portion of this item should, strictly speaking, be allocated to Group IV as an Investment Expense. Methods of doing this will readily suggest themselves, as for example, on a rental basis.

Item 2.— Although the division shown has been made on Salaries, &c., of a departmental basis, in practice it will probably be more convenient to allot the salary of each individual to that group with which his work is mainly concerned. On the average, the application of this rule should yield good results, since the compensating factors at work will practically nullify any error in the final results. So far as possible, the splitting of a salary between two groups should be avoided, although this may sometimes be necessary, under exceptional circumstances.

All staff expenses, such as National Health Insurance, or any allowance towards the payment of insurance premiums, may be treated in the same manner as salary. Where this rule cannot be applied; as for example, the cost of teas or lunches, it will be quite sufficient for our purpose if the cost charged—say to Initial Expenses—is obtained from

$$\text{Total Cost of Allowance} \times \frac{\sum \text{Salaries allotted to Group I.}}{\sum \text{Total Salaries.}}$$

Item 7.— The cost of this item is divided into two parts :
 Printing,
 Stationery, &c.

(a) Cost of Circulars, Prospectuses, and other Publicity matter.

(b) Cost of General Office Stationery,
 the first of which is treated as a form of Advertising, and as such allocated to Group V—Oncost.

As regards the second, with the aid of the vouchers, no difficulty will be found in allocating the cost of the larger items to the correct group, or to the department by which they were ordered. When the larger items have been dealt with, a small residue will remain, representing the cost of numerous small articles, and if this cannot be divided without undue labour, it can either be assigned to Group V *en bloc*, or divided up more or less approximately.

Item 9.— Here again it should not be difficult to sort the
 Travelling
 and Similar
 Expenses. vouchers into five groups, allotting each individual item to the same group as the salary of the member of the staff by whom the expense was incurred. For the most part, this item will consist of payments made to Inspectors, Special Agents, &c., and as separate records will probably be kept of these, the work will be much reduced, and only a small part of the total amount will require analysis.

Item 10.— It will be observed that this item is subdivided
 Stamps
 and Postages. into five parts, the data for which will, however, be readily obtained, since policies and receipts are usually only stamped at regular intervals in large quantities. The cost of posting renewal premium notices, circulars, &c., can also be easily obtained, since these again are issued periodically in large batches, if the Postage Book is examined at weekly or monthly intervals. The balance of the total amount may then be safely regarded as General Postages, and transferred to Group V.

Item 11.— It is almost impossible to suggest any general
 Legal Expenses. manner of dealing with these expenses, since such great variation of practice exists, both as regards the payment of legal costs by the assured, and the manner in which the official solicitors are remunerated. Methods will, however, readily present themselves when the practice of the office in question is known. In any event, only Groups III, IV, and V will usually be affected, and hence it will be almost sufficient if the amount

incurred in connection with claims only can be separated, either exactly, or approximately, from the total cost.

Item 12.—
Valuation
Expenses.

If an annual valuation is made the expenses involved should be treated as part of the "Oncost."

Where a valuation is made only once in five years, and in the case of a distribution of Surplus, the attendant expenses may be treated in a similar manner; although another method would be to regard these special expenses as a charge on the surplus disclosed, instead of as a part of the normal expenses of management.

This second course has certain advantages, although it throws the burden upon the "With Profit" policyholders.

In any event, special expenses of this nature should be shown, and treated, separately, so as not to disturb the comparison of the results of the analysis year by year.

Item 13.—

Superannuation. The treatment of this item will vary very much in different offices, and three distinct methods of dealing with it present themselves:

- (a) A separate Superannuation Fund may be built up out of the profits of the Company, upon which all pensions are charged. This course eliminates Superannuation from consideration, so far as our present purpose is concerned.
- (b) A Pension may be treated as deferred Salary, and added to the salaries of that department to which the pensioner last belonged, or to which he was attached during the greater part of his service.
- (c) The whole cost of this item may be allocated to Group V—Oncost.

In practice, Methods (b) and (c) will probably produce almost identical results, since the pensions of any retired higher officials will outweigh those paid to other retired members of the staff.

IX.—BRANCH EXPENSES.

So far only Head Office Expenses have been dealt with. Although the same principles apply, it is impossible to deal with the subject of Branch Expenses, except in a very general manner, owing to the difficulty of devising methods which will be equally applicable to, say, the London Branch of a large Scottish Office, with a staff and expenses greater than

the whole of those of a small Company ; and to a small Branch, whose entire staff consists of a manager and one clerk.

The analysis in the case of Branch Expenses has also to serve a double purpose since :

- I. The figures have to be combined with those of the Head Office, and
- II. The expenses of each Branch have to be considered in relation to the benefit accruing to the Company from its existence ; and in relation to the expenses of other Branches.

In the case of a large Branch, the expenses can probably be analysed to nearly the same extent as those of a Head Office ; while in the case of smaller Branches, the method shown in the general Scheme in Section VII will probably be sufficiently near the truth, although it is obviously only a compromise.

It is assumed that while the salary and personal expenses of a Branch manager are largely Initial Expenses, and are only to a small extent Renewal, the opposite is the case in connection with the clerical staff, which is largely occupied with book-keeping, and the collection of Renewal premiums. The plan adopted of allotting the whole of the first item to Group I, and the whole of the second to Group II, will, therefore, probably work well under average conditions. If it is not convenient to analyse the item of Branch Postages as fully as suggested, the whole item can be treated as " Oncost ", until it is deemed worth while taking steps to obtain fuller information. Branch Stationery is not dealt with, since this is usually supplied by the Head Office, and would thus be included in their figures.

X.—" UNITS " IN TERMS OF WHICH EXPENSES CAN BE EXPRESSED.

When the total expenses of management have been divided up among the five main groups, the next step is to consider in terms of what " unit " the total expenses in each Group should be expressed.

Many items of expense really vary directly with the number of contracts involved, and with no other factor. As shown by Mr. Rietschel, however, the " contract " basis is inadmissible in practice, since it leads to the assumption that the premiums per-cent should vary according to the amount assured ; or, in other words, that the assured should pay an annual contribution,

irrespective of the size of his policy, to cover these particular expenses, a scheme unworkable under present conditions. The "contract" basis is, however, useful as an auxiliary to the use of other "units" in tracing the causes of changes in the final ratios from year to year; and in ascertaining the expenses incurred under different classes of policies. These uses are dealt with later.

The elimination of the "contract" basis leaves only three "units" in terms of which expenses may be expressed. These are: Sums Assured, Premium Income, Interest Income or the mean Life Fund; the "unit" whose use is suggested for the expenses of each Group being given in the following table.

For special purposes, of course, it may be useful sometimes to adopt other "units" than those suggested. For example, the cost of advertising may also be compared with the new business obtained, as a rough test of the efficiency of the methods adopted.

Group and Nature of Expenses involved	"Unit" in proportion to which it is proposed to divide total expenses of each Group	Result yielded
Group I.— Initial Expenses.	Total new Sums Assured after deduction of Reassurances.	Initial Expenses for each £100 of New Business.
Group II.— Renewal Expenses.	Total Premium Income (including new premiums, with the exception of Single Payments).	Cost of collecting each £1 of Premium Income.
Group III.— Claim Expenses.	Total Claims paid by maturity or death (Sums assured only).	Cost of payment of each £100 claimed.
Group IV.— Investment Expenses.	Mean Fund of the year.	Rate of Interest on Funds required to provide for these expenses.
Group V.— Oncost.		

It will be obvious that not one of the "units" suggested is an ideal standard of comparison, since each is liable to variations apart from the expenses with which it is used. For instance, a heavy fall in the value of securities would cause an apparent increase in the ratio under Group V, even if the Oncost really

remained unaltered. They are, however, the best standards available, and the reasons for any alteration will be easily recognized, and given effect to, especially when the experience of, say, ten consecutive years is to be combined.

**Initial
Expenses.**

It will be observed that the "unit" used is the total *net* new sums assured, the commission received under reassurances being deducted from the total amount of initial commission paid before the latter is analysed. The adjustments necessary in respect of proposals rejected, or not taken up, and for the special features of Non-Medical business, Joint Life Policies, &c., are dealt with in Section XII, when the analysis is extended to different classes of Policies.

**Renewal
Expenses.**

It is in connection with these that the inadmissibility of the "contract" basis causes most difficulty, since any other method can only be a compromise. The "unit" suggested is the total annual Premium Income, which includes a certain amount of new business, so that the ratio obtained is hardly correct from a theoretical point of view. It has, however, the advantages that the labour of ascertaining the exact renewal premium income is avoided, and that the ratio obtained is that required in the calculation of Office rates, since the loading for these expenses is usually applied to all premiums, including the first. Moreover, a new policy does undoubtedly cause the renewal department a certain amount of extra work in placing it on the books.

Another difficulty is that the ratio obtained makes no distinction between annual premiums, and those payable more frequently. For example, if we have two assurances with premiums of £8 per annum; in the first case payable annually, and in the second case payable quarterly, each policy is charged practically the same amount, although the actual expenses in the second case are four times those in the first. No remedy, however, is possible, without the use of special books designed to give the information required; and the additional accuracy would hardly be worth the extra labour involved. The ratio obtained is small—probably under 2 per-cent. on the Premium Income—and sufficiently accurate for our purpose, if the assumptions involved are borne in mind in making use of it.

"Oncost."

The use of the Mean Fund as a basis for the division of the general expenses of management was, I believe, first suggested by the late Sir G. F. Hardy, during the discussion

on Mr. Rietschel's paper (vol. xliv, p. 451) in the following words :

“ That expenditure (*i.e.*, general expenses) could not
 “ be expressed in the form of a ratio on the renewal
 “ premiums : it was rather a ratio on the reserve values
 “ of the policies. In other words, it could be best ex-
 “ pressed by making a reduction from what might
 “ otherwise be considered a safe rate to assume. If it
 “ was assumed that the Office would make $3\frac{3}{4}$ per-cent,
 “ he thought it was a very fair suggestion that $\frac{1}{4}$ per-cent
 “ should be taken off that amount to cover the cost of
 “ obtaining the interest and managing the funds, and that
 “ the premiums should be based on the lower rate.”

By the method suggested, each policyholder—including new entrants—contributes a percentage on the reserve held against his policy, year by year, towards the general expenses of management, his contribution being thus proportionate to his interest in the Company. The method has, moreover, great advantages in connection with actuarial calculations, since it is applicable to all classes of policies, and automatically apportions a fair share of the Oncost to each. A comparison with the arithmetic mean of the Life Funds at the beginning, and at the end of the year, really gives a “force of expense”, as compared with the effective rate of interest used in Life Office calculations ; but this point is of no practical importance, although it could easily be given effect to if desired.

XI.—COMMISSION.

If the results of an analysis of this item are to be of any practical value, effect must be given to the varying scales of commission in force under different classes of policies.

This presents considerable difficulty, but as regards Initial Commission, the problem is readily solved by means of the “ Analysis of Initial Expense ” Book, described in Section XII. As there shown, this book will supply the data required in respect of each class of policy, with a minimum amount of labour. The subject of Renewal Commission is far more difficult, since for an exact analysis we require to know the Premium Income, and the Renewal Commission paid thereon, under each class of policy. As a rule, this information could not easily be obtained, even approximately, and I think that there is no alternative

but to ignore the question of Renewal Commission, and to assume that it is payable on the normal scale, or on a slightly sub-normal scale, in every case. This assumption errs on the side of safety, since those cases under which no renewal commission is payable will probably outweigh any cases under which the rate is in excess of the normal scale. The accuracy of the assumption can be roughly tested by the use of the premium income under each class of policy at the last valuation, when it will be apparent whether further investigation is necessary.

XII.—ANALYSIS OF EXPENSES ACCORDING TO CLASS OF POLICY.

So far the expenses have only been analysed in respect of the business as a whole, but if the results are to be of any practical use a further analysis must be made giving the expenses involved under each class of policy ; in the first place, because certain expenses are incurred under some classes of policies, and not under others ; and in the second place, because certain expenses vary purely with the number of contracts involved. If the latter feature is ignored, and such an expense is divided over the total Sums Assured of the whole Office, the ratio obtained would be correct if every policy were for the same amount. Hence, the ratio in question only applies to a policy for the average sum assured of the whole Office ; and as the average amount varies widely under different classes of policies, the results of a general apportionment will be, in some cases, far from the truth.

We can, however, divide those expenses which vary on a contract basis, amongst the different classes of policies, according to the number of contracts in each class ; and then proceed to apportion the amount so allotted to each class, in proportion to the total sums assured under that class. The ratios obtained are now correct for a policy of the average amount in each class.

This procedure is free from the objection attaching to a general division on a contract basis ; since, although we cannot differentiate between the rates per-cent charged for policies of various amounts in the same class, there is no objection to the rates under one Table, as a whole, being more heavily loaded for expenses than the rates under another Table.

The next step is to consider to which of the expenses this further analysis must be applied.

Group I. Initial Expenses and Initial Commission.

The individual items of which this group is comprised are :

- (a) Salaries and Expenses of New Business, and Policy Departments at Head Office.
- (b) Stationery.
- (c) Salaries and Expenses of Inspectors, &c.
- (d) Medical Fees.
- (e) Policy Stamps.
- (f) Initial Commission.

Of these items (a) and (b) vary mainly with the number of cases involved, while it is a matter for personal decision whether item (c) varies with the amount of new business completed, or with the amount for which proposals are obtained. Personally, I incline to the latter view, since, as will be seen later, a treatment on this basis gives some correction for any error involved in treating the whole of this item as an Initial Expense.

Items (a) and (b), therefore, may be divided amongst the different classes of policies, in the proportion which the number of cases in each class bears to the total number of proposals received, while item (c) may be divided in proportion to the total sums assured in each class.

The data necessary for the analysis of the remaining items are most accurately obtained by the use of a special book, giving the following details of each proposal, and divided up into sections according to the different classes of policies. It is only by some such means that the necessary adjustments for Joint Life Proposals, Non-Medical business, cases rejected, &c., can accurately be obtained, and effect given to the varying scales of commission in force.

The small amount of extra work entailed by such a book appears to be well warranted by the additional accuracy obtained. If even this labour is objected to, approximations will suggest themselves, although it is absolutely necessary to ascertain the number of contracts, and the total sums assured under each class.

ANALYSIS OF INITIAL EXPENSES BOOK.

Class of Assurance.....

Proposal No.	Policy No.	Sum Assured	Medical Fee	Policy Stamp	INITIAL COMMISSION PAID.		Other Special Expenses incurred in any particular case
					Normal Scale	Special or Overriding	

If the Chief Medical Officer is remunerated by a fixed fee, irrespective of the number of cases examined, the amount to be entered against each case examined at Head Office is obtained from

(Expenses of Head Office examinations) ÷

(No. of Examinations at Head Office).

Cases rejected, or not taken up, should be allotted a separate section of the book ; and throughout the analysis would be treated as a separate class of policy, the total initial expenses allotted to which would be finally added to, and treated, as part of the Oncost.

We thus obtain the total Initial Expenses allotted to each class of Policy ; the division of these amounts by the appropriate new Sums Assured giving the Initial Expenses per £100 of new business in each class.

Group II. Renewal Expenses.

These expenses do not vary with the class of policy involved, and the ratio obtained from the whole business is applicable to all classes alike ; although the assumptions made, and the difficulties involved, dealt with in Section X, must be borne in mind.

Group III. Claim Expenses.

As these expenses vary solely with the number of contracts involved, they may be dealt with in the manner suggested above. It hardly, however, seems worth while troubling about such a refinement, since the amount involved is so small. Moreover, any subdivision will probably lead to violent fluctuations in the ratios obtained year by year.

Groups IV. and V. Investment Expenses and Oncost.

No further division is necessary, since the method adopted automatically apportions a fair share of these expenses to every policy, and to each class.

The question of Renewal Commission has already been dealt with, so that, in practice, this additional analysis will be confined to Initial Expenses and Initial Commission.

XIII.—WHAT CONSTITUTES A “CLASS OF POLICY.”

The length to which this further analysis is to be carried, and the manner in which the policies should be divided in classes, must be settled by each Office for itself. Apart from certain assurances, such as those under Non-Medical, and Joint Life Tables, and those under which distinct commission scales are in force, the determining factor in deciding whether or not it is necessary to form a separate class for a certain kind of policy will be that the average sum assured thereby is distinctly different from that under other Tables. It is further necessary that a class should be sufficiently comprehensive to ensure that enough new business will be obtained thereunder, year by year, to prevent violent fluctuations in the ratios. Bearing these points in mind, it should not be difficult to divide the new business into broad groups, classing together all Tables in the prospectus under which the average sums assured do not usually vary one from another by £25-£50.

Special policies, such as Issue Risks, of which only one or two are granted from time to time, may conveniently be grouped together, irrespective of their nature, since they are usually for large amounts. It will probably be convenient to include with these Short Term Assurances issued for periods of one year or less, since under this class of assurance the average sum assured probably decreases as the term increases.

This course will do much to remove the difficulty that, if all Short-Term Policies are grouped together, the Initial Expenses applicable to the class as a whole cannot possibly be covered by any premium which can be charged for very short terms.

It may also be convenient to group together all policies under the ordinary Tables for very large amounts, since this course will enable the question of special rates for such assurances to be investigated easily, and will tend to render the Initial Expense ratios under other classes more stable from year to

year. On the other hand, this procedure will tend slightly to increase the share of those expenses varying on a contract basis allotted to the remaining classes ; since the average sums assured thereunder are decreased.

XIV.—ANNUITIES.

In addition to the expenses arising out of the payment, &c., of Annuities, the following expenses will fall to be allotted to the Annuity business of a Company.

INITIAL EXPENSES AND COMMISSION. Annuities should be treated as a separate class in the analysis of this group, although the amount allotted to annuities in respect of the expenses of Inspectors, &c., must be to some extent arbitrary, since no real basis of division exists.

RENEWAL EXPENSES are not incurred under Immediate Annuities, although this ratio will be used in connection with the premiums for Deferred Annuities.

CLAIM EXPENSES. Not incurred under Annuities.

INVESTMENT EXPENSES AND ONCOST. Annuities should bear their full share of these expenses, the general ratio for the whole office being used.

An apportionment on these lines should be made whether a separate Annuity Fund is set up or not, since in the first case it will be necessary to ascertain how much of the Life Expenses should be charged to the Annuity Department.

XV.—USES OF THE RESULTS OF THE ANALYSIS IN CONNECTION WITH THE CALCULATION OF RATES.

When the analysis is complete we shall be in possession of the following information in regard to the expenses for a particular year ; the figures inserted in the second column being, of course, imaginary.

Nature of Expense	Cost
*Initial Expenses for Whole-Life Policies.	1·190 per-cent of the New Sums Assured.
*Initial Commission for Whole-Life Policies.	1·246 per-cent of the New Sums Assured.
Renewal Expenses	1·109 per-cent of the Premium Income.
Renewal Commission	†2·500 per-cent of the Premium Income.
Claim Expenses	·425 per-cent of the Sums Assured Paid in Claims.
Investment Expenses and Oncost ...	·356 per-cent of the Mean Fund for the year.

* The cost of these items will vary according to the Class of Policy.

† Assumed in accordance with Section XI.

Bringing these items together into the formula for the Office Premium for a Whole Life, Non-Participating, Policy, we have the following expression :

$$\frac{100}{100 - 3.609} \left(100.425 \times \pi_x + \frac{1.190 + 1.246}{1 + a_x} \right)$$

Renewal Expenses and Commission.	Claim Expenses.	Initial Expenses and Commission.
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Assuming that $3\frac{1}{2}$ per-cent is the rate normally assumed in calculations, the rate of interest used in the above formula should be $(3.500 - .356)$ per-cent, or 3.144 per-cent, in order to provide for the Oncost and Investment Expenses.

The formula then reduces to

$$1.03744(102.861 \times \pi_x + 2.436 \times .03048^*)$$

or

$$106.712 \times \pi_x + .077$$

[At age 40, with O^[M] Select Mortality the premium given by the above formula (£2 13s. 2d.) is that given by $106.712\pi_x + .174$ on a $3\frac{1}{2}$ per-cent basis, or 9d. less than the rate given at $3\frac{1}{2}$ per-cent by Mr. Andras's formula

$$1.05(100\pi_x + \frac{2}{1 + a_x} + .125) = 107.1 \times \pi_x + .202.]$$

On these lines a skeleton set of rates for quinquennial, or even decennial, ages can be constructed, using the formula for each class of policy obtained from the results of the analysis. A comparison of these rates with those in use will then show the extent to which the working formulæ are justifiable, and the margin allowed by them for profit, and fluctuations, over and above that provided by the assumptions made as to Mortality and Interest.

Although in some cases it may be necessary to use a fractional rate of interest (probably lying between $3\frac{1}{2}$ per-cent and 3 per-cent) in calculating the trial rates, the net premiums at the rate required can easily be obtained by the use of a first difference interpolation: an investigation showing that the error involved is practically negligible. Moreover, although an annual analysis of the expenses is made, it will probably only be necessary to calculate such trial rates at long intervals, when a decided, and apparently permanent, change is found to have occurred in the weight or incidence of the expenses.

* d when $i = .03144$.

It is not suggested that the results of an analysis should be actually used in the calculation of Office premiums, but simply that the skeleton set of rates mentioned would be a most valuable guide in fixing the formulæ to be used in practice. Over and above the loading required to cover the actual expenses which will probably be involved, a further provision must be made, varying with the profit anticipated from each class of policy, competition, and the general views of the management. For example, the rates for complicated Survivorship Assurances will naturally be loaded very heavily, on account of the extra trouble involved, and the small amount of profit possible. Although I have only mentioned the use of an analysis of the expenses of a single year, it will be obvious that the results will gain in value year by year, as experience is accumulated, and it becomes possible to eliminate casual fluctuations in the different ratios by a rough graduation.

An analysis of expenses would not supplant the methods at present in use, but merely supplement them, and even when the subject has been brought to a fine art, I do not anticipate that the formulæ in daily use will be found to be defective. The difference will be that we shall then have an accurate knowledge of the past experience in respect of *all* the main factors involved, instead of only two of them, and so be far better able to provide for the future.

The methods of using the results of an analysis in the investigation of surrender values and other functions will be obvious, and need not be dwelt on here.

At the end of the paper I give a few short tables showing the loading given by a reduction of $\frac{1}{4}$ per-cent, and $\frac{1}{2}$ per-cent, in the rate of interest used in calculating the net premiums. These are only a portion of a fairly complete set for several rates of interest, but are sufficient to show the general tendency, and to prove that the method proposed for loading for general expenses, or Oncost, applies equally well to all classes of assurances. This feature is especially brought out by the Table of Survivorship premiums.

XVI.—THE QUESTION OF CHARGING SPECIAL RATES FOR VERY LARGE ASSURANCES.

Space forbids the discussion of the ethics of this subject ; or such questions as the relative value to an Office of large and small assurances, and the rates of mortality under large cases,

which have to be considered in practice. It will, however, be interesting to examine the extent to which an analysis of expenses would be of practical use in arriving at the special rates to be quoted for very large cases. As already indicated, if this course has been decided upon, it will be necessary to consider these policies as a class apart, and to ascertain the Initial Expenses incurred thereunder. Apart from Commission these are likely to be very small, on account of the size of the average sums assured.

Since Renewal and Claim Expenses vary purely on a contract basis, they are practically negligible in the case of very large assurances; although, if this view is adopted, the "units" used in the general division of these expenses must be suitably adjusted by the omission of the premiums, &c., under policies of this special class. This will be a very easy matter, and will have no appreciable effect on the accuracy of the general ratios.

I consider that these policies should contribute their full share towards the expenses of Investment and Oncost. In addition, therefore, to these last, it will only be necessary to load for Initial Expenses, and for the scale of Commission payable.

If trial rates are constructed on these lines, the actuary will be in a position to judge what additional loading for profit should be imposed, and to arrive at working formulæ for practical use. A very narrow margin for fluctuations will be required, since the Initial Expenses are definitely known and the weight of the other two items is hardly likely to increase to a serious extent. Conversely, it will be possible to judge the maximum commission which can be paid under a given scale of premiums.

In fixing the sum assured to which these special rates are to apply, it is necessary to bear in mind that sufficient policies of this class must be obtained to form an average; and also that the larger the average sum assured, the lower the rates of premium produced. From general considerations, I am inclined to think that in many cases £5,000 might prove a suitable amount for the lower limit, but this is a matter for individual investigation. Only the sums actually retained by the Company are, of course, to be taken into account.

XVII.—WORK INVOLVED BY THE METHOD SUGGESTED.

The extra work involved by the method will necessarily vary greatly in different Offices, according to the system of book-keeping adopted, and the extent to which the expenses of

management are already analysed. Assuming, however, that these are habitually split up into their chief component parts, the only main items under which additional work will be involved are Salaries, Travelling Expenses, Stationery, Stamps, Postages, and perhaps Legal Expenses. None of these involves an extraordinary amount of work, and once an annual analysis has been decided upon, it will be an easy matter to devise a few simple additions to the books already in use, which will greatly simplify the process, and render the data required more readily obtainable. At the outset, too, approximations may be substituted for more exact methods in dealing with some of the items.

The further analysis according to classes of policies involves practically no additional work, beyond keeping up to date the book suggested in Section XII.

Once the data have been obtained in the form required, little extra labour is essential, although further investigation of the results may be pushed to almost any extent, and will probably well repay the trouble taken.

When it is remembered that the work can be carried on continuously throughout the year, and that the data will be collected from several different departments, it will, I think, be recognized that an annual analysis of expenses would involve infinitely less work than, say, a comparison of the actual and expected claims, while the results obtained would be far more valuable.

XVIII.—ACCURACY OF THE METHOD.

An ideal method of analysis should allot an absolutely equitable share of the expenses to every policy, irrespective of its class or amount. Like all ideals, this is unattainable, and in considering the extent to which any system suggested falls short of this standard, it is necessary to remember that, in the first place, the results of an analysis are only intended to be used as a guide, and not as the actual basis for numerical calculations; and in the second place, that our calculations throughout depend upon the realisation of average results over large masses of data.

From this point of view, the treatment of those expenses which really vary solely with the number of contracts involved, appears justifiable, since the resulting ratios are correct for the average sum assured under each class of policy.

The method departs furthest from the truth in the cases of Renewal Expenses and Commission, although here the errors arise

mainly from the necessity of avoiding undue labour, and from the form in which Insurance Accounts are at present kept. These inaccuracies, however, and certain other assumptions, such as that made in regard to Branch salaries, only affect comparatively small items of expense, so that any adjustments would hardly affect the final results.

The errors involved do not, therefore, appear to be of great importance, and the method thus places us, with comparatively little labour, in possession of certain average results, which are sufficiently near the truth to be of considerable value, and which will form a basis for further research, when experience and data have been accumulated.

XIX.—APPLICATION TO COMPOSITE COMPANIES.

As an example of the adaptation of the method to special circumstances, it may be interesting to consider the extreme case of a Composite Office, in which the whole of the Life expenses—excluding commission—are met by the Fire Department, in return for an annual contribution of, say, 10 per-cent on the Life premium income; an arrangement which it is not considered desirable to amend.

An examination of the schedule in Section VII will show that the actual amount of many of the separate items of which the life expenses are made up can be easily determined. When the total sum of such expenses has been obtained, and deducted from the 10 per-cent contribution, a balance will remain, representing the life department's share of such items as the following :

Rent and Maintenance	Directors' Fees & Expenses.
of Head Office.	Auditors' Fees.
Salaries and Expenses of :	Advertising.
General Manager.	*Legal Expenses.
Secretary's Dept.	*Salaries, &c., of Inspectors
Investment Dept.	and Special Agents.
Accountant's Dept.	*Branch Expenses.
Agency Dept.	
Typists.	

[In some cases, of course, the "Life" share of the Legal Expenses can be ascertained exactly.]

Of these, all but the last three—marked *—are Oncost expenses, so that no great harm would be done if the whole of the balance in question were treated as the Life Oncost.

A further improvement, however, will be made if a definite share of the comparatively large item of Inspectors' Salaries, &c. can be allotted to the Life Department, and so treated as Initial Expenses. This, perhaps, cannot be done exactly, but if the salary list is examined, it should be a simple matter to determine whether the business produced by any special Agent is mainly Fire, or mainly Life, in order that the rule suggested for the salaries of the inside staff (see Section VIII) may be applied. One or two cases may, of course, need special consideration and treatment. Another basis of division would be the amount of Commission paid by each Department to each Inspector. A similar course may be followed in the case of Inspectors attached to Branches; but I am afraid that, for lack of any proper basis of division, the remainder of the item of Branch Expenses must be treated as part of the Life Oncost.

Comparing the results of an analysis of expenses under these special conditions with the case of a purely Life Office, we see that:

- I. Initial expenses are reduced by the amount of Branch Managers' Salaries.
- II. Renewal expenses are reduced by the amount of Salaries of Branch Clerical Staffs.
- III. Claim expenses may be reduced by the omission of Legal Expenses.
- IV. Investment Expenses are throughout combined with the Oncost.

Thus the general effect is to increase the number of items of which the Oncost is composed—though not necessarily to increase its total amount—and to render it impossible to split up this group into its component parts. The Oncost ratio, too, may vary more from year to year than in a purely Life Office; since, in the special case under consideration, the amount of the Oncost depends jointly on the premium income, and the expenditure under other groups, and is expressed in terms of the Mean Fund.

These points, of course, arise entirely from the fact that an attempt is being made to analyse expenses which are definitely fixed by an arbitrary rule, which bears no relation to their actual weight, or incidence. Even under these adverse conditions, the necessary adjustments do not appear to be by any means fatal to the usefulness of the method; although possibly more care may be necessary in making use of the ratios obtained.

XX.—IS THE PAST EXPERIENCE IN REGARD TO EXPENSES ANY GUIDE TO THE FUTURE ?

Although I consider that an annual analysis would be worth the trouble involved, if the only result obtained were a knowledge of the annual variations in expenses, much of its value would be lost if the experience of the past proved to be no guide to the future. This, however, will only be the case if expenses are found to be subject to rapid and violent changes ; or if the ratios obtained vary so widely and erratically from year to year, that no comparison, or graduation, is possible. Both of these contingencies appear unlikely to occur in practice, since the grouping suggested appears wide enough to ensure a reasonable amount of stability.

Alterations and fluctuations are most likely to occur in the Initial Expenses. These, however, are incurred before a policy is issued, and hence future fluctuations in the ratio obtained from this group can be disregarded when considering the present practice of the Office, since the Initial Expenses under the policies now being issued will be definitely known. In any event, we need not look more than a few years ahead.

The ratios obtained from the other main Groups will probably be far more stable ; and on the whole, appear more likely to decrease than to increase. It is even possible that Initial Expenses will not tend to increase to any great extent ; since, for example, improvements in the efficiency of an agency system may more than counterbalance any increase in the scales of commission, &c., paid.

If it is found that expenses tend to increase—the only change of really vital importance—the tendency will be recognized long before the matter has become serious, and proper allowance can then be made for it.

For these reasons, I consider that probably the past experience in regard to expenses will be at least as valuable as a guide to the future, as the experience of the past in regard to mortality ; although this point can only be definitely settled after an annual analysis has been tried.

XXI.—CONCLUSION.

I much regret that there are no data available which would enable me to show the results of an analysis on the lines suggested, and the premiums produced ; although the disguised

figures of any particular Office, or of an imaginary Office, would have little value beyond being a numerical example of the methods suggested, and would afford little or no indication of the results which would be obtained were the method applied to a second Company. An analysis of expenses can only be made by one familiar with the Office under examination, and the results are only of value when applied in the light of a knowledge of its general practice.

LOADING GIVEN BY A REDUCTION OF $\frac{1}{4}$ PER-CENT IN RATE OF INTEREST USED IN CALCULATING NET PREMIUMS.

Values of $100\{\pi. 3\frac{1}{4} \text{ per-cent} - \pi. 3\frac{1}{2} \text{ per-cent}\}$.

The values of $100\pi. 3\frac{1}{2}$ per-cent are also given in Italic type for comparison.

ENDOWMENT ASSURANCES.

O^[M] Table.

Age at Entry	TERM								
	10 years	15 years	20 years	25 years	30 years	40 years	50 years	60 years	Whole Life
20	0.113	0.104	0.094	0.086	0.079	0.065	0.056	0.050	0.048
	<i>8.473</i>	<i>5.299</i>	<i>3.755</i>	<i>2.864</i>	<i>2.300</i>	<i>1.672</i>	<i>1.389</i>	<i>1.286</i>	<i>1.265</i>
40	0.114	0.102	0.093	0.083	0.077	0.066	0.063	...	0.063
	<i>8.626</i>	<i>5.519</i>	<i>4.052</i>	<i>3.251</i>	<i>2.791</i>	<i>2.403</i>	<i>2.333</i>	...	<i>2.329</i>
60	0.110	0.096	0.085	0.078	0.075	0.074
	<i>9.561</i>	<i>6.856</i>	<i>5.815</i>	<i>5.428</i>	<i>5.316</i>	<i>5.295</i>

WHOLE-LIFE - LIMITED PAYMENTS.

O^[M] Table.

Age at Entry	NUMBER OF PAYMENTS								
	Single Premium	10 years	15 years	20 years	25 years	30 years	35 years	40 years	Whole Life
20	2.217	0.228	0.154	0.117	0.095	0.080	0.070	0.062	0.048
	<i>27.224</i>	<i>3.227</i>	<i>2.363</i>	<i>1.943</i>	<i>1.700</i>	<i>1.547</i>	<i>1.445</i>	<i>1.376</i>	<i>1.265</i>
40	2.393	0.236	0.156	0.117	0.094	0.079	0.071	0.066	0.063
	<i>40.784</i>	<i>4.897</i>	<i>3.630</i>	<i>3.032</i>	<i>2.705</i>	<i>2.518</i>	<i>2.413</i>	<i>2.359</i>	<i>2.329</i>
60	2.014	0.183	0.119	0.092	0.078	0.077	0.073	...	0.074
	<i>61.027</i>	<i>7.898</i>	<i>6.248</i>	<i>5.612</i>	<i>5.377</i>	<i>5.306</i>	<i>5.296</i>	...	<i>5.295</i>

LOADING GIVEN BY A REDUCTION OF $\frac{1}{2}$ PER-CENT IN RATE OF INTEREST USED IN CALCULATING NET PREMIUMS.

Values of $100\{\pi, 3 \text{ per-cent} - \pi, 3\frac{1}{2} \text{ per-cent}\}$.

The values of $100\pi, 3\frac{1}{2} \text{ per-cent}$ are also given in Italic Type for comparison.

SHORT TERM ASSURANCES.

O^[NM] Table.

Age at entry	TERM								
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	10 years
20	0·002	0·003	0·003	0·003	0·004	0·004	0·004	0·005	0·005
	<i>0·406</i>	<i>0·481</i>	<i>0·537</i>	<i>0·576</i>	<i>0·604</i>	<i>0·625</i>	<i>0·640</i>	<i>0·653</i>	<i>0·674</i>
40	0·003	0·004	0·004	0·005	0·006	0·007	0·007	0·008	0·010
	<i>0·630</i>	<i>0·726</i>	<i>0·808</i>	<i>0·878</i>	<i>0·936</i>	<i>0·983</i>	<i>1·025</i>	<i>1·062</i>	<i>1·131</i>
60	0·010	0·012	0·013	0·015	0·019	0·022	0·025	0·028	0·035
	<i>1·966</i>	<i>2·156</i>	<i>2·424</i>	<i>2·666</i>	<i>2·893</i>	<i>3·091</i>	<i>3·269</i>	<i>3·435</i>	<i>3·749</i>

CONTINGENT SURVIVORSHIP ASSURANCES.

$\pi_x^1 : y$, x by O^[NM] Table, y by O^[af] Table.

Values of $100\{\pi, 3 \text{ per-cent} - \pi, 3\frac{1}{2} \text{ per-cent}\}$.

AGES		Loading and $3\frac{1}{2}\%$ Rates	AGES		Loading and $3\frac{1}{2}\%$ Rates	AGES		Loading and $3\frac{1}{2}\%$ Rates	AGES		Loading and $3\frac{1}{2}\%$ Rates
x	y		x	y		x	y		x	y	
20	20	0·057 <i>1·113</i>	40	20	0·103 <i>2·249</i>	60	20	0·134 <i>5·543</i>	75	20	0·156 <i>12·261</i>
...	40	0·025 <i>0·912</i>	...	40	0·075 <i>1·991</i>	...	40	0·125 <i>5·354</i>	...	40	0·153 <i>12·117</i>
...	60	0·009 <i>0·742</i>	...	60	0·029 <i>1·470</i>	...	60	0·087 <i>4·711</i>	...	60	0·138 <i>11·684</i>
...	80	0·004 <i>0·600</i>	...	80	0·009 <i>0·994</i>	...	80	0·032 <i>3·216</i>	...	80	0·083 <i>9·472</i>
...	90	0·003 <i>0·512</i>	...	90	0·006 <i>0·802</i>	...	90	0·019 <i>2·501</i>	...	90	0·056 <i>7·739</i>

ABSTRACT OF THE DISCUSSION.

Mr. H. J. RIETSCHER said that the author suggested an analysis of the expenses into five groups, and then proceeded to ascertain the "units" in terms of which each of those groups of expenses could be expressed. He (the speaker) agreed that the new business expenses should be compared with the new business sums assured, and the investment expenses with the amount of the funds. With regard to the renewal expenses, he inclined to the view that their amount was dependent upon the sums assured in force. The labour and expense entailed in collecting the renewal premiums depended upon the number of premiums which had to be collected, or what was practically the same thing, upon the number of contracts in force. The elimination of the contract basis necessitated, as with the new business expenses, substituting the sums assured for the number of contracts.

The "Oncost" group of expenses consisted principally of expenditure incurred in the search for new business, and the author measured it against the mean funds, which enabled it to be expressed in terms of the rate of interest. As the method of dealing with this group might be said to constitute the vital part of the author's paper, he proposed to confine his remarks to that method, and he would use the term "Oncost" as meaning that part of the group of expenses which was expended directly or indirectly in the procurement of new business. In the first place, he should like to quote the extract given in the paper from the late Sir George F. Hardy's speech made during the discussion on a paper read by himself in 1910 (*J.I.A.*, vol. xliv, p. 451): "The work of the directors was largely concerned with the management of the insurance fund. . . . That expenditure could not be expressed in the form of a ratio on the renewal premiums: it was rather a ratio on the reserve values of the policies. In other words, it could best be expressed by making a reduction from what might otherwise be considered a safe rate to assume. If it was assumed that the Office would make $3\frac{3}{4}$ per-cent, he thought it was a very fair suggestion that $\frac{1}{2}$ per-cent should be taken off that amount to cover the cost of obtaining the interest and managing the funds and that the premiums should be based on the lower rate." It was clear from those words that Sir George Hardy's remarks were only intended to apply to the expenses of investment and collection of the interest, and did not contemplate the application of the suggestion to other expenditure, such as Oncost.

In order to consider Mr. Maltby's treatment of the Oncost there were two considerations essential to an accurate analysis and apportionment of expenses. The first condition was that there must exist some definite relationship between the particular group of expenses and the "unit" with which it was compared; such a relationship undoubtedly existed between the new sums assured and the initial commissions, or between the cost of advertising and the new business thereby secured. The second condition was

consequential, namely, that if the expenditure was duly measured against its correct unit a comparison of the resulting ratios from year to year should indicate the relative productivity in those years of that expenditure. Thus a fall in the renewal expense ratio would indicate that the office was obtaining a greater value than formerly for each £1 expended in managing the renewal business. The author's method of treating the Oncost did not seem to satisfy either of those conditions. It was difficult to understand in what manner the Oncost was dependent upon the mean funds or the mean funds upon the Oncost, and if that was so must it not be concluded that the wrong unit had been employed?

Take, for example, the case of a new office with very small reserves, and obtaining a satisfactory amount of new business in return for its expenditure. As the office had only very small funds the ratio of Oncost to the funds would be very large indeed. Suppose that for some years the office continued to spend the same amount in its search for new business and secured a like result. By the end of that time it would have accumulated considerable reserve funds and in consequence the ratio of Oncost expense to the funds would have decreased although the productivity of the expenditure had remained the same. It was evident from that example that the Oncost expenditure bore no relationship to the amount of the funds.

In a new business, with no present interest income, how could the Oncost expenditure be met out of interest? It was evident that the office had to look to the new business obtained to liquidate that expenditure, and if the interest income was to bear the charge then it must be the future interest income from the new business, and the unit for comparison would be the present value of the future interest income to be derived from the new business; that was to say, the Oncost became a function of the new business, but measured against the future interest from new assurances, instead of against the new sums assured. Was that admissible? What relationship existed between the present Oncost and the future interest income to be derived from the new business obtained as a result of that Oncost? Certainly no relationship per policy or per class of policy. If the premiums were so constructed as to meet that Oncost, whether by adoption of a lower rate of interest or by a level loading to provide the initial expenses per-cent of the sum assured, it was obvious that the past new business either had borne or was bearing its due share of past Oncost, and one must, therefore, in equity apportion any new Oncost expenditure to new business in respect of which it had been incurred.

The position would appear to be similar in the case of an old-established office which had decided to make a strong effort to increase its new business. With that view it increased its expenditure on office accommodation, branch rents, agency department, secretarial department, and circularising and advertising. The hoped-for increase in new business resulted, but the ratio of the Oncost to the fund exhibited a sudden and violent increase, which in a few years

showed a steady decrease due to the increase in the reserve funds created from the premiums derived from the larger new business.

Those examples of two hypothetical offices showed that Mr. Maltby's method of analysis and apportionment of the expenses failed to fulfil either of the two conditions which were essential. First, they showed that the Oncost bore no relationship to the funds, and, secondly, the ratios did not give any indication of the productivity of that expense. He had already endeavoured to prove that the Oncost must be definitely apportioned to the new business, and that if the interest income was to meet the Oncost expense then one must look to the future interest income from the new business, with which interest income, however, it seemed to have no very direct relationship. There was, it appeared, a clear relationship between the new sums assured and that part of the Oncost which was incurred to attract new business, and the ratio between those two items would indicate the relative productivity of those expenses. He was, therefore, of opinion that that—the larger—part of the Oncost should be compared with the new sums assured. In other words, it should be added to the new business expenses and the comparison of those total initial expenses with the new sums assured would give the initial expenses per unit of sum assured. If the office premiums were then loaded to provide for those initial expenses and the renewal and investment expenses, an office would be in the position that the new business of any period would support the whole of the expenses incurred in connection therewith and would not in any way draw upon the old business for assistance—a position every office should endeavour to create.

Mr. S. G. WARNER said that the paper should be read in connection with Mr. Rietschel's of April, 1910. Mr. Rietschel and Mr. Maltby stood together in recommending to the notice of the Institute a suggested method of analysing life office expenses, and the arguments upon which the scheme was based appeared to be the following: That it constituted a third factor in life office operations (the other two being interest and mortality); that less was known about it than about either of the other two; that if the method were adopted it might do something to modify the evils of rate-cutting and excessive commission; and that it would secure that economy should not "degenerate into a mere process of cutting down the most obvious items of expense." The paper followed in principle the lines of Mr. Rietschel's, but there was a difference of opinion between the two authors with regard to the precise allocation of certain items of expense. On the whole, Mr. Maltby's paper might be read as a kind of sequel to Mr. Rietschel's, or an extension of it; with the one significant exception that an attempt was made to put into practice Sir George Hardy's suggestion that certain expenses might be regarded as a charge on the interest income. That had actually been done in the one specimen of a premium contained in the paper.

With regard to the arguments cited, it was perfectly true that expenses did constitute a third factor in life assurance operations, standing side by side with mortality and interest, although it would

be seen on reflection that it was a factor of a very different kind. It was remarked that there was less known about that factor than any of the others, but how did Mr. Maltby know that? He said that there was very little published about it, but did not recommend that the results of an analysis on his lines should in any case be published. He thought Mr. Maltby meant that no office knew very much about any other office's expenses, and that was perfectly true. But was it not possible that every well-managed office knew a good deal about its own? He did not know what general enquiry Mr. Maltby might have made, on the results of which his statement was based, but it had rather surprised him to learn that no attempt was made to analyse expenses. Of course, if such attempts were made, the question at issue simply became one of substituting the method suggested by the author for others in operation, but the force of the argument for it as a substitute was rather weakened by the reservations and saving clauses with which its recommendation was accompanied.

He viewed with considerable doubt the possibility that the new method would check such evils as might exist in the way of cutting rates and paying excessive commission. There was no peculiar magic in any method of considering expenses which would lead anyone once resolved unjustifiably to cut a rate or increase a commission to pause and see the error of his ways. He would be probably quite aware that he was incurring undue expenditure, and there was nothing in any particular form of analysis which was likely to succeed in providing him with a conscience.

All such investigations, useful and well meant as they were, laboured under the great disadvantage that the foundation upon which they rested was absolutely riddled and honeycombed with assumptions. The widely-varying circumstances of different offices reflected themselves in all the methods employed, and differentiated at once this factor of expense from the other two. While it was literally true that it was a third factor in the business, standing by the side of mortality and interest, the suggestion that it could be similarly dealt with seemed, therefore, a fallacious one. Mr. Maltby had obviously felt these difficulties, and had shown much skill in safeguarding his position with saving clauses. In one part of the paper, for example, he said: "Over and above the loading required to cover the actual expenses which will probably be involved, a further provision must be made, varying with the profit anticipated from each class of policy, competition, and the *general views of the management*." "The general views of the management" constituted, he feared, an element that defied analysis. Sometimes the saving clause afforded so large a proportion of loophole that there was practically no wall left. For instance, early in the paper, when dealing with one of the most difficult and crucial points of the whole subject, the question of the composite offices, the author said: "I am afraid that from its very nature the problem is only capable of an empirical solution; although under normal conditions a division based in one way or another on the funds belonging to each section

is probably as free from objection as any other system which can be suggested." Probably there was no method of any sort which could not be said to be based "in one way or another" on the funds.

Tabulated suggestions were set out in the paper, including not only the chief analysis but a subsidiary one; and then the writer reached well-known ground, being once more among the familiar and much-tortured formulas which endeavoured to set forth the ideal office premium. A result was shown for one age only, age 40, with O^[M] mortality, and it was ninepence per-cent less than the result obtained on other and simpler lines by Mr. Andras. That consummation was reached by the reader with a sense of relief, because the machinery called into operation had been so formidable that one was rather alarmed as to what the result might be. When, however, it was found that all the meticulous manipulation of figures, the anxious analysis which extended its operation to postage stamps and tea money, brought about a result differing only by ninepence from what Mr. Andras found so much more easily a good many years ago, it led one to think that after all there was nothing so very serious involved. It was an important point, because whatever might be said about the various incidental benefits which might arise from the adoption of methods of the kind suggested it was impossible to avoid the conclusion that they were really directed towards the formation of premiums—the determination of premium rates. Implicitly involved in that was the suggestion that in some way or other expenditure could be made to dominate rates. If there was one thing more certain than another about life assurance business it would appear to be that expenditure did not and would not dominate rates, but that rates did and would dominate expenditure. The reason was plain. The two other factors which had been mentioned, the main factors of rate production, mortality and interest, were practically certain in their operation; interest absolutely certain; mortality, with the information now available, practically certain. Those factors were public property. There was no mystery about net premium rates; tables were given in public books of reference, and the public were quite aware of them, and knew very well what the cost basis of a premium should be. But the other element was subject to all kinds of fluctuations. The difference of opinion, for instance, between Mr. Rietschel and Mr. Maltby as to whether "advertising" was an initial expense, a general expense, or partly both, involved what might almost be called a metaphysical discussion, in which different individual views might lead to far larger variations in the premiums arrived at than the ninepence per-cent of Mr. Maltby's specimen. So might half-a-dozen other differences of opinion on points raised in the paper.

He hoped Mr. Maltby would understand that while he made these criticisms he very much appreciated the paper itself, and the labour that its production had involved. Mr. Maltby had done the Institute a service in bringing forward a subject which was of great practical interest, and would no doubt agree that frank and friendly criticism, offered in good faith, was the best compliment that could be paid

to his or to any other paper. There was only one protest he wished to make, and that was with regard to the word "Oncost." Mr. Maltby said that there were objections to other familiar terms in use for the same purpose, but as he had not specified those objections it was impossible to deal with them. There was surely, however, a very grave objection to saddling the literature of the profession with an unfamiliar and uncouth word. The thing itself had often been the subject of discussion and had been described in various ways, and what had been meant had been always understood. Personally he should think that "general expenses" was an adequate expression for the purpose, and one which nobody would be likely to misunderstand.

The PRESIDENT, in moving a vote of thanks to Mr. Maltby for his Paper, expressed regret that owing to the exceptional circumstances so few Members had been able to attend the meeting and to join in the discussion. Their cordial acknowledgments were due to Mr. Maltby for the time and thought he had devoted to the subject.

He had been surprised to hear Mr. Warner say that one thing which was absolutely certain in the calculation of premiums was the future rate of interest. He felt that he must have failed to grasp Mr. Warner's meaning, for if there was one thing the course of which it was difficult to predict over any long period it was, in his experience, the rate of interest on the funds of life assurance institutions.

Mr. WARNER said he intended to say that interest was a force which was certain in its operation—when one had decided on the rate.

The vote of thanks having been carried unanimously,

Mr. MALTBY, in reply, said that Mr. Warner, in his rather severe criticism of the Paper, accused him of too hastily forming the conclusion that offices did not know anything about their own expenses. Personally, he doubted whether many did. Possibly certain companies had some system of analysis, but he knew of other offices with no system at all. Unless an office made an analysis of its expenses he did not see how it was going to get at the proper loadings to be placed on its premiums; since without it they bore absolutely no relation to the expenses actually incurred, or likely to be incurred. In practice the calculation of rates too often came down to taking the rates of half-a-dozen other companies, calculating the average and then using a formula which reproduced those rates, or something less. That method brought out very good rates from a competitive point of view, but was hardly scientific. An office might just as well write the rates down straight away, without any basis at all. He certainly thought there was ground for saying that expenses ought to be analysed. He did not hold any brief for his own method of analysis; probably everybody would prefer a different method, especially in dealing with such subjects as advertising. If advertising and similar expenditure were treated as initial expenses, a difficulty would arise when an attempt was made to divide expenses according to the class of policy; it would probably be found that the initial expenses so

obtained were too heavy for certain classes of policies, such as short-term assurances, and adjustments would have to be made, and some part of those initial expenses transferred to the other classes. He quite admitted that in the case of a new office the initial expenses must be met from some outside source, and that the system suggested would not apply.

[Mr. Maltby has asked us to publish the following further comments on the points raised in the discussion of his paper.—EDS., *J. I. A.*]

The differences of opinion between Mr. Rietschel and myself mainly concern the particular “units” with which certain groups of expenses should be compared. In practice it would only take a few minutes to calculate both sets of ratios, and undoubtedly this would be useful.

My chief objection to Mr. Rietschel's method of dealing with that part of the Oncost incurred in searching for new business is that if this is added to the Initial Expenses, the ratio of the latter to the new sums assured becomes so large that it cannot be borne by many classes of policies, and that hence any allocation of these expenses amongst the different classes of policies can only be made on an empirical basis.

In the two particular cases given by Mr. Rietschel I think that the greater portion of the heavy expenditure should be borne by those who presumably hope to benefit by it; in the first case by the founders of the Company, and in the second case by the Company as a whole. It should not be wholly charged on the new business. I do not think Mr. Rietschel would suggest that the premiums of a new Company should be loaded to cover the heavy initial expenditure necessary during the first few years.

The changes in the Oncost ratio in such cases as these hardly seem important since their cause will readily be recognized; especially if the ratio of the Total Reserves to the Total Sums Assured is calculated.

I think that the Renewal Expenses are more nearly related to the amount to be collected, than to the Total Sums Assured, and for this reason I prefer my own method. Besides, the Total Sums Assured would need to be adjusted for Single Premium Policies, &c., a somewhat troublesome matter.

Mr. Warner's main objections to all such analyses appear to be that:

- I. Expenses were a factor of a very different kind from mortality and interest—mortality and interest were practically certain in their operation, but the other element (expenses) was subject to all kinds of fluctuations.
- II. “The foundation upon which they rested was absolutely riddled and honeycombed with assumptions.”
- III. “Expenditure did not and would not dominate rates, but that rates did and would dominate expenditure.”

As regards the first of these objections, it is true that expenses differ from mortality in so far as they are under human control—

at any rate theoretically—but in this respect they do not differ from salary scales, rates of withdrawal, &c., which are admittedly fit subjects for actuarial investigation. The effects of assumed rates of mortality, interest, and expenses, are all certain in their operations; in the case of all these factors, the rates actually experienced will differ—often widely—from those assumed, and I fail to see why these differences should not be as carefully studied in regard to expenses as in regard to mortality and interest. In the case of the last two factors, the evidence at our disposal points to the fact that, on the whole, our estimates err on the side of safety. Is this the case in regard to expenses? If it is not, the conclusion must be drawn that expenses should be more carefully analysed than either of the other two factors. Expenses are not likely to be more affected by accidental variations than either mortality or interest.

As regards the second objection, while assumptions and reservations must be made if a Paper dealing generally with a subject of this nature is to be kept within reasonable limits, I fail to see how any assumptions necessary vitiate the analysis by an Office of its own expenses. The foundation of such an analysis seems at least as sound as that upon which a mortality investigation is based, and the truth of any assumptions made can be readily tested.

The third objection means much the same as the criticism made in the discussion on Mr. Rietschel's paper, that "a better method is to adjust the loadings with an eye to past and future expenses, and having so fixed them, to endeavour to keep the expenses within the estimates." This is no reason for not making an analysis, since an analysis is essential before the expenses allowed for by the loadings can be compared with those actually incurred, in order to see whether the estimates are exceeded. The exact relation between rates and expenditure ought to be known, no matter which of the two is regarded as the dominant factor.

In addition to the calculation of rates, such questions as surrender values, and the proper reserve to be made for limited payment policies—to mention only two out of a large number—involve assumptions as to expenses. Should not, therefore, steps be taken to see how far the assumptions made agree with the facts?

The solitary rate in Section XV was intended solely as an illustration of the process of linking up the results of an analysis with the premiums charged. As it is based on imaginary data it is obviously not put forward as an ideal office premium.

Graduation by Reduction of Mean Square of Error. (III.)

By W. F. SHEPPARD, SC.D., LL.M.

(Continued from Vol. XLVIII, p. 412.)

24. TO graduate the u 's so as to obtain as regular a sequence of 3rd differences as possible, differences of U above

the 3rd being supposed negligible, we proceed as in § 18. The formula for v_r , using $m \equiv 2n + 1$ u 's, will be of the form

$$v_r = u_r + c_4 \delta^4 u_r + c_6 \delta^6 u_r + \dots + c_{2n} \delta^{2n} u_r;$$

and therefore the 3rd difference will be

$$\delta^3 v_{r+\frac{1}{2}} = \delta^3 u_{r+\frac{1}{2}} + c_4 \delta^7 u_{r+\frac{1}{2}} + c_6 \delta^9 u_{r+\frac{1}{2}} + \dots + c_{2n} \delta^{2n+3} u_{r+\frac{1}{2}}.$$

It is clear that the problem of determining the c 's so that the mean square of error of $\delta^3 v_{r+\frac{1}{2}}$ shall be a minimum is the same as that of determining the "best" value of $\delta^3 v_{r+\frac{1}{2}}$ from $2n+4$ u 's for the case of $j=5$ or 6.

- (i) The formula in terms of central differences, for moderately large values of m , is easily obtained from Table I.B (ii) on pp. 381 and 383 of "Reduction." Suppose, for instance, that $m=13$. Then v_r is to be of the form

$$v_r = u_r + c_4 \delta^4 u_r + c_6 \delta^6 u_r + \dots + c_{12} \delta^{12} u_r;$$

and the 3rd difference of the v 's will be of the form

$$\begin{aligned} \delta^3 v_{r+\frac{1}{2}} = & \delta^3 u_{r+\frac{1}{2}} + c_4 \delta^7 u_{r+\frac{1}{2}} + c_6 \delta^9 u_{r+\frac{1}{2}} \\ & + \dots + c_{12} \delta^{15} u_{r+\frac{1}{2}}. \end{aligned}$$

This will involve 16 u 's; and the values of the c 's which make the mean square of error of $\delta^3 v_{r+\frac{1}{2}}$ a minimum are those given in the above-mentioned table (for $j=5$ or 6) under $m=16$, i.e.

$$\begin{aligned} & \delta^3 v_{r+\frac{1}{2}} \\ = & \left\{ \delta^3 - \frac{75}{26} \delta^7 - \frac{40}{13} \delta^9 - \frac{45}{34} \delta^{11} - \frac{84}{323} \delta^{13} - \frac{25}{1292} \delta^{15} \right\} u_{r+\frac{1}{2}}, \end{aligned}$$

or

$$\begin{aligned} 16796 \delta^3 v_{r+\frac{1}{2}} = & \{ 16796 \delta^3 - 48450 \delta^7 - 51680 \delta^9 \\ & - 22230 \delta^{11} - 4368 \delta^{13} - 325 \delta^{15} \} u_{r+\frac{1}{2}}, \end{aligned}$$

so that

$$\begin{aligned} 16796 v_r = & 16796 u_r - 48450 \delta^4 u_r - 51680 \delta^6 u_r \\ & - 22230 \delta^8 u_r - 4368 \delta^{10} u_r - 325 \delta^{12} u_r. \end{aligned}$$

(ii) For the formula in terms of sums, we write

$$M \equiv \frac{1}{5670} (n+1)(n+2)(n+3)(2n-1)(2n+1) \\ (2n+3)(2n+5)(2n+7)(2n+9)$$

and then, with the notation of § 18 (ii),

$$Mr_r = -\frac{1}{3} (n^2-1)(4n^2-1)(2n+3) \{ \sigma^4 u_{r \pm (n+2)} \} \\ + \frac{2}{3} (n+1)(4n^2-1)(17n-6) [\sigma^5 u_{r \pm (n+1\frac{1}{2})}] \\ - \frac{10}{3} (4n^2-1)(25n-3) \{ \sigma^6 u_{r \pm (n+1)} \} \\ + 20(2n-1)(34n-1) [\sigma^7 u_{r \pm (n+\frac{1}{2})}] \\ - 1540(2n-1) \{ \sigma^8 u_{r \pm n} \} \\ + 3080 [\sigma^9 u_{r \pm (n-\frac{1}{2})}].$$

(iii) The formula in terms of u 's is

$$v_r = b_0 u_r + b_1 u_{r \pm 1} + b_2 u_{r \pm 2} + \dots + b_n u_{r \pm n}$$

where, M being as above,

$$b_t = b_0 + \frac{1}{2} t^2 \delta^2 b_0 + \frac{1}{24} t^2 (t^2-1) \delta^4 b_0$$

$$+ \frac{1}{720} t^2 (t^2-1)(t^2-4) \delta^6 b_0$$

$$+ \frac{1}{40320} t^2 (t^2-1)(t^2-4)(t^2-9) \delta^8 b_0,$$

$$M b_0 = + \frac{1}{144} (n+1)^2 (n+2)^2 (n+3)^2 (3n^2+12n-4),$$

$$M \delta^2 b_0 = - \frac{1}{18} (n+1)(n+2)^2 (n+3)(5n^2+20n-3),$$

$$M \delta^4 b_0 = + (n+1)(n+3)(7n^2+28n-2),$$

$$M \delta^6 b_0 = -20(9n^2+36n-1),$$

$$M \delta^8 b_0 = +3080.$$

The results in (ii) and (iii) above are obtained from general formulæ given in "Fitting I" and "Fitting II."

25. These formulæ are not convenient for practical purposes. In the first place, the formula in (ii) involves successive sums up to the 9th inclusive. This inconvenience can be mitigated by calculating the 2nd or 4th differences of the v 's and then performing successive additions (cf. § 9 (iii)); but even so it is necessary to form some of the 9th sums in order to obtain the first few v 's. In the second place, the coefficients are always large. The following are the formulæ in some of the simplest cases:

$$(13 \text{ } u\text{'s}) \quad 16796v_r = -325 + 832 - 910 + 580 - 220 + 40,$$

$$(23 \text{ } u\text{'s}) \quad 4032015v_r = -17250 + 24978 - 15640 \\ + 5595 - 1155 + 110,$$

$$(25 \text{ } u\text{'s}) \quad 4032015v_r = -13455 + 17949 - 10350 \\ + 3404 - 644 + 56,$$

$$(27 \text{ } u\text{'s}) \quad 2966700v_r = -7830 + 9675 - 5175 \\ + 1575 - 275 + 22;$$

the coefficients on the right-hand side being those of

$$(13 \text{ } u\text{'s}) \quad \{\sigma^4 u_{r \pm 8}\}, [\sigma^5 u_{r \pm 7\frac{1}{2}}], \dots [\sigma^9 u_{r \pm 5\frac{1}{2}}],$$

$$(23 \text{ } u\text{'s}) \quad \{\sigma^4 u_{r \pm 13}\}, [\sigma^5 u_{r \pm 12\frac{1}{2}}], \dots [\sigma^9 u_{r \pm 10\frac{1}{2}}],$$

$$(25 \text{ } u\text{'s}) \quad \{\sigma^4 u_{r \pm 14}\}, [\sigma^5 u_{r \pm 13\frac{1}{2}}], \dots [\sigma^9 u_{r \pm 11\frac{1}{2}}],$$

$$(27 \text{ } u\text{'s}) \quad \{\sigma^4 u_{r \pm 15}\}, [\sigma^5 u_{r \pm 14\frac{1}{2}}], \dots [\sigma^9 u_{r \pm 12\frac{1}{2}}].$$

For actual calculation, adaptations may be made; thus for the fourth of these formulæ the calculations will be simplified by multiplying both sides by 4.

Although the formulæ are not convenient, their investigation is useful as affording a standard by which to judge the success of any formula that aims at regularizing the 3rd differences on the assumption that 4th differences are negligible. If we denote by R^2 the ratio of the mean square of error of the new to that of the original 3rd difference, so that R is the "smoothing coefficient", then the value of R^2 for the formula of § 24 is

$$R^2 = (9n^2 + 36n - 1)/M,$$

where M has the value given in § 24 (ii). This gives the following values of $1/R$:—

No. of u 's in r .	$1/R$.	No. of u 's in r .	$1/R$.
5	4	19	153
7	9	21	208
9	17	23	276
11	30	25	358
13	49	27	457
15	75	29	574
17	109	31	710

No other formula can give a larger value of $1/R$, for the number of u 's involved, than is shown in this table. Spencer's 21-term formula, for instance, is

$$350r_r = [5][5][7]\{[3][3] - [7]\}u_r,$$

and gives $1/R=160$, which is less than the value (208) in the table. It is, however, slightly better than the value for my 19-term formula.

26. For purposes of comparison, I give in Example 4 (p. 157) the results of applying Spencer's formula and my 23-term formula (the coefficients in the 21-term formula being too clumsy) to the data of Example 2, the graduated table running from $x=56$ to $x=81$. The degrees of smoothness in the two cases may be roughly compared by comparing the sums of the squares of the 4th differences. As it happens, Spencer's formula appears on the whole to give a better result than mine. But it will be seen that this is due to the last four or five values; if these are left out of account, my 3rd differences will be found to be a good deal more regular.

The v 's in this example have been given to 6 decimal places. But this is only done for the purpose of comparison of 3rd differences. The data are not sufficiently numerous to justify us in going to more than 4 or 5 places; and there is the further question whether a 6-place table will not be vitiated by the introduction of the elements due to the 4th-difference term in the graduating formula.

27. This leads to the further question :—How are we to tell what differences are negligible? As has been pointed out in § 4, all graduation methods which consist in replacing u_r by a linear compound of u 's are based on the implicit assumption

that there are some orders of differences which may be neglected; and the assumption ought to be checked before the graduated table is adopted.

The proper method of testing is to obtain improved values for the first order of negligible differences. If, for instance, we assume that $j=3$, we must obtain improved values for the 4th differences. The process is analogous to that of improving the 2nd differences for $j=2$ or 3 (§§ 15, 16); and it does not seem necessary to go into it fully here. If we use $2n+1$ u 's for improving $\delta^4 u_r$, and if we write

$$M \equiv \frac{1}{22680} (n-1)n(n+1)(n+2) \\ (2n-3)(2n-1)(2n+1)(2n+3)(2n+5),$$

then the improved value of $\delta^4 u_r$ is given by

$$M\delta^4 u_r = \frac{1}{6} (n-1)n(2n-3)(2n-1) [\sigma^4 u_{r \pm (n+\frac{1}{2})}] \\ - \frac{5}{3} (n-1)(2n-3)(2n-1) \{\sigma^2 u_{r \pm n}\} \\ + 15(n-1)(2n-3) [\sigma^2 u_{r \pm (n-\frac{1}{2})}] \\ - 35(2n-3) \{\sigma^4 u_{r \pm (n-1)}\} \\ + 70 [\sigma^5 u_{r \pm (n-1\frac{1}{2})}] ;$$

and, if the ratio of the mean square of error of $\delta^4 u_r$ to that of $\delta^4 u_r$ is R^2 , then $R^2=1/M$. The following are values of $1/R$ corresponding to values of $2n+1$:—

$2n+1.$	$1/R.$	$2n+1.$	$1/R.$
5	1	19	904
7	7	21	1429
9	27	23	2165
11	71	25	3165
13	156	27	4490
15	304	29	6210
17	542	31	8403

The formulæ which involve 17, 25, and 31 u 's are fairly convenient; the σ 's involved being

$$(17 \text{ } u\text{'s}) \quad [\sigma^4 u_{r \pm 8\frac{1}{2}}], \quad \{\sigma^2 u_{r \pm 8}\}, \quad \dots \quad [\sigma^5 u_{r \pm 6\frac{1}{2}}], \\ (25 \text{ } u\text{'s}) \quad [\sigma^4 u_{r \pm 12\frac{1}{2}}], \quad \{\sigma^2 u_{r \pm 12}\}, \quad \dots \quad [\sigma^5 u_{r \pm 10\frac{1}{2}}], \\ (31 \text{ } u\text{'s}) \quad [\sigma^4 u_{r \pm 15\frac{1}{2}}], \quad \{\sigma^2 u_{r \pm 15}\}, \quad \dots \quad [\sigma^5 u_{r \pm 13\frac{1}{2}}]$$

and their coefficients in the formula for $\delta^4 v_r$, being given by

$$(17 \text{ } u\text{'s}) \quad 8398\delta^4 v_r = +52 - 65 + 39 - 13 + 2,$$

$$(25 \text{ } u\text{'s}) \quad 2861430\delta^4 v_r = +3036 - 2530 + 990 - 210 + 20,$$

$$(31 \text{ } u\text{'s}) \quad 2017356\delta^4 v_r = +783 - 522 + 162 - 27 + 2.$$

28. I have applied the formula which involves 31 u 's to the data of Example 2, with the following result:

x	$10^7\delta^4 v$	x	$10^7\delta^4 v$
60	+ 7	69	+ 39
61	+ 27	70	+ 17
62	+ 28	71	- 3
63	+ 35	72	+ 12
64	+ 16	73	- 30
65	+ 15	74	+ 36
66	+ 14	75	- 13
67	+ 6	76	- 49
68	+ 9	77	- 78

The later values are rather irregular, but, so far as the earlier ones are concerned, it will be seen that we must allow for the existence of a 4th difference amounting to 3×10^{-6} . This is multiplied, in the graduating formula, by a factor which may be as great as 10 or 20. Spencer's formula, for instance, is equivalent to

$$v_r = u_r - 12 \cdot 6 \delta^4 u_r - \&c.;$$

and the formula used in § 11 for Example 2 is ("Reduction", p. 379)

$$v_r = u_r - 18 \delta^4 u_r - \&c.$$

These formulæ may therefore introduce into v a mistake amounting (numerically) to 4 or 5 in the 5th decimal place. Obviously we cannot give v to 5 places; and, even if we take it to 4 places, there will in 40 or 50 per-cent of the values be a mistake of 1 in the final figure.

It will be seen from Example 4 that, if we take v to 4 places only, the 3rd difference will be so small that it is hardly worth while tabulating it. This illustrates the weakness of the whole theory of graduating so as to smooth the highest non-negligible differences, except in the very unusual case in which these differences alter so slowly that the next

difference is very small in comparison with them. In ordinary cases, if the smoothing of the differences is successful, the smoothed values will indicate the existence of a difference of the next higher order; which is contrary to the hypothesis that this difference is negligible. The contradiction will be due to our having taken the values to a larger number of decimal places than is justified. To avoid this, we must take them to a smaller number of places; and then the true values of the differences we are smoothing will be so small that the smoothing will not be worth while.

29. The only point that remains to be dealt with is the treatment of the u 's at the two ends of the range. This is referred to in § 10 (b); as the process is not of special importance in actuarial investigations, it will be sufficient to treat it briefly.

Suppose that our data are u_1, u_2, \dots , and that for improving u_7, u_8, \dots we use 13 u 's in each case. We could use the first 11 u 's for improving u_6 ; the first 9 for improving u_5 ; and so on. But we may as well use 13 throughout; i.e., use u_1, u_2, \dots, u_{13} for improving not only u_7 but also u_6, u_5, \dots, u_1 . It is found—as already stated at the beginning of § 9—that the improved values are those that would be obtained by fitting a polynomial of degree j to these 13 u 's by the method of least squares. The resulting v 's will have a constant difference of order j , and their values can be constructed if we know those of $v_1, \Delta v_1, \dots, \Delta^j v_1$. General formulæ for doing this are given in "Fitting I" and "Fitting II."

Suppose that we are using the first m u 's. Then we construct the Hardy sums as shown in the following scheme:

$$\begin{array}{ccccccc}
 & & \Sigma'' u_1 & & & & \\
 u_1 & & & & \Sigma''^2 u_2 & & \\
 & & \Sigma'' u_2 & & & & \Sigma''^3 u_3 \\
 u_2 & & & & \Sigma''^2 u_3 & & \\
 & & \Sigma'' u_3 & & & & \Sigma''^3 u_4 \\
 u_3 & & & & \Sigma''^2 u_4 & & \vdots \\
 & & \Sigma'' u_4 & & \vdots & & \vdots \\
 u_4 & & \vdots & & \vdots & & \vdots \\
 \vdots & & \vdots & & \vdots & & \vdots \\
 \vdots & & \vdots & & \vdots & & \vdots \\
 \vdots & & \vdots & & \vdots & & \vdots \\
 u_m & & \Sigma'' u_m & & \Sigma''^2 u_m & & \Sigma''^3 u_m \\
 & & & & \Sigma''^2 u_{m+1} = 0 & & \Sigma''^3 u_{m+1} = 0 \\
 & & \Sigma'' u_{m+1} = 0 & & & &
 \end{array}
 \quad \&c.$$

in which each term is the difference, with sign changed, of the two next it on the right. The values of v_1 , Δv_1 , $\Delta^2 v_1$, . . . can then be conveniently expressed in terms of $\Sigma'' u_1$, $\Sigma''^2 u_2$, $\Sigma''^3 u_3$, . . .

For finding the initial v 's in Example 1 (the values in italic type on p. 184 of vol. xlviii) it has been assumed that $j=2$. Writing

$$M = \frac{1}{12}(m-2)(m-1)m(m+1)(m+2),$$

the values of v_1 , Δv_1 , $\Delta^2 v_1$ are then given by

$$\begin{aligned} M v_1 &= +A_{1,1} \Sigma'' u_1 - A_{1,2} \Sigma''^2 u_2 + A_{1,3} \Sigma''^3 u_3, \\ M \Delta v_1 &= -A_{2,1} \Sigma'' u_1 + A_{2,2} \Sigma''^2 u_2 - A_{2,3} \Sigma''^3 u_3, \\ M \Delta^2 v_1 &= +A_{3,1} \Sigma'' u_1 - A_{3,2} \Sigma''^2 u_2 + A_{3,3} \Sigma''^3 u_3, \end{aligned}$$

where

$$\begin{aligned} A_{1,1} &= \frac{1}{4}(m-2)(m-1)(3m^2-3m+2), \\ A_{1,2} &= A_{2,1} = (m-2)(m-1)(3m-4), \\ A_{2,2} &= 4(m-2)(4m-7), \\ A_{1,3} &= A_{3,1} = 5(m-2)(m-1), \\ A_{2,3} &= A_{3,2} = 30(m-2), \\ A_{3,3} &= 60. \end{aligned}$$

For $m=13$ this gives

$$\begin{aligned} 1001 v_1 &= +517 \Sigma'' u_1 - 154 \Sigma''^2 u_2 + 22 \Sigma''^3 u_3, \\ 1001 \Delta v_1 &= -154 \Sigma'' u_1 + 66 \Sigma''^2 u_2 - 11 \Sigma''^3 u_3, \\ 1001 \Delta^2 v_1 &= +22 \Sigma'' u_1 - 11 \Sigma''^2 u_2 + 2 \Sigma''^3 u_3. \end{aligned}$$

The calculation of v_2 , v_3 , . . . v_6 is then very simple.

A similar process is adopted with the u 's at the other end of the range.

ERRATUM.

J.I.A., vol. xlviii, p. 177, line 4 from bottom :

for $-23(u_{11}-u_3)$ read $-24(u_{14}-u_3)$.

EXAMPLE 4.—*Graduation of data of Example 2 by Spencer's 21-term formula and by new 23-term formula.*

x	OBTAINED BY SPENCER'S FORMULA				OBTAINED BY NEW FORMULA			
	r	$10^6 \delta r$	$10^6 \delta^2 r$	$10^6 \delta^3 r$	r	$10^6 \delta r$	$10^6 \delta^2 r$	$10^6 \delta^3 r$
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		+				+	+	
56	·006581	431	·006562	448
7	·007012	533	+ 102	0	·007010	584	136	— 34
8	·007545	635	102	+ 22	·007594	686	102	— 43
9	·008180	759	124	— 32	·008280	745	59	— 32
60	·008939	851	92	— 36	·009025	772	27	+ 7
1	·009790	907	56	— 55	·009797	806	34	— 19
2	·010697	908	+ 1	— 31	·010603	821	15	+ 5
3	·011605	878	— 30	— 7	·011424	841	20	+ 28
4	·012483	841	— 37	+ 60	·012265	889	48	+ 20
5	·013324	864	+ 23	+ 52	·013154	957	68	+ 36
6	·014188	939	75	+ 100	·014111	1061	104	+ 45
7	·015127	1114	175	+ 71	·015172	1210	149	+ 46
8	·016241	1360	246	+ 58	·016382	1405	195	+ 47
9	·017601	1664	304	+ 11	·017787	1647	242	+ 14
70	·019265	1979	315	— 11	·019434	1903	256	+ 7
1	·021244	2283	304	— 69	·021337	2166	263	— 28
2	·023527	2518	235	— 44	·023503	2401	235	0
3	·026045	2709	191	— 18	·025904	2636	235	+ 38
4	·028754	2882	173	+ 39	·028540	2909	273	+ 2
5	·031636	3 94	212	+ 77	·031449	3184	275	+ 57
6	·034730	3383	289	+ 100	·031633	3516	332	— 7
7	·038113	3772	389	+ 63	·038149	3841	325	+ 83
8	·041885	4224	452	+ 58	·041990	4249	408	+ 116
9	·046109	4734	510	+ 56	·046239	4773	524	+ 81
80	·050343	5300	+ 566	...	·051012	5378	605	...
1	·056143		·056390	

The Medico-Actuarial Investigation of the Mortality of American and Canadian Life Assurance Companies.

[Communicated by MR. ARTHUR HUNTER, A.I.A., F.F.A., Chairman of the Central Bureau.]

THE final volume (V) of the Report on the Medico-Actuarial Mortality Investigation has recently been published. A synopsis of the principal contents of the volume is here given.*

FAMILY HISTORY OF TUBERCULOSIS.

Policyholders with a family history of tuberculosis were divided into seven classes, but in the three following classes the amount of data was too small to be of value.

1. Tuberculosis in both parents and one or more brothers or sisters ;
2. Tuberculosis in one parent and two or more brothers or sisters ;
3. Tuberculosis in both parents.

When a member of the family suffered from tuberculosis at date of application for insurance, it was considered to have the same weight as a death in the family from that disease.

Cases treated as sub-standard for physical condition or personal history were excluded, but cases sub-standard for family history alone were included.

TUBERCULOSIS IN ONE PARENT AND A BROTHER OR SISTER.

When all the insured in this class ranging from 45 pounds underweight to 45 pounds overweight, and from 5 feet 3 inches to 6 feet 2 inches were put in a single group, the mortality was only 11 per cent. in excess of the normal (the M.A. Select Table). The relative mortality for the three main plans of insurance, Ordinary Life, Limited Payment Life, and Endowment, was 114 per cent., 107 per cent., and 111 per cent. respectively. As it was known that weight was an important factor in connection with the mortality of persons with a tubercular family history, the entire data were then divided into four weight groups, of which the following is a synopsis :

* Abstracts of the contents of vols. i-iv will be found in vol. xlvii of this *Journal*, pp. 384-394; vol. xlviii, pp. 414-421 and 539-548; vol. xlviii, pp. 186-197.

Ages at Entry	WEIGHT GROUP -25 TO -45 LBS.			WEIGHT GROUP -5 TO -20 LBS.		
	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	3	1.29	233%	33	24.28	136%
30-44	42	31.81	132%	122	110.02	111%
45 & over	44	46.30	95%	78	83.30	94%
Total ...	89	79.40	112%	233	217.60	107%

	WEIGHT GROUP AVERAGE TO +20 LBS.			WEIGHT GROUP +25 TO +45 LBS.		
	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	43	25.43	169%	6	3.52	170%
30-44	95	93.32	102%	42	30.92	136%
45 & over	82	85.00	96%	38	31.88	119%
Total ...	220	203.75	108%	86	66.32	130%

While the statistics are not sufficiently extensive to justify positive deductions, the foregoing table indicates that a history of tuberculosis in one parent and a brother or sister is accompanied by a decided increase in the mortality at the younger ages at entry.

TUBERCULOSIS IN TWO OR MORE BROTHERS OR SISTERS.

The following is a synopsis of the results of the investigation into the above class:

Ages at Entry	WEIGHT GROUP -25 TO -45 LBS.			WEIGHT GROUP -5 TO -20 LBS.		
	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	5	2.18	229%	41	29.31	140%
30-44	39	38.95	100%	208	172.15	121%
45 & over	64	83.62	77%	145	163.68	89%
All ages	108	124.75	87%	394	365.14	108%

	WEIGHT GROUP AVERAGE TO +20 LBS.			WEIGHT GROUP +25 TO +45 LBS.		
	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	44	33.08	133%	4	6.79	59%
30-44	145	157.19	92%	65	54.97	118%
45 & over	144	159.00	91%	59	49.36	120%
All ages	333	349.27	95%	128	111.12	115%

The mortality in the lightest weight group is lower than in the next group—a result largely due to more severe selection; evidence of which selection appears in the different distribution of the cases by age at entry.

TUBERCULOSIS IN ONE BROTHER OR SISTER.

This is a large group, consisting of more than 114,000 entrants, among whom there were 6,317 deaths. The statistics were divided into the four weight groups already mentioned, and sub-divided according to the three main plans of insurance—Ordinary Life, Limited Payment Life, and Endowment Insurance. There was a much larger proportion of entrants at the younger ages of entry under the higher priced, than under the lower priced plans. No serious distortion of the facts results from combining the various plans of insurance, and accordingly a synopsis is given for the four weight groups, all plans of insurance being combined:

Ages at Entry	WEIGHT GROUP -25 TO -45 LBS.			WEIGHT GROUP -5 TO -20 LBS.		
	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	76	40.08	190%	644	454.64	142%
30-44	365	365.91	100%	1,230	1,285.77	96%
45 & over	330	441.74	75%	681	921.27	74%
Total ...	771	847.73	91%	2,555	2,661.68	96%
	WEIGHT GROUP AVERAGE TO +20 LBS.			WEIGHT GROUP +25 TO +45 LBS.		
	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	522	432.57	121%	70	66.22	106%
30-44	1,001	1,105.18	91%	353	338.80	104%
45 & over	735	844.66	87%	310	274.84	113%
Total ...	2,258	2,382.41	95%	733	679.86	108%

The statistics of which the foregoing table is a synopsis indicate that for ages at entry above say 35 the death of one brother or sister from consumption may be disregarded. It should be remembered, however, that a more severe selection was probably exercised among those underweight than among those of or above average weight. In the detailed tables it is

shown that the mortality ratios among the underweight groups tend to decrease with the increasing duration of the insurance, while the reverse holds good among overweights. The Committee points out that this should be carefully considered in ascertaining the extra premium to meet the additional hazard.

The high death rate from tuberculosis of the lungs among the underweights may be seen from the following tables :

Ages at Entry, 15 to 29.

Variation from Average Weight	Total Deaths	Deaths from Tuberculosis of Lungs	Ratio
- 25 to - 45 pounds	76	39	51%
- 5 to - 20 "	644	309	48%
0 to + 20 "	522	176	34%
+ 25 to + 45 "	70	6	9%

Ages at Entry, 30 to 44.

- 25 to - 45 pounds	365	97	27%
- 5 to - 20 "	1,230	322	26%
0 to + 20 "	1,001	119	12%
+ 25 to + 45 "	353	9	3%

In this class, and also the one comprising a family history of tuberculosis in one parent, an investigation was made of the effect of *height* on mortality. If the number of pounds departure from the average weight is considered, it appears that tall men at the younger ages at entry among underweights experienced a relatively higher mortality than short men.

TUBERCULOSIS IN ONE PARENT.

In this class there were nearly 95,000 entrants, and 4,405 deaths. While a subdivision was made of the data into the three main plans of insurance already mentioned, the synopsis for all plans of insurance only is given, as there did not appear to be a marked variation in the mortality under the different plans. As already noted in connection with the preceding class some of the companies limited the poorer risks to the high premium plans, which is shown by the large proportion of higher priced policies at the young ages at entry.

ALL PLANS COMBINED.

Ages at Entry	WEIGHT GROUP -25 TO -45 LBS.			WEIGHT GROUP -5 TO -20 LBS.		
	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	59	46.64	127%	658	532.48	124%
30-44	302	281.96	107%	911	1,001.31	91%
45 & over	156	222.45	70%	357	465.19	77%
Total ...	517	551.05	94%	1,926	1,998.98	96%
	WEIGHT GROUP AVERAGE TO +20 LBS.			WEIGHT GROUP +25 TO +45 LBS.		
	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	475	454.53	105%	71	62.55	114%
30-44	670	761.45	88%	216	215.65	100%
45 & over	359	449.78	80%	171	145.29	118%
Total ...	1,504	1,665.76	90%	458	423.49	108%

The striking feature of the above table is the very good mortality at the older ages at entry except in the overweight groups. It is probable that this low mortality was largely due to a severe selection on account of the family history. The same conditions were found with regard to the young ages at entry as in the class of those with a family record of one brother dead or suffering from tuberculosis—namely, a high mortality among the light weights. It is apparent from the above that tuberculosis in one parent seems to be of comparatively little moment except at the younger ages at entry, provided as severe a standard of selection is practised in the future as in the past.

The average age at entry in the group of those 25 to 45 pounds under the average weight is much higher than in the group of those 5 to 20 pounds under the average weight. This indicates that the companies delayed the acceptance of many risks distinctly underweight until it was thought that the hazard from the tuberculous family history was past.

An endeavour was made to determine whether a family record of tuberculosis in a parent or in a brother or sister was the more serious. A comparison of the experience in the two classes shows that the incidence of mortality is similar. At the younger ages of entry a family record of tuberculosis appears to be of more

consequence in the case of a brother or sister than of a parent, but this may be due to a more rigid selection in the latter class. At ages at entry 15 to 29 in the group of men from 5 pounds to 20 pounds under the average weight, 52 per cent. of the total deaths during the first ten policy years were due to tuberculosis of the lungs in the class of the insured with a family history of tuberculosis in one brother or sister; while the corresponding percentage was 48 in the case of those with a family record of tuberculosis in one parent.

MORTALITY IN SOUTHERN STATES OF THE UNITED STATES.

It has generally been believed that the mortality experience of insurance companies in the Southern States was higher than in the other States of the United States. The companies were requested to supply information regarding their experience in seven Southern States, but as this was optional, only fourteen of the forty-three companies gave their material. The seven States investigated were Alabama, Arkansas, Florida, Georgia, Louisiana, Mississippi and Texas. With the exception of Georgia, parts of these States border on the Gulf of Mexico, and are generally low-lying. The same is true of the counties alongside the great rivers, such as the Mississippi. These are generally known as the Delta Counties.

Alabama, Florida, Louisiana and Texas were divided into (1) Gulf Counties, (2) All Other Counties; Arkansas into (1) Delta, (2) Other Counties; Georgia into (1) Seacoast, (2) Other Counties. Mississippi was divided into (1) Gulf, (2) Delta, (3) Other Counties; but the amount of data received by the Committee was not large enough to make a division into Gulf and Delta Counties, and, accordingly, the statistics of these counties were combined. With the exception of the State of Texas, the mortality of the various sections ranged from 131 per cent. to 178 per cent. In Texas it was 117 per cent. in the Gulf Counties, and 112 per cent. in other counties, which generally consist of great plains or prairies.

The data were subdivided by period of issue, in order to determine whether there had been an improvement in the mortality. In the majority of the States there had been a substantial improvement.

The mortality in the Southern States from typhoid fever was about $1\frac{1}{2}$ times the standard, and from malarial fever about 7 times the standard.

MALARIAL FEVER.

Reports are given of the mortality among men who gave a history of having had malarial fever. Three different types of malarial fever were investigated: (a) pernicious, (b) remittent, (c) all other kinds of malarial fever, including ordinary chills and fever. There was a large amount of data under (c), sufficient to enable a division according to States, and frequently according to sections of States. This matter is not of a great deal of moment to those living outside the United States, and it is therefore only necessary to mention that a distinctly high mortality prevailed among residents of the seven Southern States who had been accepted within two years of an attack of ordinary malaria. A single attack of malaria seemed to be of little moment in other parts of the United States and in Canada; in the case of men who had one attack within two years of application, the mortality was 105 per cent., and among those with two or more attacks, 110 per cent.

It does not appear that a history of malarial fever in any Southern State has resulted in a materially higher mortality than among the insured as a body in that State, but the history of *recent* malarial fever seems to have been of importance. The improvement in mortality in cases with a history of malarial fever in the Southern States as a whole is much greater than among similar cases in other parts of the United States and Canada.

MORTALITY UNDER JOINT LIFE POLICIES.

There were seven classes of policies investigated which were issued on the Joint Life plan, but in only four of these were the data sufficient to justify publication. A synopsis is as follows:

Classes 80 and 81—Joint Life, males.

Plan	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Ordinary Life	58	52.14	111%
Limited Payment Life and Endowment	30	31.31	96%

Joint Life, male and female.

Ordinary Life	334	272.63	123%
Limited Payment Life and Endowment	215	203.53	106%

The foregoing groups showed traces of selection adverse to the company. The data relating to the Joint Life Insurance on a man and a woman were subdivided, and the relative mortality among the women was determined. It was noted that there was a high mortality among the women at the younger ages at entry, but that at the middle and older ages they had a better relative mortality than men. A notable feature of the insurance of a man and a woman on Joint Life plans was a high death rate from tuberculosis of the lungs at the young ages at entry on the Ordinary Life plans, and a heavy death rate from child-birth among the women.

The Committee in their concluding remarks pay a tribute to the efficiency of the system of perforated cards and electric sorting and tabulating machines. These recent mechanical devices have enabled an investigation covering about three million policies to be handled by a force of women clerks which never exceeded fifteen. It is significant of the zeal of the members of the Committee that after concluding an investigation which has involved much labour to them, one of the last sentences of their report should be: "The results of this investigation have been so important that the companies may decide to conduct a similar one in the future"; and they express a hope that "the companies will maintain their records in such shape that an investigation can be made without having to refer again to the applications and the reports of the medical examiners."

LEGAL NOTES.

By WILLIAM CHARLES SHARMAN, F.I.A., *Barrister-at-Law*.

THE question as to whether the Public Trustee may accept the trusteeship of any settlement other than an English one recently came before Mr. Justice Eve and was decided by him in the negative. The case is *In re Hewitt's Settlement, Hewitt v. Hewitt* and is reported 1915 L.R. Ch. 228.

Public Trustee.
Power to accept
trusteeship of
foreign settlement.

The plaintiffs were the present trustees of a marriage-contract entered into by one H. M. Hewitt, who was a domiciled Englishman, and his wife, who was domiciled and resident in Scotland. The marriage-contract when executed was in form and substance

a Scotch settlement. H. M. Hewitt died 11 January 1914, leaving his wife and two children under age 21, all now resident in England. The securities in which the husband's and wife's funds were invested were English and included freehold ground rents in England which the trustees desired to retain.

The plaintiffs were all desirous of retiring from the trusts of the settlement and of appointing the Public Trustee to be the sole trustee. The Public Trustee was willing to accept the trust if the Court held that he had power to do so.

On 21 October 1914 the trustees took out a summons to which the widow and the Public Trustee and an infant child of the marriage were made respondents, asking, *inter alia*, whether the plaintiffs, as such trustees, had power under the provisions of the Public Trustee Act, 1906 or otherwise, to appoint the Public Trustee to be a trustee of the property subject to the trusts of the deed either alone or jointly with the plaintiffs.

Eve, J., in giving judgment said: "I think that both in
" form and substance this was intended to be a Scotch settle-
" ment to be governed and construed by the law of Scotland, and
" if so, it still remains a Scotch settlement, notwithstanding
" the facts that the beneficiaries and the persons who now claim
" to be the trustees are all resident in England, and that the
" whole of the trust fund is invested in securities in England.
" These circumstances, although they cannot alter or affect
" the intention of the parties when the deed was executed in
" 1890 (see per Vaughan Williams, L.J., in *In re Fitzgerald*
" (1904) 1 Ch. 573, 578), constitute a sufficient reason for ap-
" pointing the Public Trustee to be sole Trustee of the settlement
" if he is eligible, and subject to the question of his competency
" he has intimated his willingness to accept the trust. In order
" to ascertain whether he can do so it is necessary to consider
" the provisions of the Act by which the office was established
" and its power defined, and in my opinion that Act contains
" nothing which warrants the appointment of the Public Trustee
" to be a trustee of what I will call a foreign settlement—that is
" to say, a settlement construed and governed by the law of any
" foreign country, or of any part of the United Kingdom to
" which the Act itself does not extend. . . . Not only is there
" a complete absence from the Act of any enabling words, or
" even provisions, from which we could gather an intention on
" the part of the Legislature to extend the powers of the office
" to the acceptance of foreign trusts, but there are many

“ indications pointing to the contrary conclusion. It is enough
 “ for me, I think, to say that in the absence of statutory power,
 “ either express or necessarily to be implied from what is expressed,
 “ I cannot hold that the Public Trustee can accept the trustee-
 “ ship of a foreign settlement, and I decide that no such statutory
 “ power at present exists. This conclusion does not, of course,
 “ preclude him from executing the trusts of an English settle-
 “ ment involving the management or realization of, or the
 “ intermeddling with, property outside the jurisdiction. That
 “ is quite a different thing.”

The case of *In re the Tean Friendly Society (Distribution of Assets)* deals with the interesting point as to what constitutes an equitable distribution of assets on the winding-up of a Friendly Society.

Friendly Society.
 Equitable distri-
 bution of assets
 on winding-up.

The case, apparently, has not been reported, and I am indebted to Mr. T. G. Ackland for the following particulars of the facts and for the material part of the Order.

The Society was founded more than a hundred years ago, and provided for its members medical aid, sickness benefits up to age 70, a pension after that age, and death benefits to members and their wives, in consideration of contributions ceasing at age 70. It was not registered under the Friendly Societies Acts. An actuarial valuation was made as at 31 December 1880, which disclosed a surplus of £443, part of which the Actuary recommended to be applied in increasing the sick benefits and reducing the contributions of members existing at the date of the valuation. These extensions of benefit and reduction of contributions were in fact applied also to all members entering after that date. In June 1894 an additional rule was passed, providing that, at the end of every three years, the “ surplus income ” of the period should be divided among the existing members, in proportion to the time that each had been a member ; and acting upon this rule the increase in the assets, or so called “ surplus income ” (irrespective of actuarial liabilities) was divided up among the members at triennial intervals from 1896 to 1911 inclusive, the total amount so divided being £2,419. A valuation, made as at 31 December 1910, but not completed until after the 1911 “ bonus ” had been distributed, disclosed a deficiency of £873. A further valuation was made as at 29 June 1912, by the Actuary appointed by the Court, when an increased

deficiency of £1,047 was disclosed. No actuarial valuation appears to have been made between 1880 and 1910.

After the passing of the National Health Insurance Act, 1911, the members decided to dissolve the Society and divide the available funds amongst the members; and, on 29 June 1912, the payments of contributions ceased, benefits being, however, continued until 5 April 1913. On 7 December 1912, a resolution was passed that, in the event of dissolution, all the assets should be distributed amongst the members, in proportion to the aggregate contributions of such members, excluding fines, and after deduction of the amounts received by them in respect of distributions of "surplus income" by way of "bonus." On 5 April 1913, the above resolution was rescinded, and it was resolved that the Society should be dissolved, if the following conditions should be agreed upon, namely, "That every member "be paid out the sum-total of contributions he has paid into the "Club, not including entrance fees, and after deducting such "part of his contributions as represented medical fees; that "members on old-age pay shall have the choice of an annuity, "or their sum-total of contributions, exclusive of the entrance "and medical fees as above; that the balance of the Funds, "after paying out the sum-total of contributions to members "and providing annuities for those entitled to them, be divided "equally amongst those members who do not elect to receive "annuities." These conditions were assented to by 268 out of 277 members, but one of the objecting members commenced proceedings against the Trustees of the Society and obtained an injunction against their distributing the assets, and subsequently presented a petition to wind-up, upon which a winding-up order was made on 11 November 1913.

The case came in the first instance before Mr. Justice Astbury, who consented to the appointment of an Actuary to prepare a scheme for equitable distribution of the assets amongst the members. In his report, dated 14 July 1914, the Actuary submitted a scheme for distribution, based upon the net liabilities in respect of each member, but providing in the first instance for annuities (on the Post Office scale) for the 10 members over 70 years of age. The Actuary considered that the "bonuses" of 1908 and 1911 should be brought into account, as not in the nature of true "surplus", but, in the absence of any information as to the actuarial position of the Society between 1880 and 1910, left it to the Court to determine whether and to what extent

the bonuses, from 1896 to 1905 inclusive, should be similarly brought into account.

The case came before Mr. Justice Neville on 15 and 16 December 1914, when the interests of the younger members, the older members, and the pensioners, were separately represented by Counsel. Ultimately the Judge decided that, in determining an equitable basis for the distribution of the assets, he was not bound to have regard to actuarial bases or recommendations, but rather to settle a scheme in accordance with the laws of equity, as interpreted by reported decisions quoted in the present arguments (*Crunnack v. Edwards*, 1895, 1 Ch. 489 ; do. in Appeal Court, 1896, 2 Ch. 681 ; *The Printers and Transferrors, &c., Society*, 1899, 2 Ch. 184 ; *The Lead Companies Workmen's Fund Society*, 1904, 2 Ch. 196). These cases favoured the view that the distribution of funds should be on the basis of the contributions paid by the members, without regard to other considerations. His Lordship therefore made the following Order : " That subject to the costs and expenses of Liquidation, " the balance of the funds in the hands of the applicant as such " Official Receiver and Liquidator ought to be distributed among " persons who were members of the Society on 29 June 1912. " or those claiming under them in proportion to their contributions " and entrance fees paid by them respectively (excluding fines), " without any deduction for triennial bonuses or benefits received " up to that date, but allowance being made for benefits (if any) " received since that date."

Courts
(Emergency
Powers Act, 1914)
Mortgagee in
possession.

It has been questioned whether the exception introduced in Section 1 (1) (b) of the Courts (Emergency Powers) Act 1914 " except by way of sale by a mortgagee in possession " refers only to a mortgagee in possession whose security is real property or also to a mortgagee in possession of personal property. This point was considered by the Court of Appeal in the case of *Ziman v. Komata Reef's Gold Mining Company (Limited)* on 1 March last. The case is of interest in view of the question as to whether a mortgagee of a policy of assurance can be deemed to be a mortgagee in possession for the purposes of the Act.

A report of the case will be found in the *Times* newspaper of 2 March, but as it has not yet appeared in any of the recognized Law Reports it is not proposed to deal with the matter

fully on this occasion. It will be sufficient to state that the Court decided that the exception refers equally to personal property and real estate.

[Indexes of the subjects and cases dealt with in the "Legal Notes" included in vols. xlv to xlviii will be found on pp. 182-192. We are indebted to Mr. E. W. Phillips, F.I.A., and to Mr. E. W. Townley, F.I.A., for the preparation of the indexes.—EDS., *J.I.A.*]

REVIEW.

Stone's Insurance Cases. By GILBERT STONE and KEITH GRIMBLE GROVES, *Barristers-at-law.*

(2 Vols. London: The Reports and Digest Syndicate, Limited.)

THESE volumes are something of a novelty in Insurance literature. The legal text-book may take many forms. It may be in the nature of a digest or a commentary. A digest at its best is on the pattern of a code where the results of existing statute and case law are consolidated in concise statements, logically arranged and usually divided into short paragraphs or sections. To a digest of this kind there is often added at the end of each section or group of sections a summary of the more important cases. The commentary, on the other hand, is on an entirely different pattern from the digest. It is essentially discursive, considers and discusses doubtful points, and deals with the more important cases critically and in detail. The volumes now before us do not purport to be a commentary. As the learned authors state in their preface, they have been careful throughout the work to rely on the decisions of the judges and to eliminate almost to extinction their own opinion. The object aimed at has been to collect together, in logical order, all the English, Scotch, Irish, most of the Colonial and all the Indian cases on the various branches of insurance law apart from marine. The Canadian cases, in particular, we are told have all been included. The manner in which the individual cases are presented to the reader differs very considerably either according to the degree of importance which the authors attribute to the decision in question, or according to the necessity or otherwise of stating the facts in detail so as to give the reader an accurate conception of the point or points decided and the limitations of the decision. Thus, cases like *Canning v. Farquhar*, *Joel v. Law Union and Crown Insurance Co.*, *Anderson v. Fitzgerald* and *Thomson v. Weems*, occupy space to the extent of some two to six pages each, the facts are fully stated and considerable quotations

are given from the opinions of the judges, whereas other cases are presented in the form of a brief proposition of law occupying two or three lines without any reference to the particular facts of the case or any quotation from the opinions of the Court. The case of *Griffiths v. Fleming*, for instance, is disposed of with the statement, "A policy of insurance effected by a husband on the life of his wife may be enforced by him without giving affirmative evidence of any insurable interest therein." The cases are grouped in chapters or parts, and in an introductory note to each part the authors have given a very brief outline of the law relating to the group of cases which follows. These preliminary statements are intended merely to act as finger-posts to the cases and not to be accepted as an exposition of the law. The book, therefore, does not purport to contain a digest of the law of insurance any more than it purports to be a commentary. It is merely a digest of cases on an improved principle which will make it more useful than the ordinary type of law digest. As supplementary to a good text-book it ought to prove extremely valuable, but it is not a work which is designed to be or ought to be used in lieu of a text-book.

As far as those interested only in life insurance are concerned, all that they want will be found in the first volume. This volume contains life and fire cases, while the second volume contains cases relating to accident, fidelity guarantee, and other risks and workmen's compensation cases. We regret to observe that the life cases are not always grouped separately or even distinguished from the fire cases. Thus, in the parts dealing with "Formation of the Contract", "Scope of Agent's Authority", "Contracts *Ultra Vires*", "Mistakes", "Payment of Premiums", life cases are mixed with fire cases, and frequently there is no indication whether the risk in question was fire or life. On the other hand, in dealing with "Illegal Contracts", "Insurable Interest", "Misrepresentation", "Warranties", "Commencement and Duration of Risk", and "Assignment", the life cases are separated from the fire cases, and we think that it would have been more satisfactory if this system had been adopted throughout.

On the whole the work appears to have been very carefully compiled, and the authors have attained a high standard of accuracy. It is very doubtful, however, whether they have from the point of view of the manager of a life office made their collection of insurance cases sufficiently comprehensive. We are told in the preface that they have not included company cases "which are really foreign to insurance law." We had hoped that this statement was not intended to exclude all cases under the Assurance Companies Acts, but we find that it practically does so. There is no reference to cases such as *In re Life and Health Assurance* (1910) 1 Ch. 458, *Nelson & Co. v. The Board of Trade* (1901) 17 T.L.R. 456, and other important cases relating to the liability of a company carrying on one of the five classes of insurance business to make the statutory deposit. Cases relating to amalgamation or transfer of insurance

companies and insurance business are also excluded. It is difficult to appreciate why such cases are said to be foreign to insurance law. Even if insurance law is to be confined to cases which relate directly to the contract of insurance between insurer and assured, all the cases on the statutory deposit ought to have found a place in this collection, since the failure to make the deposit has the effect of rendering illegal and void the contract of insurance with the company. Questions of transfer of insurance business and proof in winding up are also so directly related to the right to enforce the contract of insurance that they might well have been represented by the inclusion of at least the more important decisions. There is not, perhaps, the same reason for criticizing the absence of many important cases upon Income Tax, Powers of Investment, Stamp Duty, Voluntary Settlements, *Bona Fide* Purchasers for Value, Money-lenders Act, Annuities, Rights of Co-sureties, &c., because these cannot strictly be classed as insurance cases. Many of them, so far as the actual decision is concerned, have nothing to do with insurance policies or insurance companies, but at the same time the principles involved are of first-rate and peculiar importance to those engaged in transacting life insurance business. We cannot help thinking that this collection would have been much more useful if, instead of including every trifling Canadian case, the authors had included a more generous selection of cases which, although not insurance cases in one sense, are yet indispensable to anyone who has to deal with insurance claims.

NOTICES OF NEW PUBLICATIONS.

On the graduation of mortality tables by G. F. Hardy's modification of method of moments. By J. F. STEFFENSEN.

From *Scenska Aktuarieföreningens Tidskrift*, 1915.

DR. STEFFENSEN gives us in this paper a discussion of the graduation of the values of $\log p_x$ on Makeham's hypothesis, using the method of moments or rather the modification of that method due to the late Sir George Hardy. Dr. Steffensen first gives a proof that the method of moments and Hardy's modification are identical in result, and then obtains expressions for the successive sums of $a + \beta c^x$ which he solves ingeniously by the use of interest functions. The method is then applied to the Danish experience excluding the first five years of assurance for (1) the whole experience; (2) ages 20 to 64; (3) ages 30 and over; and (4) ages 30 to 79. These results* differ considerably from one another and differ also from the successful graduation of the same experience given by Dr. Steffensen in the Proceedings of the Fifth International

* The accuracy of the work has been assumed.

Congress of Mathematicians, where he applied Makeham's hypothesis by the method of least squares, giving proper weight to the observations. Dr. Steffensen therefore concludes that "the method of moments is a less universal instrument than sometimes assumed."

As will be seen the method adopted is substantially the same as that used by Mr. Lidstone (*J. I. A.*, vol. xxxviii, p. 11) in graduating part of the O^M Table in connection with the group valuation of endowment assurances, but, while Mr Lidstone used graduated values of $\text{colog } p_x$ and merely regraduated them. Dr. Steffensen has worked on the ungraduated material. Clearly Mr. Lidstone was not attempting to produce a graduation of rough data, or he would presumably have followed Sir George Hardy and operated on the actual exposed to risk and deaths, thereby giving weight to the observations. Dr. Steffensen's results do not prove that the method of moments is a less powerful instrument than the method of least squares; they prove that the method of moments applied to a function without taking the number of observations into account may give poor results as compared with the method of least squares when the latter is used in a form that brings in the number of observations.

The results reached by Dr. Steffensen afford a timely warning that the application of the method of moments, or for that matter of almost any other curve-fitting operation, to $\text{colog } p_x$ may give unsatisfactory results if no account is taken of the weights of the observations.

W. P. E.

Tafels der Engelsche Maatschappijen.

(3 Vols.). Amsterdam, 1913-4.

THIS extensive collection of tables, based on the British Offices Life Tables, 1893, has been compiled, under the immediate supervision of Mr. P. Th. de Rijk, acting on the general instructions of a Committee representing a number of the principal companies transacting life assurance and annuity business in Holland, and the Committee have courteously recognized their indebtedness to the Institute and the Faculty (whose consent to the publication of the tables had been readily given) by presenting four copies of the work to the library of each body. Some of the tables relate to special assurance and annuity contracts which are unknown in the United Kingdom, and the rates of interest throughout—4 per cent. for assurances (with a few unimportant exceptions) and $3\frac{1}{2}$ per cent. for annuities—would preclude the tables from being of any general use in the valuation work of British Companies. But cases may arise in research, and occasionally in practice, when they will prove of considerable utility, and the following synopsis (from which we omit the special tables referred to above, and tables already available in the publications of the Institute and Faculty) may therefore be of interest:

Vol.	Function tabulated	Range	Mortality and Interest
I	${}_n P_x$	$x=20$ to 75 ; $n=5$ to $80-x$	OM 4%
	$A_{x:n}, {}^1 P_{x:n}$	$x=20$ to 75 ; $n=1$ to $80-x$	"
	$A_{x:n}$	$x=15$ to 24 ; $n=25-x$ to 50 $x=25$ to 34 ; $n=1$ to 50 $x=35$ to 84 ; $n=1$ to $85-x$	"
	${}_P x:n$	$x=15$ to 19 ; $n=25-x$ to 50 $x=20$ to 34 ; $n=5$ to 50 $x=35$ to 65 ; $n=5$ to $85-x$	"
	D_{xy}, \bar{N}_{xy}	$x=20$ to ω ; $x-y=0$ to 60	"
	$a_{xy}, A_{xy}, {}_P x$	" "	"
	$a_{xy:n}, A_{xy:n}$	$\left\{ \begin{array}{l} x=20 \text{ to } 70-n \\ y=20 \text{ to } 70-n \end{array} \right\} n=2$ to 40	"
	${}_P xy:n$	" $n=5$ to 40	"
II (1)	$a_{[x]:n}, {}_n E_{[x]}$	$x=20$ to 65 ; $n=1$ to $70-x$	$O^{(am)} \& O^{(af)} 3\frac{1}{2}\%$
	${}_P \frac{1}{[x]:n}$	$x=20$ to 65 ; $n=5$ to $70-x$	"
	$n \hat{a}_{[x]}^{(4)} (\text{approx.})$	$x=20$ to 65 ; $n=1$ to $70-x$	"
	$n \hat{a}_{[x]}^{(4)} / a_{[x]:n}$	$x=20$ to 65 ; $n=5$ to $70-x$	"
	$D_{[x]+t[y]+t},$ $\bar{N}_{[x]+t[y]+t},$ $a_{[x]+t[y]+t}$ (all extended)	$x=20$ to ω ; $x-y=0$ to 60	" (two males)
	"	"	" (two females)
	"	"	"
	"	"	"
II (2)	"	"	" (male and female)

It may be mentioned that the policy-values (which are not included in this synopsis) are given to one more significant figure than in the British tables.

All the calculations are stated to have been made in duplicate. The only typographical error we have noticed is in Vol. II (i), where

throughout a considerable section $[x] + n$; $[y] + n$ is erroneously printed $[x + n$; $y + n]$.

The Assurance Magazine. Vol. I.

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(C. & E. Layton, London.)

THE publication of this reprint will be of great interest to actuarial book-lovers. In form it is a very successful "counterfeit presentment" of the original. The text of the latter has been reproduced by a photographic process, and both paper and binding are pleasantly reminiscent of the old Vol. I. It will therefore well serve the purpose of making up Journal sets in which this volume is either missing or defective. At the same time it respects the vested interests of the original by frankly announcing itself as a reprint on the title page and on the first page of each number.

The interest of the volume is of course mainly literary and historical. It contains one or two papers of permanent scientific value, such as Mr. J. A. Higham's on Selection, but the actuary who takes it occasionally from the shelf will probably do so as a rule for the curiosities of the Miscellanea—for the notes on Corpulence in connection with Life Assurance, the Number of People at the Resurrection, and the Fertility of Women in Iceland—or for the sidelight it throws on those leisurely times when an actuary could introduce a numerical illustration with a Latin quotation.

It is unfortunate that it is impossible to insert diagrams by photography. In our copy the diagram showing the mortality amongst officers of the Bengal Army has strayed into Mr. Higham's paper on Selection.

Obituary.

Mr. H. W. Manly.

By the death of Mr. Henry William Manly, on 29 December 1914, the actuarial profession, not only in the United Kingdom, but throughout the world, has lost one of its commanding figures, and many of its members a staunch personal friend.

Mr. Manly was born on April 12, 1844, and educated at the City of London School, where his father was Master of writing, arithmetic and book-keeping. When 15 years old, in 1859, he became a clerk in the London Discount Company, but he did not remain long there. In December, 1860, or January, 1861, there was a vacancy in the small staff of the then London and Provincial Law Assurance Society and Mr. Manly received the appointment. At that time the late Mr. Archibald Day was Actuary of the Company, the late Mr. W. P. Pattison the Chief Clerk, the late Mr. G. W. Berridge the

second Clerk, and Mr. T. E. Young, who is still with us, the junior Clerk, and it was on Mr. Young's transfer elsewhere that Mr. Manly took his place. It is remarkable that all these five men should have achieved distinction in the insurance world, three of them attaining to the Presidency of the Institute of Actuaries, and that two of them, Mr. Pattison and Mr. Manly, should have won, while on the staff of the Company, the Messenger Prize on the first and the second occasions respectively of its being offered for competition.

In 1870 Mr. Day left the London and Provincial Law to become London Representative of the Scottish Widows' Fund, and the late Mr. R. P. Hardy, who was Chief Clerk of the Mutual Life Office, succeeded him. This produced a vacancy at the Mutual and Mr. Manly was appointed. In 1875 he became Assistant Actuary and on 1 January 1876, Actuary and Chief Officer. He remained with the Mutual until 1893 and did excellent work in reorganizing the Actuarial Department of the Company and its affairs generally, and one of the papers which he wrote for the Institute of Actuaries dealt with the special method of distribution of surplus which he devised for the Company. In 1893 he was appointed to the proud post of Actuary of the old Equitable Life Assurance Society, where he remained until 1905, when he was unfortunately compelled to retire on account of ill-health. His health for a year or two was precarious, but to a large extent he recovered his strength, and during the remaining years of his life he carried on a considerable practice as Consulting Actuary, more particularly in connection with staff superannuation funds.

Immediately on entering the service of the London and Provincial Law Mr. Manly became a member of the Institute of Actuaries, being elected Associate on 28 January 1861, but apparently he had been permitted to sit for the first examination the previous December, because he passed the examination in 1860. He pursued his actuarial studies, but was in no hurry to get through the examinations, sitting for the second examination in December, 1862, and for the third in December, 1865, and it is worthy of note that in both these examinations he took first place.

Immediately on passing the third and then final examination of the Institute of Actuaries he was entitled to be transferred to the class of Fellow, but he took this promotion only in 1870, and meantime he had served the Institute in the honourable post of Auditor, and had made his name as an Actuary by winning the Messenger Prize in 1868, his subject being "A comparison of the values of policies as found by means of the various tables of mortality, and the different methods of valuation in use among Actuaries." The essay is a memorable one, and showed considerable originality of mind on the part of the author, because for the first time, and by ingenious methods, he brought out clearly the great differences that may arise in Life Office reserves through the employment of different data, and it had considerable influence in days gone by on the finances of the companies.

On account of his geniality and unostentatious bearing Mr.

Manly was always popular with the insurance fraternity, and his professional progress began early, and was rapid and permanent. He was first elected to the Council of the Institute of Actuaries in 1875, and served in all 35 years as a member of that body. He also filled all the posts of honour which his professional brethren could bestow. As already mentioned, he began by being Auditor in the years 1868-1870. He was elected Honorary Secretary in 1881, and served to 1885; he was Vice-President for two terms, 1886-7 and 1895-7, and Treasurer during the years 1890-1893, and in 1898 he was elected to the Presidency, the highest post of all. Also, in what may be called the actuarial social life, he was always *persona grata*. He was elected a member of the Institute of Actuaries Club in 1878, and in 1890 also a member of the Actuaries Club, there being at that time two of these clubs, mostly of a social character, which were merged as the Actuaries' Club in 1899. He was Secretary of the Institute of Actuaries Club from 1888 to 1894, a post of great influence and responsibility, and he was President from 1894 to 1895. In 1895 Mr. Manly received the honour of election to the membership of the Actuarial Society of America.

Mr. Manly's name figures largely in the index of this Journal. We have already referred to his Messenger Prize Essay, and to his paper upon Distribution of Surplus. As a corollary to the Prize Essay he contributed a paper upon the Construction of Tables of Policy Values, which was read before the Institute in 1869, and some years later he submitted another important paper upon Tontine and Assessment Insurance Schemes. When he had occasion in his professional activities to solve problems of general interest he freely gave to the world the result of his labours. Among such problems was the application of bonus to convert whole life policies into endowment assurances and to shorten the endowment term, and in vols. xxvii and xxviii there are two papers upon this subject with voluminous tables to render the office work short and simple. He also investigated the bearing of consumptive family history upon the acceptability of lives for assurance, and contributed a paper on the subject which appears in vol. xxx. He was President for two years, and delivered two presidential addresses.

It was, however, in the field of staff pension funds that in his later years Mr. Manly mostly worked, and the *Journal* contains many monumental contributions from his pen, with enormous masses of tables to illustrate the principles he evolved. He was the first to publish any systematic essays on this subject, although sporadic papers by other authors had previously appeared from time to time. His work was original, and we believe that he derived but little assistance from other investigators, although others were at the same time in the course of professional practice moving in the same direction, and it is remarkable how closely the principles independently evolved by different thinkers agree, and how even the formulas and notation are very similar. To Mr. Manly, however, belongs the credit, in so far as the public are concerned, of practically creating the mathematics of staff pension funds.

Mr. Manly did not view staff pension funds in any limited way, but included in his researches widows' and orphans' funds, and funds of similar character. The arithmetical work involved in such enquiries was stupendous, and the younger members of the Institute were always delighted to assist Mr. Manly, and in doing so to sit at the feet of a great master. He recognized the value of their help by coupling their names with his own in the titles of a number of his papers.

Among other contributions was one of great value which he wrote in collaboration with Mr. T. G. Ackland on the staff pension funds of Metropolitan Boroughs. His last contribution was a short paper entitled "A Novel Superannuation Scheme," which appeared in the *Journal* in January, 1914.

Mr. Manly attended regularly the International Actuarial Congresses, and we believe that he was present at all the seven which have been held, and he was Treasurer and Secretary for Home Correspondence of the London Congress in 1898. He, however, did not take very active part in the deliberations, there being only two papers from him in the *Transactions*. One of these he submitted to the London Congress, its subject being "The Solution of some problems which frequently arise out of the rules of Pension Funds and Friendly Societies." This created great interest, especially among the foreign delegates. His second Congress paper was read at the Congress held in Amsterdam in 1912, when he dealt with a comparison between the mortality experience of the Equitable Life Assurance Society at the beginning and the end of the 19th century.

Although, as we have said, not a frequent contributor to the *Transactions*, Mr. Manly was always an honoured member, and through his social qualities added much to the success of the Congresses. He was thus better known amongst the actuaries in the British Dominions and in foreign countries than the majority of his confreres.

Mr. Manly's home was a happy one. He married about 45 years ago and was a good husband and kind father, and he has left his life companion and many children and grand-children to mourn his loss.

G. K.

The late SIR G. F. HARDY, K.C.B.

[Mr. Lidstone has kindly communicated for publication, as an Addendum to his Memoir in the January number of the *Journal* the following account by Sir Lionel Abrahams, K.C.B., Assistant Under Secretary of State for India, of Sir George Hardy's connection with the India Office.]

A branch of Government work that absorbed much of Sir George Hardy's attention during the last five years of his life was undertaken at the request of the Secretary of State for India.

His connection with the India Office had indeed begun much earlier, since, as already mentioned, he carried out important investigations into the age distribution and rates of mortality of the population of India as shown by the Indian Census figures of 1881, and did similar work in connection with the figures of 1891 and 1901. (See *Report on the Census of British India* for 1881, volume I, chapter vi, published by Eyre and Spottiswoode, London, 1883; *Census 1891, General of India Tables for British Provinces and Feudatory States*, volume II, pages 143 to 185, same Publishers, 1893; and *Memorandum on the Age Tables and Rates of Mortality of the Indian Census of 1901* by G. F. Hardy, published by the Superintendent of Government Printing, Calcutta, 1905). He had also prepared in 1903-4 for the Secretary of State for India and the Managers of the Bengal Uncovenanted Service Family Pension Fund (a fund not managed by Government but in receipt of Government assistance) Tables of Mortality and Marriage experience which were used as the basis of a new and larger Fund for the Presidencies of Bengal and Madras, established with Government approval and assistance in 1904.

But from 1909 until his death his connection with the India Office was much closer and practically continuous. The India Office, unlike most other Government Offices, has undertaken the establishment and management of several funds by means of which members of the various Indian services provide pensions for their widows and orphans; the most important being maintained respectively on behalf of the Indian Civil Service and the officers of the Indian Army. Subscription being compulsory, the contributors naturally allege occasional grievances against the Government in respect of the rates of subscription and the amount and scope of the benefits. In 1909 Lord Morley appointed a Committee under the presidency of Lord Welby to examine the Indian Military Service Family Pension Scheme; and Sir George Hardy became a member. He subsequently carried out one of the periodical valuations both of that scheme and of the Indian Civil Service Family Pension Scheme. The chief result, in the strictly professional sense, of this work was that by using a much wider experience of mortality, marriage, remarriage and issue than had previously been brought under investigation, he was able to recommend in the case of one scheme that, in spite of a reduction from $4\frac{1}{2}$ to 4 per cent. in the rate of interest allowed by the Government on the balance at the credit of the scheme, the rebates of

subscriptions granted when the $4\frac{1}{2}$ per cent. rate was in force should nevertheless be continued ; while in the case of the other scheme he recommended the grant of valuable new privileges to the subscribers. In both cases his recommendations were adopted. In addition, an entirely new fund, on a more scientific basis, was established for officers joining the Indian Army on and after 1st January, 1915.

But the professional result was not the only result of his connection with the India Office. He found a great pleasure and interest in assisting the work of a busy Government department ; while to the other members of Lord Welby's Committee, all men of financial experience, but without any special actuarial knowledge, it was a liberal (and also a practical) education to discuss the somewhat difficult problems arising out of a Government pension scheme with a colleague so eminent a master of the subject, but so modest, lucid, and patient in understanding and removing the difficulties of non-professional colleagues.

THE INSTITUTE OF ACTUARIES.

Sessional Meetings.

The Council propose that in future an abstract of the paper to be submitted at a sessional meeting shall, as a general rule, be circulated to Fellows and Associates before the meeting. Any Fellow or Associate requiring a full copy of any particular paper may obtain it from the Assistant Secretary after the abstract has been circulated. Fellows or Associates who wish to have full copies of all papers should intimate their desire to the Assistant Secretary at the beginning of each session.

The Council propose, also, that at the meetings papers shall always be taken as read, and that the author shall either read the circulated abstract or introduce the paper in a short speech based on the abstract. It is hoped that this procedure will tend to promote the full discussion which should form one of the most useful and instructive features of the meetings.

The Council take this opportunity of reminding members that they are always glad to receive for consideration papers suitable for submission to, and discussion at, the sessional meetings. Such papers should be as concise as is consistent with adequate treatment

of the subject, and should not as a rule exceed 20 *Journal* pages in length. They may be of any of the following descriptions :

- (a) Papers on new developments or applications of actuarial science and method, or dealing on original lines with existing developments or applications.
- (b) Concise summaries of theory and practice in regard to actuarial questions of importance in professional work.
- (c) Papers on statistical, legal, financial and other subjects of general interest to the profession.

Papers may, if preferred, be submitted to the Council for consideration in abstract. The Hon. Secretaries will be glad at any time to hear from any member who has a paper in preparation, and to discuss the matter with him.

R. TODHUNTER, }
A. D. BESANT, } *Honorary Secretaries.*

March, 1915.

INTERNATIONAL CONGRESS OF ACTUARIES, 1915.

SINCE the opening meeting of the Session, when the President of the Institute made the announcement which will be found on p. 30, of the January number, an official intimation has been received that the Congress is indefinitely postponed.

ERRATUM.

J.I.A., vol. xliii, p. 354, line 20,

for $\frac{1}{M} \{ -\log_{10} s - (\log_{10} g \log_{10} c) c^x \},$

read $\frac{1}{M} \{ -\log_{10} s - \left(\log_{10} g \cdot \frac{\log_{10} c}{M} \right) x \}$

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Practical Points in Connection with the Formation and Valuation of Pension Funds, with a note on Group Assurances. By JOSEPH BURN, F.I.A., F.S.I., Actuary of the Prudential Assurance Company, and FRANK PERCY SYMONS, F.I.A., of the Prudential Assurance Company.

[Read before the Institute, 22 February 1915.]

IT is with some diffidence that we venture to submit to the Institute a paper on the subject of Pension Funds; indeed, after again perusing the masterly essays on this important subject which Mr. Manly and Mr. King have given to us it appears almost presumptuous to add anything further. Valuable papers have also been contributed by other members of our Institute. Thus Mr. James McGowan in vol. xxxvii of the *Journal* dealt with the subject of Pension Funds more or less generally, and supplied us with a very large number of formulæ. Mr. James Bacon in vol. xlii dealt with the construction of Salary Scales, whilst Mr. E. C. Thomas and Mr. Thomas Tinner (see *J.I.A.*, vols. xlviii and xlv) both supplied formulæ intended to meet difficulties arising in certain events. With regard to benefit societies, Sir Alfred Watson's paper (*J.I.A.*, vol. xlv) on this important branch of the subject may be taken as a text book.

Everyone who has valued Pension Funds will acknowledge how deeply they are indebted to these gentlemen for the formulæ they have originated and for the methods of valuation they have so carefully explained. We, however, have come to the conclusion that there are a great many difficulties which cannot be overcome by any formulæ, and that the actuary consulted is expected

to give advice on many practical matters which he might well be excused for considering as entirely beyond the scope of actuarial science.

It is quite probable that some of the difficulties with which we personally have attempted to deal might have been approached from entirely different standpoints by other actuaries with equally good, or perhaps better results, but it is our hope that some account of our attempts to solve the problems presented may prove of interest.

The subject naturally resolves itself into two parts :

- (1) The inception of a fund ;
- (2) the valuation of an existing fund.

We presume that all actuaries who have advised on Pension Funds will agree that the experience of each fund has characteristics peculiar to itself ; in fact, so varied and unexpected are the peculiarities that it is most dangerous to give any opinion until the past experience has been investigated.

We have found that not more than one out of six applications has resulted in the institution of a scheme. The most usual difficulty is that of "Cost"; indeed, we have never yet had a case where the cost was not far greater than had been anticipated. This difficulty should be boldly faced, and there should be no hesitation in submitting the true facts of the case in a plain and straightforward manner ; the actuary must insist upon a sound financial basis being adopted, even if such a course should result in his services not being further required.

Even where the applicants are able to face the truth as to the probable cost, the actuary will generally find that he is subjected to constant pressure in order to induce him, if possible, to agree to some greater benefits being granted than the contributions warrant. Then there are the cases where the number of prospective members is so limited as to render any scheme based on averages both dangerous and unsuitable. Generally speaking, Pension Schemes are not practicable unless they have the support of the employers, in which case it is desirable from many points of view that they should undertake all cost of management and so relieve the fund from a very real difficulty. The cost of management is an item which is hardly ever taken into account when a scheme is first suggested.

At the outset it is of importance to consider the form in which the Pension Fund should be initiated. Practically all Life

Assurance Companies grant deferred annuities at fixed ages, with return of premiums in the event of death before the attainment of the specified pension age, and these benefits are similar to the more important benefits obtained in a Pension Fund. It is true that the cost may be greater than that shown in some cases of large and well-managed schemes where the working expenses are borne by the employer. On the other hand, however, there is the great advantage of an absolute guarantee that every liability will be promptly met, a guarantee that is painfully lacking in a considerable number of Pension Funds. Why then should the employer incur the anxieties that must be associated with a Pension Fund, when with very little more expense he can shift his responsibilities on to the broad and willing back of an Assurance Company? Not only does he burden himself with managerial worry and responsibility, but he may also find himself confronted with serious financial responsibilities in the event of the fund becoming insolvent. Where the members joined the fund on the invitation of the employer, and the promises held out by the rules are not fulfilled, they may argue that the employer is morally, if not legally, bound to make up some portion of the deficiency. If for no other reason, the above considerations should make it clear to all employers that they should on no account permit themselves to become involved in any such undertaking without in the first place insisting upon competent actuarial advice. In many cases huge deficiencies are steadily accumulated through ignorance of the most simple considerations, and when at length the desperate financial position becomes manifest an actuary is called in in much the same manner as a doctor to the death-bed of a patient who has blindly trusted in the faith healer.

It is certain that Pension Funds or Pension Schemes are becoming increasingly popular with large employers, and we think that the reasons for this are not difficult to find. In the first place there is a natural feeling that to take the best years of a man's life and then eventually to cast him off when his failing abilities render him an expensive servant, is so repugnant to the modern relationship of employer and employee that even where no fund or scheme is in existence it is more the rule than the exception to make some benevolent allowance. At first, the men who receive these allowances are few in number, since they are survivors of a period when the business of the Company required only a small staff. Sooner or later it becomes apparent to the management

that the number of men who will attain old age in the Company's service and who will expect the same liberal treatment, must rapidly increase, and such allowances will form a serious drain on the Company's profits. It will then be obvious that the pensions must be placed on an organized basis.

Another consideration which is becoming increasingly manifest is that if the best service is to be obtained, the employees should be satisfied and be desirous of remaining in the Company's employment.

Although there are many ways of securing continuity of service, one of the most powerful incentives that can be put forward is the certainty that a suitable provision for old age is assured. It has the advantage from the employer's point of view that it is not a present benefit, but one which is to be granted in the future, and will therefore be forfeited by those who leave the service. From the financial standpoint it may certainly be argued that the employer's contribution to the Pension Fund is an excellent investment, since it results in a much more effective service than could be obtained by merely giving increases in wages.

It is evident that there are many reasons in favour of employers setting up separate funds instead of assisting their members to obtain deferred annuities from a Life Assurance Company. It is advisable that the factor of personal good feeling between master and servant shall be strongly in evidence. The fixed conditions which an Assurance Company must necessarily require, destroy to a large extent the personal element which is so attractive to the members. Again, as the employer's contribution is in effect a portion of the salary, it is desirable that it should bear a constant ratio thereto, and this is difficult if not impossible to arrange by means of premiums paid to an Assurance Company.

To look at the matter from the member's point of view, it is to be remembered that contributions are deducted from his salary, and, therefore, since he has never received his salary in full he does not feel the loss of the deduction. Moreover, his contributions adjust themselves to his means, being insignificant when he is in receipt of a small salary and only becoming considerable when he is in a position to make large payments.

Let us, however, suppose the case where an actuary is asked to advise as to the desirability of instituting a Pension Fund of which the probable membership will be 100 or less. With so small

a number of members the working expenses and the difficulty in selecting suitable investments for the slowly accumulating funds will be considerable disadvantages, quite apart from dangerous fluctuations in the experience. In such circumstances it may be that the actuary has no alternative to recommending a suitable arrangement with an Assurance Company. The main difficulty in the adoption of such a course is in respect of withdrawals. The contributions made by the employer are intended for the benefit of those remaining in the service until the attainment of the pension age. The accumulation of the employer's contributions which is forfeited by the members who withdraw, is of vital importance to practically all successful Pension Schemes. When, for instance, the employer pays an appreciable proportion of the contributions and the number of withdrawals is considerable, not only at the younger but also at the middle or older ages, it will necessarily follow that the pensions obtained by those who remain in the service will appear highly satisfactory, whereas when the employer pays only a small contribution or the withdrawal rate is insignificant, the benefits will appear unsatisfactory.

A scheme which has been suggested as suitable for a small membership, and one which we think has certain advantages, is as follows :

The pension is split into two equal portions, and two deferred annuity policies are issued on each employee's life to cover these portions. One of the policies is issued to and is under the control of the employee, who can thus deal with it as he wishes should he leave the service. On the death of the employee before the attainment of pension age, all premiums paid on both policies accumulated at compound interest (the rate may be $2\frac{1}{2}$ per-cent or 3 per-cent) are returned. In the case of the employee's policy the returned premiums are in the ordinary way payable to his representatives, and if the employer also undertakes to pay over the amount returned under the policy held by him, the result will be that the total death benefit will be equal to twice the contributions paid by the employee accumulated at compound interest. Thus the death benefit secured is somewhat better than that granted under the rules of most Pension Funds. On the surrender of either policy the Assurance Company undertakes to pay the same amount as would be paid at death. The employee can at his option continue to pay the premiums on his own policy after withdrawal from the service. The employer

will retain the surrender-value of the policy held by him, and such amount will be available to carry to a fund out of which the pensions of those who remain in the service to the pension age can be supplemented. Where there is any considerable rate of withdrawal this fund should be sufficient at least to double the pensions provided by the policies.

The scheme can also be adjusted to give an ill-health pension by applying the surrender-value to the purchase of a pension. This may be at special rates, in view of the fact that the member is retiring from business on account of impaired health. The premiums in the ordinary case would be level throughout the duration of the policies (although there is no theoretical objection to grading them), and for this reason probably the plan is more suitable for those whose wages are more or less constant than for the salaried classes. The great weakness of the whole scheme is that the amount which will accumulate from withdrawals will in any ordinary experience eventually amount to so large a sum as to form no small proportion of the total funds, and the benefits which members may expect to obtain therefrom cannot with any certainty be forecast. Then again, it is not likely that the Assurance Company would guarantee always to issue policies such as those suggested.

Returning to the consideration of the ordinary type of Pension Fund, in some few cases the employer may decide to limit the pensions to the amount which his own contributions will provide, and not call for any contribution from the staff. There are many advantages in this method, the principal one being that the employer retains complete control in his own hands, for in such circumstances the employees would not be allowed any voice in settling the conditions of the scheme. If the scheme is to be contributory, the members must almost certainly be allowed to share in the management, and the employer will be called upon from time to time to conduct negotiations with the representatives of the staff; these negotiations will probably arise at a time when relations are somewhat strained, and the employer may for diplomatic reasons be under the disagreeable necessity of acceding to various demands. On the other hand, unless he is prepared to pay a very heavy contribution, the pensions he can safely afford to promise will be in most cases comparatively small, and will not be an adequate provision for the declining years of an old servant. There is a further point against non-contributory schemes which, whilst applying to

both forms, is more common in non contributory schemes. In only a small proportion of non-contributory schemes is adequate provision made for the accruing liabilities under the scheme, and as a consequence the security of the members is very uncertain. In this connection we may refer to the very grave warning contained in the report of the Departmental Committee on Railway Funds. The Committee say :

“ We cannot but feel very grave doubts whether the extent of the liabilities undertaken by the companies in connection with the guarantee of the funds has been properly appreciated . . . The apprehensions that have been expressed as to the way in which the operation of the guarantee may react on the conditions of the staff, while in the service of the company, may or may not be well founded, but in any case the effect which is feared by the members of the fund would be felt apparently to the greatest extent by the future shareholders of the company ; and it is for the shareholders to consider what liability the guarantee may involve and what measures should be taken to provide for meeting that liability. . . . We would suggest that the matter should be brought to the serious notice of the railway companies concerned. Not only is it important that the members of the funds should be enabled to ascertain to what extent they are relying for their pensions upon the resources of the companies, but it would also seem indisputable that ordinary business prudence requires that the exact position of the matter should be ascertained by the best means available. Therefore we consider that periodical valuations should be made even in the case of guaranteed funds.” While the reference is only to railway companies, the principle is equally applicable to all companies granting pensions.

Having settled the point as to whether the scheme is to be contributory or non-contributory, the question of the benefits to be given must next be considered. As a contributory scheme is the more general it will be more useful to examine the benefits given under this form of fund. When the actuary is first approached there may be some fairly definite idea as to the amount of contribution to be paid, but as a rule the ideas on the subject of the benefits desired are very far from being definite. They have one decided characteristic, however, namely, that while already much too large they tend to grow still larger at each successive interview.

The question will then arise as to whether the pension is to

be based on final or average salaries. In either case, if the contributions are assessed as a percentage of the wage or salary, it necessarily follows that the advantage will rest with those members whose wages or salaries are subject to the greatest increases at ages most nearly approaching the pension age, when contributions cease to be payable. This inequality is, of course, much greater when final salaries are made the basis of pensions, and unless, therefore, the employer is prepared to bear the cost of the pensions to the more highly paid servants, such method should only be countenanced in exceptional circumstances. Only a small minority of the staff rise to the highly paid posts, and in the great majority of cases the high salaries are not attained until late in life. The larger contributions are therefore only paid for a few years, so that the value of the contributions are considerably less than the value of the pensions to be received. Even when the pensions are based upon average salaries it will generally be found that the more highly paid officials are somewhat more generously treated than the rank and file. Here, however, the employer may well argue that he is not under any obligation to allow equal benefits to all his employees. He is giving a large sum of money to the staff, and may hold the opinion that those who have risen in the service are the most worthy to receive the greatest benefit from his benevolence. It is true that this same argument can be used in favour of basing pensions upon final salaries, but in these circumstances it may happen that some of the lower salaried employees would receive nothing from the employer's contributions, or even less than the value of their own contributions, and no scheme of this description could be legitimately put forward. There is, however, no reason why both methods should not be jointly adopted, the pension given on account of the member's contribution being based on an average salary, and the pension on account of the employer's contribution being based on either the final salary or a modification of the final salary. In this way the scheme may be made to adjust itself to the wishes of both parties. With regard to the subsidiary benefits little need be said, for they will vary according to the circumstances of each fund. We, however, feel that these benefits should be restricted as much as possible, for they must reduce the pension which, after all, is the main object of the fund. The principal of these minor benefits are the allowances given on withdrawal or death. The first may conveniently be in the form of a return of the member's contributions, for in

no case can the member who leaves the service expect to receive the employer's contributions, which were given with the intention of providing a suitable reward for long and faithful service. With regard to the death benefit, there is ample justification for returning the employer's as well as the member's contributions. We do not consider it desirable to enter into such refinements as the allowance of interest accumulations on returned contributions.

The actuary is often called upon to advise as to the most suitable arrangements, where it is desired to initiate a Pension Scheme in place of an existing Benevolent Fund which in the past has been conducted upon generous but somewhat indefinite lines. As an example of this and other points already referred to it will, we think, be useful to give some detailed account of the Union Castle Line Superannuation Fund Association. In this case there was already a Benevolent Fund in existence with assets of about £100,000, which had been set aside from time to time out of the profits of the Company. There had been in the past no fixed rules for the administration of this Benevolent Fund, and the only definite liabilities consisted of some 15 or 20 pensions of a capital value of about £40,000. It was desired to place the Fund on an organized basis.

The staff whom it was proposed to bring under the scheme numbered about 1,000. It is almost invariably found that the number of beneficiaries coming under any proposed scheme will steadily increase as the matter progresses, and the present instance proved to be no exception to the rule. More than once our most careful estimates were upset owing to the fact that the claims of some other members of the Staff could not be ignored. Before any scheme could be constructed it was necessary to extract the experience, and fortunately very complete details of the staff, both past and present, were available. It was decided to analyse the experience of the previous 10 years. It very soon became evident that the conditions of service varied very considerably as regards the clerical staff on the one hand and the sea-going staff on the other, and further investigation showed that some sub-division of the sea-going staff would also be necessary. Eventually it was decided to group the staff in 3 classes, namely, (1) Clerical, (2) Deck Officers and Pursers, (3) Engineers and Stewards. This sub-division rendered the data on which to base the rates of mortality and withdrawal as well as the salary scales somewhat scanty. This disadvantage was, however, certainly

less than the danger which would have been incurred by combining such widely different classes. For instance, the average age at entry for the clerical staff was 15, and what, in the absence of a better expression, we will call the average maximum salary (that is, the maximum according to our graduated salary scale) was over £500, whilst in the case of the Engineers the average age at entry was 26 and the average maximum salary less than £300. The rates of withdrawal also showed marked variation in the three classes.

The methods employed by us for tabulating the data for the most part followed the lines which we discuss later. The Engineers' section, however, showed that safer results would be obtained by neglecting the age and constructing the withdrawal rates on the basis of duration alone. The average age at entry was therefore obtained, and the assumption was made that all entries had actually taken place at this age, and that all withdrawals from the service would be at the rates shown by the select withdrawal table formed as above.

Thus, the average age at entry was found to be 26, and the rate of withdrawal from 26 to 27 was assumed to be that experienced in the first year of service irrespective of age; from 27 to 28 the rate of withdrawal was assumed to be that for the second year of service, and so on.

The mortality shown during the 10 years was extraordinary amongst all classes of members, even after allowing for fluctuations due to the paucity of the data. Out of 7,650 total years of exposure there were only 55 deaths. In the case of the Engineers and Stewards the result were still more remarkable, for out of about 2,000 years of exposure due to entrants during the 10 years there was only one death. In these circumstances it was decided to assume $O^{(M)}$ mortality up to age 42 and the $O^{(M)+1}$ mortality after that age, 42 being the age at which withdrawals were assumed to cease for all classes. That is to say, the mortality rates for the first year of assurance were used for all ages up to 42, and the rates for the second year of assurance after that age.

The rates of mortality and withdrawal having been settled, there only remained the salary scale. This, as is nearly always the case, presented certain novel features and many difficulties. In the clerical staff we found that it was not by any means an unusual occurrence for a member to be transferred to some foreign station and receive at once a large increase in salary. Such transfers occurred at comparatively young ages, and

consequently resulted in much more rapid increases in the salary scale between ages 20 and 30 than is usually shown. Most careful enquiries were made as to the probable future course of the Company in this matter before deciding upon the graduated scale to be adopted for the purposes of valuation.

It was decided that no contribution should be made and no benefit granted in respect of salary in excess of £1,000, and this considerably lessened our difficulties as to the more highly paid members. Generally speaking, we have found it advisable before preparing a salary scale to exclude not only all cases of exceptionally high salaries, but also the probable successors of those in receipt of high salaries. These special cases we have then valued individually by means of suitable assumptions as to the future, and thus avoided any dangerous exceptions in the salary scale. Mr. Geo. King on page 140 of *J.I.A.*, vol. xxxix, gives a very lucid explanation of one method of using salary scales in the valuation of Pension Funds. By this method the actual salaries payable at any given age are valued upon the assumption that they will be subject to proportionate increases in accordance with the salary scale. Whilst fully appreciating Mr. King's arguments, we personally are of opinion that more correct results are obtained by making two sets of valuations. The first valuation is based on the assumption that the present salaries will continue unchanged throughout the remainder of active service. The second valuation deals only with *future increases* of salaries, which are assumed to follow in exact accordance with the first differences of the salary scale.

The most notable point in the salary scale of the Engineers' Staff was the comparatively high salaries at the early ages and the very steady rate of increase up to about age 50.

With the Officers and Pursers there was also a steady increase up to about age 40, but, as might be expected, the rate of increase after that age was more rapid and the maximum salary considerably higher than in the other classes.

Having settled the salary scales, we were in a position to formulate a scheme. In the case of the Union Castle, the initial difficulty, as with all schemes, was the question of the existing members of the staff, who ranged from newly appointed juniors to men just on the eve of retirement. With the material available the problem was to produce a scheme under which (*a*) reasonable pensions were secured to the older members. (*b*) inequitable treatment between different classes and also as between individual

members was avoided, (c) simplicity of outline was preserved, and (d) the cost was covered by the funds available.

A plan had been suggested by the Company under which the members under age 51 were to be allowed to count their past service since age 40, and members aged 51 and over were to be allowed to take credit for 11 years past service. This ingenious proposal, it will be noticed, graduated the pensions and avoided any awkward junctions. A weak point, however, was that the members dating back their service in this way would be required to pay their back contributions either in one sum or by a small number of instalments, and this would have been sufficient to prevent a considerable number of members from joining. The pensions of the older members also were not satisfactory. These disadvantages, in conjunction with the fact that the scheme on investigation required funds that were just double the amount available, caused it to be discarded.

Further attempts were then directed to basing the pensions on average salaries in accordance with the original scheme put forward by the Directors, but it was found quite impossible to provide a simple scheme which, whilst sufficiently attractive to induce members to join, also so graduated the pensions as to give the older members retiring allowances which seemed adequate to their requirements. Further difficulties arose when attempts were made to make the scheme consistent as between the three different classes of members.

It must be remembered that with pensions based upon average salaries, curious fluctuations in the liability are always possible. An example of this occurred in connection with the Union Castle scheme. Some twelve months having elapsed since our original investigations of the data had been made, it was thought advisable, in order to bring the facts up to date, that new figures as to membership and salaries should be obtained. New salary scales were constructed on the revised figures, as they showed larger increases for certain groups of ages than those allowed for in our original scale. On valuing the Fund it was surprising to find that the larger salaries, so far from increasing the liability, had actually decreased it. This was due to the fact that the additional increases had been principally amongst members under 40, and the excess contributions, which had a very considerable period to accumulate, more than counterbalanced the small increase in the pension based on average salary.

As it was not found practicable to base the scheme entirely

on average salaries, the final salary basis was next investigated, and a scheme was constructed dependent partly upon average salaries and partly upon final salaries, as follows :

All members to be entitled to pensions of an amount equal to $2\frac{1}{4}$ per-cent of total salary received from the inception of the Fund until the attainment of age 65 (the pension age).

All members over 45 years of age to be entitled to receive additional pensions based on final salaries as follows :

Present age :

45	..	2 per-cent	54	..	11 per-cent
46	..	3 „	55	..	13 „
47	..	4 „	56	..	15 „
48	..	5 „	57	..	17 „
49	..	6 „	58	..	19 „
50	..	7 „	59	..	21 „
51	..	8 „	60	..	23 „
52	..	9 „	61	..	24 „
53	..	10 „	62 to 64	..	25 „

The contributions of the members to be as follows :

For members under 45	...	$3\frac{3}{4}$ per-cent of salary
„ 45 to 46	...	4 „ „
„ 47, 48, 49	...	$4\frac{1}{4}$ „ „
„ 50, 51, 52	...	$4\frac{1}{2}$ „ „
„ 53, 54, 55	...	$4\frac{3}{4}$ „ „
„ 56 to 64	...	5 „ „

The Company to contribute in all cases an amount equal to that paid by the member.

In order to carry out this scheme it was found that an increased grant would be required from the Company, and, moreover, there was great disappointment when it was found that the pension age must be 65 instead of 63, as desired. For these reasons it did not meet with approval, and the matter was put on one side for some time.

In December 1911, Messrs. Donald Currie & Co. and Sir Owen Phillips entered into an agreement for the sale of Union Castle shares, as a result of which the Royal Mail Steam Packet Co. and the Union Castle Mail Steamship Co. came under one control.

In this agreement it was provided as follows :

“The Purchasers shall procure the Company to establish a Pension Fund for the benefit of past and present employees of the Company and of the Firm and all future

employees of the service by the Company setting apart out of the profits of the Company for the year ending Thirty-first December One thousand nine hundred and eleven the sum of Two hundred and twenty thousand pounds and the Purchasers shall procure the Company to thereafter make an annual contribution to the said Fund not less than Seven thousand five hundred pounds until the total amount of such annual contributions shall reach the sum of One hundred thousand pounds. The said Fund shall be managed and controlled by trustees appointed by the Firm and by the Purchasers under a scheme to be agreed by the Firm and the Purchasers in which due regard shall be had to the special circumstances of the case.

“ The present Benevolent Fund taken out of the past profits of the Company and now in the hands of the Firm or members thereof amounting to One hundred and twenty thousand pounds or thereabouts shall remain in the hands of the Firm to be applied in their absolute discretion for the benefit of past and present employees of the Company and the balance (if any) of such Fund remaining after all such claims of past and present employees as the Firm may think fit have been satisfied shall be handed over and added to the above-mentioned Pension Fund.”

In addition to the above amounts, Messrs. Donald Currie and Co. contributed a further sum of about £50,000 to be specially ear-marked for the benefit of certain members of the staff. Thus the amount available for starting the scheme was over £450,000. With this sum we were enabled to construct a scheme, giving adequate pensions to all existing members and with reasonable rates of contribution. With regard to the latter, however, the Company's contribution was restricted to future entrants and to members who were under age 40 at the inception of the scheme.

The scheme which was eventually accepted was as follows :

Membership.

Compulsory on all future entrants into the Company's service, whose age at the date of admission does not exceed 35 years.

Optional to all those in the service whose age did not exceed 63½ years on the 1 July 1912 (the date of the inception of the Fund).

Contributions.

By the Company. A sum equal to $2\frac{1}{2}$ per-cent per annum on all salaries paid to the staff who (1) were then in the service and under age 40, and (2) should subsequently join the service.

By the Staff.—(a) Officers in the service on 1 July 1912.

For all ages up to 39 years	...	£3 15s. 0d.	per-cent
For age 40 attained	...	£3 17s. 0d.	„
„ 41	„	£3 19s. 0d.	„
„ 42	„	£4 1s. 0d.	„
„ 43	„	£4 3s. 0d.	„
„ 44	„	£4 5s. 0d.	„
„ 45	„	£4 7s. 0d.	„
„ 46	„	£4 9s. 0d.	„
„ 47	„	£4 11s. 0d.	„
„ 48	„	£4 13s. 0d.	„
„ 49	„	£4 15s. 0d.	„
„ 50	„	£4 17s. 0d.	„
„ 51	„	£4 19s. 0d.	„
„ 52	„	£5 1s. 0d.	„
„ 53	„	£5 3s. 0d.	„
„ 54	„	£5 5s. 0d.	„
„ 55	„	£5 7s. 0d.	„
„ 56	„	£5 9s. 0d.	„
„ 57	„	£5 11s. 0d.	„
„ 58	„	£5 13s. 0d.	„
„ 59	„ and upwards	£5 15s. 0d.	„

(b) Officers who join the service after 1 July 1912.

(1) Home, Colonial and Foreign Clerical Staff.

Entry ages up to 21 inclusive	...	£3 15s. 0d.	per-cent
Entry age 22	...	£3 16s. 0d.	„
„ 23	...	£3 17s. 0d.	„
„ 24	...	£3 18s. 0d.	„
„ 25	...	£3 19s. 0d.	„
„ 26	...	£4 0s. 0d.	„
„ 27	...	£4 1s. 0d.	„
„ 28	...	£4 2s. 0d.	„
„ 29	...	£4 3s. 0d.	„
„ 30	...	£4 4s. 0d.	„
„ 31	...	£4 5s. 0d.	„
„ 32	...	£4 6s. 0d.	„
„ 33	...	£4 7s. 0d.	„
„ 34	...	£4 8s. 0d.	„
„ 35	...	£4 9s. 0d.	„

(2) Deck Officers and Purser.

Entry ages up to 28 inclusive	...	£3 15s. 0d.	per-cent
Entry age 29	...	£3 16s. 0d.	„
„ 30	...	£3 17s. 0d.	„
„ 31	...	£3 18s. 0d.	„
„ 32	...	£3 19s. 0d.	„
„ 33	...	£4 0s. 0d.	„
„ 34	...	£4 1s. 0d.	„
„ 35	...	£4 2s. 0d.	„

(3) Engineers, Chief Stewards and Second Stewards.

Entry ages up to 28 inclusive	...	£3 15s 0d.	per-cent
Entry age 29	...	£3 16s. 0d.	„
„ 30	...	£3 17s. 0d.	„
„ 31	...	£3 18s. 0d.	„
„ 32	...	£3 19s. 0d.	„
„ 33	...	£4 0s. 0d.	„
„ 34	...	£4 1s. 0d.	„
„ 35	...	£4 2s. 0d.	„

It was further provided that for the purposes of the scheme, no contribution was to be made or any benefit granted in respect of salary in excess of £1,000, whilst officers who at 1 July 1912 were aged 55 years and upwards were not to contribute any percentage or receive any benefits in respect of increases in salary after that date.

Pensions.

For those in the service on 1 July 1912.

(a) 1 per-cent of final salary in respect of every year of service up to but not exceeding 35 years.

(b) 1 per-cent of all salary upon which contributions have been paid.

Age of Retirement : Optional at 63 ; Compulsory at 65.

For those who joined the service after 1 July 1912.

(a) 2 per-cent on all salary upon which contributions have been paid.

(b) Such further pension (if any) as the actuary shall certify may be allowed out of and having regard to an accumulating fund (at $3\frac{1}{2}$ per-cent) of £10,000 set aside specially for these officers.

Age of Retirement 65.

For those in the service 1 July 1912, who are members of the Fund, but do not contribute.

Officers aged 40 and upwards : 1 per-cent of final salary in respect of every year of service up to but not exceeding 35 years.

Age of Retirement : Optional at 63. Compulsory at 65.

Officers under age 40 : No benefit under the Scheme.

Death Benefit.

In the event of the death of an Officer at any time before the superannuation allowance becomes payable, there shall be paid out of the Fund to his representatives the full amount of the sums he has contributed, together with the sums contributed by the Company in respect of such Officer, but without interest in respect of any such sums whether paid by the Officer or the Company.

In the event of the death of an Officer after the superannuation allowance becomes payable, but before the total amount of superannuation received by him equals the total amount of the sums he has contributed together with the sums contributed by the Company in respect of such Officer, the difference between such total amount and the amount of superannuation paid to him shall be paid to his representatives, but without any interest on such difference.

Withdrawal Benefit.

Any Officer (except an Officer in the service and of the age of 55 years and upwards on the 1 July 1912) retiring from the Service before superannuation, *bonâ fide*, of his own accord for any reason other than ill-health (whether arising from bodily incapacity or mental infirmity), and not in order to escape dismissal for fraud or dishonesty, or misconduct, shall be entitled to receive back from the Fund the whole amount of his contributions, but without any allowance by way of interest thereon, and shall have no further claim upon the Fund.

Any Officer in the service on the 1 July 1912 may at any time cease to contribute to the Fund and thereupon shall be entitled to receive back from the Fund

the whole amount of his contributions, but without any allowance by way of interest thereon, and if under 40 on the 1 July 1912 shall have no further claim upon the Fund; but if of the age of 40 and upwards on that day shall be deemed to have always been a non-contributing employee in the Service.

Retirement on Ground of Ill-health.

Any Officer retiring from the service before superannuation on the ground of ill-health, *bonâ fide*, and not in order to escape dismissal for fraud or dishonesty or misconduct, shall

- (a) If under the age of 50 be entitled to receive back from the Fund the whole amount of his contributions, but without any allowance by way of interest thereon.
- (b) If of the age of 50 years or upwards be entitled to receive from the Fund the amount which shall be certified in writing by the actuary to be equal to his full reserve.

It will be seen that this scheme is quite simple. It avoids all awkward discrepancies between adjoining ages, and provides reasonable pensions even to those nearing the pension age. Moreover, adequate pensions are provided for those higher salaried officials for whom the Directors were anxious to provide.

We venture to think that a scheme of pensions similar to the above would frequently be applicable, since by varying the maximum number of years which are to be counted in estimating the amount of pension payable, it can be suitably adjusted according to the funds available. The costs of all benefits exclusive of those based upon full salaries can be readily estimated. Having valued the total assets, we can deduct therefrom the value of benefits based on average salaries, and the remainder will be available for benefits based upon final salaries. It only remains to decide what percentage of the final salary shall represent pensions. The value of pensions based on 1 per-cent of the final salary having been calculated, it will be merely a matter of simple proportion to find the maximum number of years' service which can be allowed to count in the calculations of that portion of pension which is based on the final salary.

With regard to ill-health pensions, there was of necessity

no data on which to construct retirement rates. In these circumstances it was provided that members retiring through ill-health after age 50 and before the pension age were to receive an amount equal to the reserve held on their behalf, such amount to be applied at the discretion of the Committee. The practice has been to allow the retiring member in these cases a pension based on his reserve, and in calculating both the reserves and the pension it has been the custom to treat the member as normal. At first sight it would appear that as the member is in bad health this should be taken into account in the annuity-value. On the other hand the state of health should be taken into account in estimating the reserve. By treating the member as normal, therefore, it will be seen that there is a balance of errors, and whilst no theoretical accuracy is claimed for this method of treatment, it has been found to produce results that were both practical in working and satisfactory to the members.

A few other practical points which are applicable to all schemes of this description may be mentioned. First, the rules as drafted by the solicitors should be submitted to the actuary in order to avoid including anything which does not accord with the scheme as submitted. Secondly, a time limit must be fixed, after which existing employees can only be allowed to join on the basis of new entrants. In this connection the actuary should make provision in his valuation for the fact that although he is valuing at a particular date, the scheme may not come into operation until 6 or 9 months later, with a consequent loss of contributions and interest. We have also found it advisable to assist the secretary with regard to the books that will be required, and also to supply him with a form of card on which the record of each member may be kept. If this is done, his work at the periodical investigations is reduced to a minimum.

We should now like to make some general remarks on the methods of tabulating and analysing the data.

In the case of a fund which has been in existence for a number of years there is a strong probability that, owing to reconstruction or alterations in the rules from time to time, several distinct classes of member have to be dealt with, and these may require separate treatment. Where possible, however, it is advisable to sub-divide the members as little as possible.

Mr. King advocates the use of the full date of birth in order to obtain the nearest age at entry, the ages at exit being the tabulated age at entry plus the curtate duration. Where the full

date of birth is available this is without doubt the best method, but frequently the only information which can be obtained is the year of birth, and in these circumstances the method of nearest ages cannot be used; we have found the following method useful in such cases, both in the formation of new funds and the valuation of old funds. The ages tabulated are obtained by deducting the year of birth from the year of entrance or exit. In the case of those who entered previous to the year in which the observations started, the year of entrance is taken as the year immediately preceding the period of observation.

Integral ages only are placed on the cards. The formula for the exposed to risk is

$$E_{x+\frac{1}{2}} = E_{x-\frac{1}{2}} + b_x + \frac{1}{2}(n_x + n_{x+1}) - (d_x + w_x + e_x + r_x)$$

where the period of observation is limited by calendar years, and

x = difference between year of birth and year of coming into or going out of observation;

b_x = members who were on the active list at the commencement of the period;

n_x = entrants during the period;

d_x = deaths during the period;

w_x = withdrawals during the period;

r_x = retirements on pension during the period;

e_x = existing at the close of observations.

The assumptions underlying this formula are that the dates of birth, entry and exit are evenly spread over the year and on the average occur on the 30 June. From the formula given above, we obtain:

$$q_{x+\frac{1}{2}}^d = \frac{d_{x+1}}{E_{x+\frac{1}{2}}}; \quad q_{x+\frac{1}{2}}^w = \frac{w_{x+1}}{E_{x+\frac{1}{2}}}; \quad q_{x+\frac{1}{2}}^r = \frac{r_{x+1}}{E_{x+\frac{1}{2}}}$$

The schedule on which the numbers are tabulated is adopted from the form given by Mr. Ackland (*J.I.A.*, vol. xxxiii), and is as follows:

at any age after 60, or before that age in the event of permanent incapacity. We would call attention to the sudden rise at age 60, followed by an immediate fall for one or two years, which again is succeeded by a rapid rise bringing the rate to unity before age 70. The same characteristic is observable in all three funds, although in one case the first maximum point is deferred for two years. This is indeed a feature which must on no account be neglected. In most cases and more especially in funds connected with the railway service, there is a distinct tendency for pensions to be taken at the earliest possible moment, and in consequence an increasing proportion of the members retire on their 60th birthday (*i.e.*, when 60, as is quite usually the case, is the normal pension age). Those who remain consist for the most part of those who are not anxious to retire, and in consequence, immediately after age 60 there is a fall in the rate for one or two years, after which the rate again rises. Any attempt to apply a summation method of graduation could not possibly fail to introduce error, for it would reduce the maximum point at age 60 and spread it on both sides. We feel that this maximum point should be retained, and we have found it advisable rather to raise it than to lower it. The advisability of this will be evident when it is remembered that the effect of deferring the age of retirement is not only to reduce liabilities but to increase the value of the contributions, and may lead to very serious under-statement of the liability.

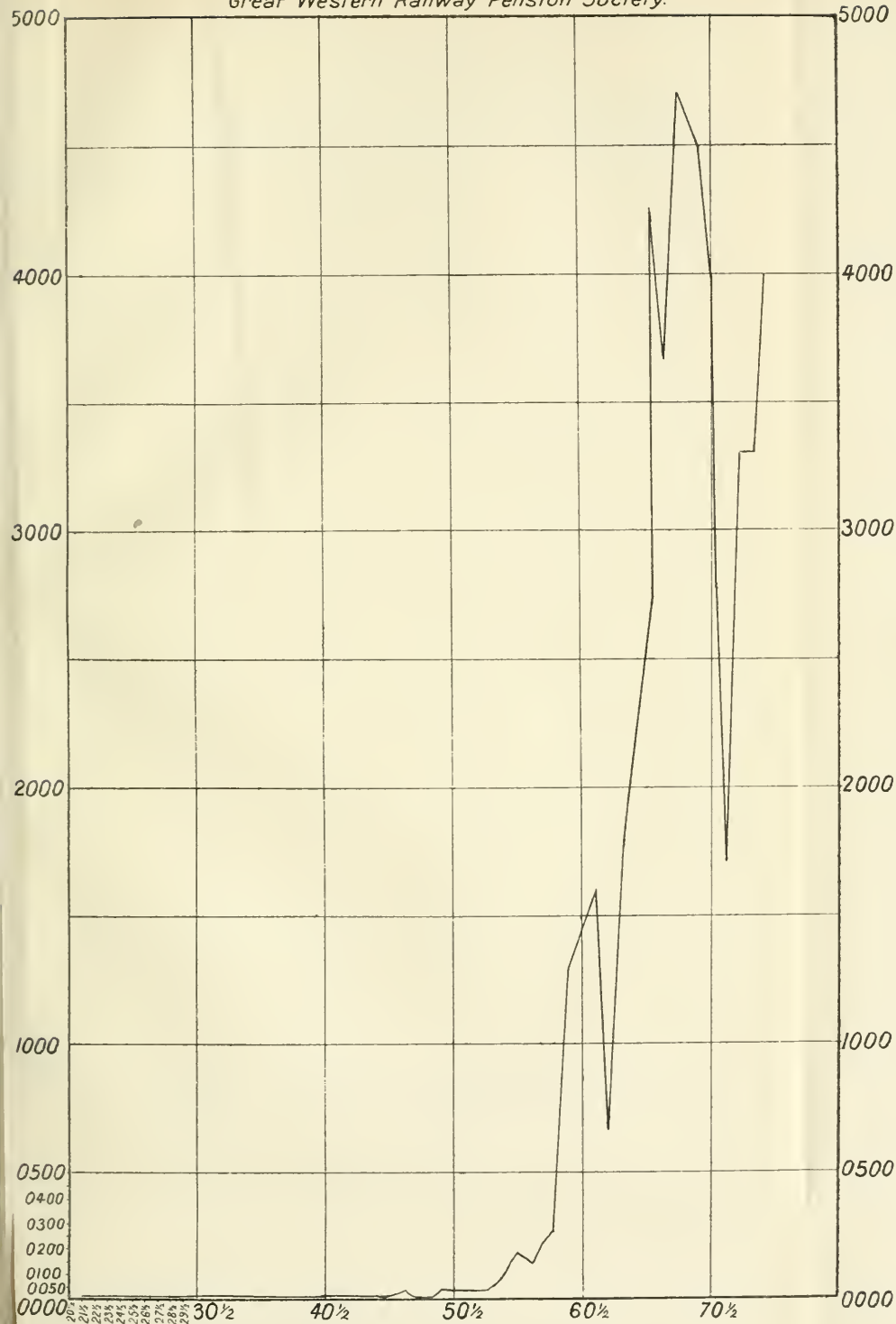
In the case of a new fund there will, of course, be no material out of which a rate of retirement can be constructed. Even in those cases where benevolent allowances have been given, the numbers will generally be found to have been so scanty and retirements so irregular that no reliance should be placed upon them. In these circumstances the actuary will have no option but to assume that all retirements are to take place at a fixed age, which must be settled by the promoters of the fund. The data, however, may be quite extensive enough to give a reliable indication of the rates of mortality and withdrawal. With regard to the rates of mortality, it has frequently been mentioned that the mortality experience of the active members of Pension Funds is extremely light. This applies to all funds where there is a heavy withdrawal rate, and where ill-health pensions are allowed; it is independent of the social status of members, and we have found the same experience amongst such widely different classes as lower grade railway

DIAGRAM I.

Rate of Retirement.

Great Western Railway Pension Society.

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amongst such widely different classes as lower grade railway

DIAGRAM 2.

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Rate of Retirement.

Great Western Railway Enginemen & Firemen's
Mutual Assurance, Sick & Superannuation Society.

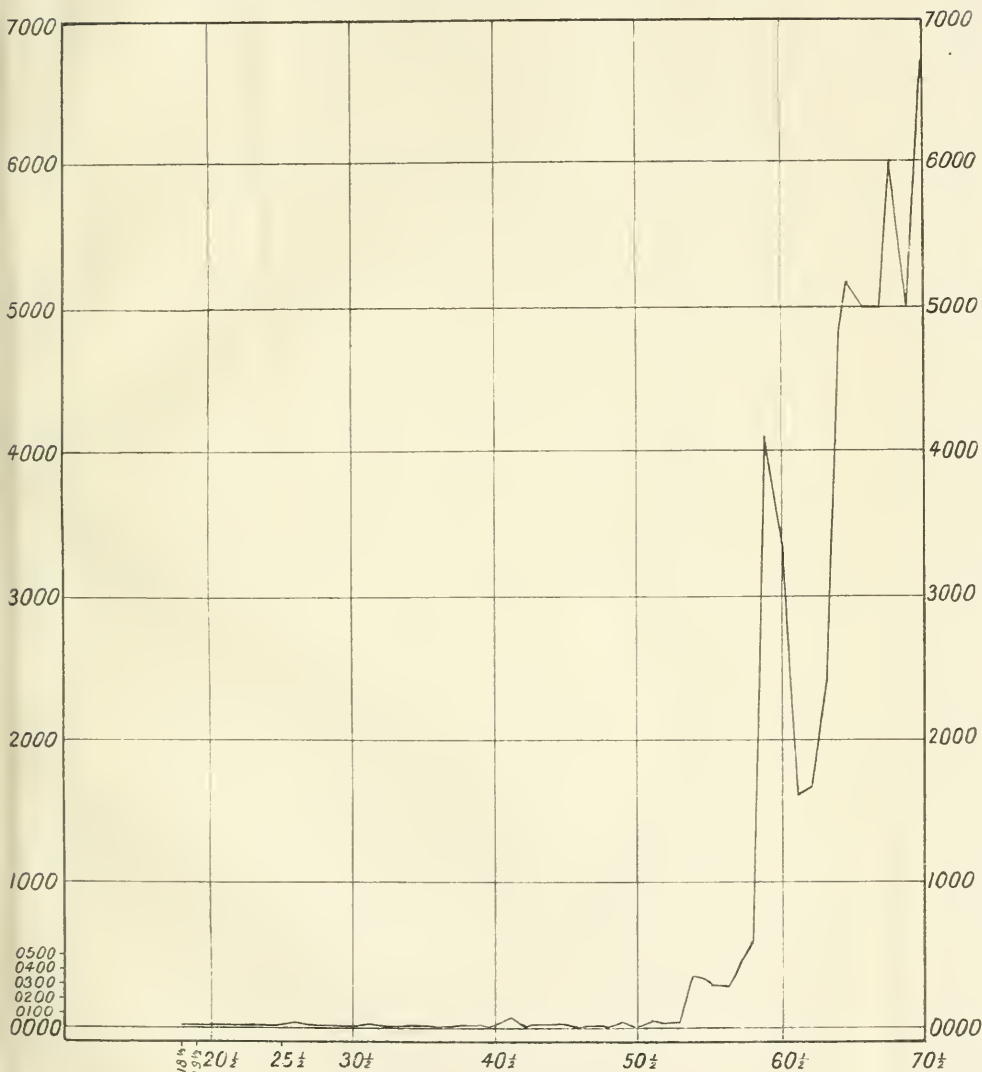


DIAGRAM 3.

Rate of Retirement.

Great Western Railway Superannuation Scheme.

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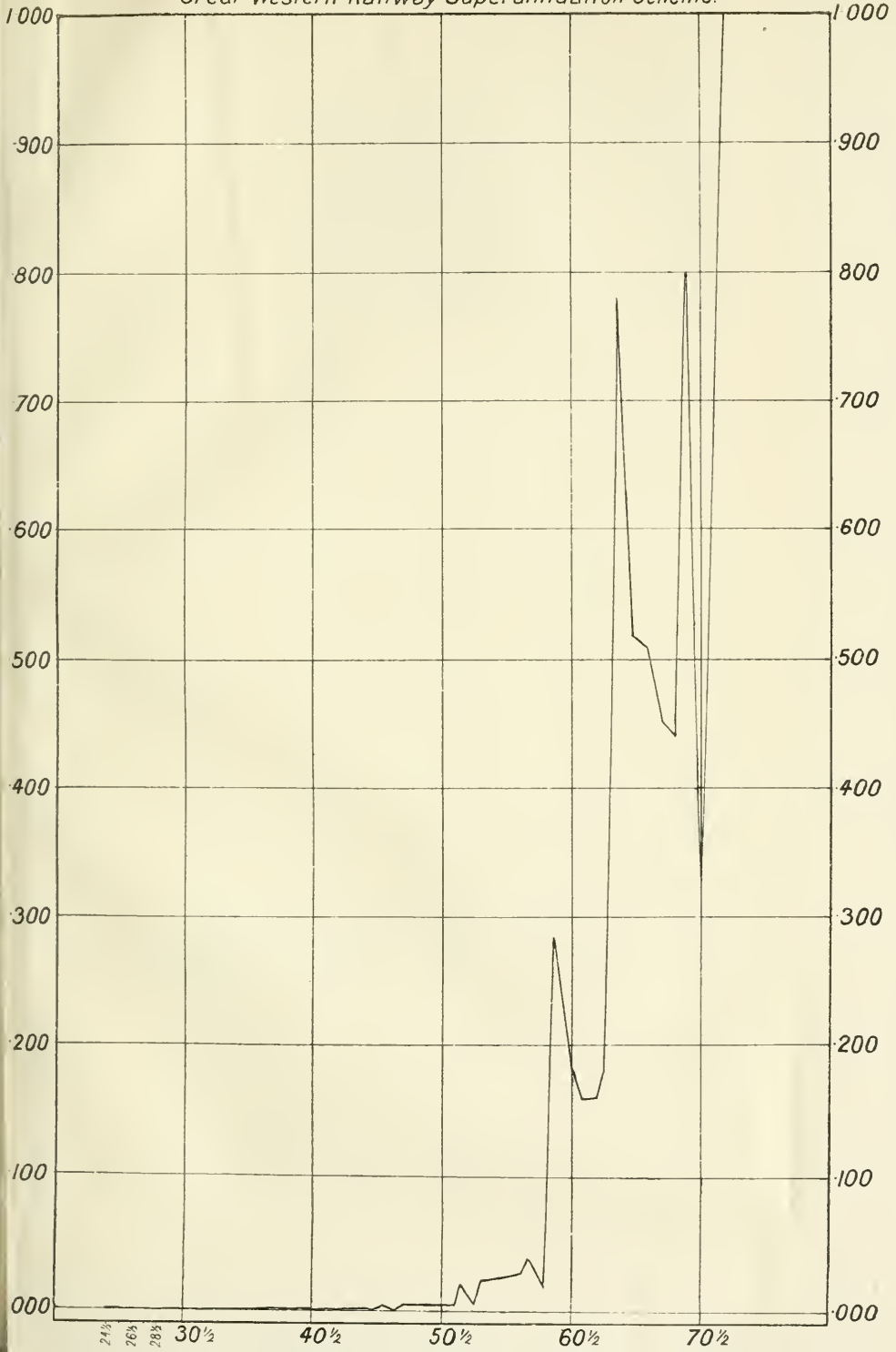


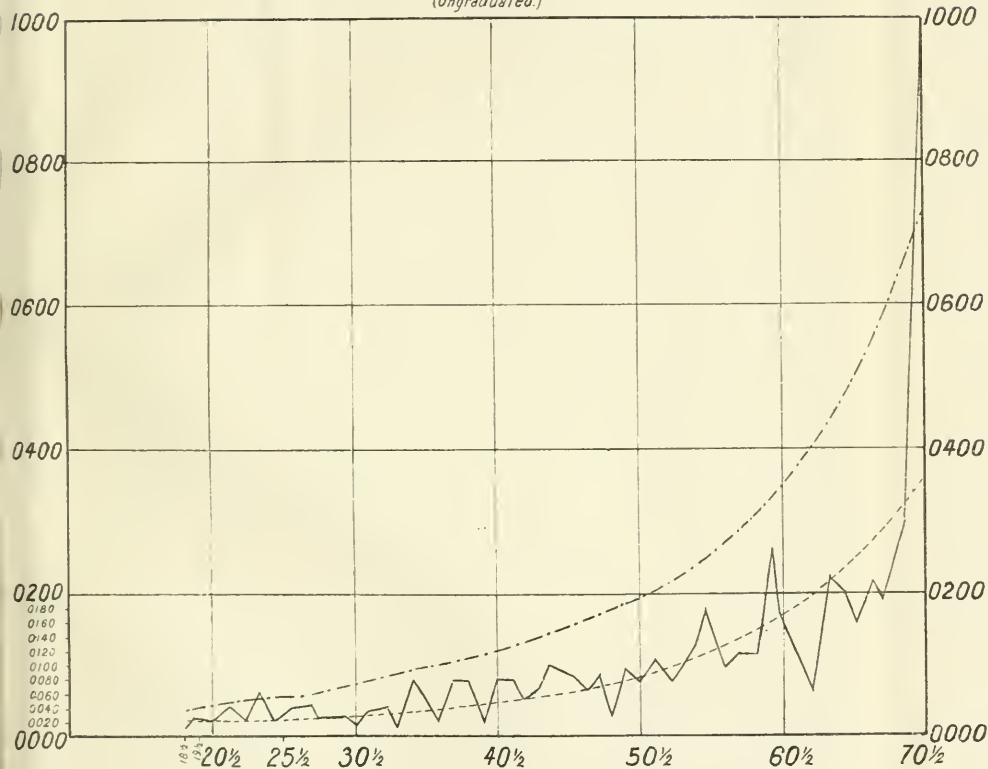
DIAGRAM 4.

Rates of Mortality.

O^(m) (for year of assurance O) -----

6th English Life -----

Gt W Ry. Pension Society -----
(Ungraduated.)



servants whose average wage does not greatly exceed £1 per week, and the clerical staff who are in receipt of considerably higher salaries. In diagram (4) we give the unadjusted rates of mortality of 20,000 employees of the Great Western Railway Company, consisting of porters, guards, and others of kindred occupations, obtained from an experience extending over the five year period 31 December 1904, to 31 December 1909. For the purposes of comparison the rates of mortality for the O^[M] and the sixth English Life Table are exhibited on the same diagram. The diagram shows that the rates of mortality are practically identical with the O^[M]; the reason for this is, of course, not that the members actually experience this extraordinary rate of mortality, but that they do not die whilst on the active list.

This feature of light mortality is not confined to the railway service for, as already mentioned, amongst the engineers of the Union Castle there has been but one death amongst those who entered during a period of 10 years.

We now come to what we consider the much more important question of the withdrawal rate. To the average student imbued with principles on which Assurance Companies' valuations are made, the practice of taking credit for the profit from withdrawals must appear very reprehensible. In the case of an Assurance Company anything which tends to impair the resources of the future is to be deprecated. With a Pension Fund, however, the conditions are altogether different. It is a mutual association which is instituted for the benefit of the members. There are no share-holders to receive dividends, and as there is little or no expense on account of salaried staff, the cost of management is usually only about $2\frac{1}{2}$ per-cent of the contributions. The constitution of the fund does not contemplate the distribution of bonuses, and, indeed, in the majority of funds the financial position is so unsatisfactory that reductions in the benefits as well as increases in the contributions have sooner or later to be faced. In all large commercial companies there must inevitably be a constant fluctuation in the staff. The conditions of trade exercise an important influence on the withdrawal rate, especially when the members belong to the wage-earning class. In seasons when trade is good the members are attracted to other occupations where the wage that can be earned is higher. In seasons of bad trade the company is forced to dispense with the labour they cannot profitably employ. It is true that in these cases the rules may enable the

member to continue his membership, but in very few cases does he avail himself of this privilege, which is inserted in the rules of all societies which come under the provisions of the Shop Clubs Act. Although it may vary, there is reason to believe that there will be a withdrawal rate, and in the general case where some or all of the contributions are forfeited on withdrawal, the financial effect on the fund will be such that it cannot be neglected in any actuarial calculation. The error involved in ignoring the mortality rate would in most cases be much less than the error involved in ignoring the withdrawal rate, more particularly in the early years.

Although there can be no doubt that the withdrawals must be taken into account, there is room for very considerable variation of opinion as to how far the known experience will be reproduced in the future. This is particularly difficult to decide in the case of a fund, the formation of which may be expected to introduce a new feature into the conditions of service and which may have a powerful effect in checking withdrawals; indeed, it may be that one of the objects in introducing the Pension Fund was to produce a more stable condition in the personnel of the staff.

The question arises, therefore, in what way is the experience to be modified in order that the fund may be started on a sound basis? With regard to younger entrants the pension age is so remote that it is unlikely to affect their decision in the event of a change of employment offering itself. With regard to the older members, however, the accumulated past service and the comparatively near approach of the pension age will probably weigh very considerably, and the institution of the Pension Fund in these cases may be expected to reduce the withdrawal rate as the age increases.

In dealing with a new fund, therefore, it is evident that whilst the experience of the past will require modification, this must not be in the form of a level percentage reduction at all ages.

We have found it convenient to fix on a limiting age which is some years earlier than the age at which, according to the past experience, the last withdrawals took place, and in graduating the experience by the graphic method we have generally found it advisable to run the curve off so as to terminate at such earlier age. By this method the graduated rate tends to depart more and more widely from the experience rate as the limiting age is approached. This is in accordance with the arguments just put forward as to the effect which the institution of the Pension

Fund may be expected to have on the future withdrawal rate.

Mr. King (*J.I.A.*, vol. xxxix) has referred to the desirability of basing estimates in Pension Fund valuations on select tables. We are in complete agreement as to the desirability of such a basis; the practical difficulty is that in the ordinary fund the necessary data for the formation of such tables are not available. If the limits of age at which the members may enter are restricted and the great bulk of entrants come in at approximately the same age, the data are practically in select form and may be used with considerable confidence. Where, however, the entrants come in over a wide range of ages some adjustment is required, if the derived rates are to represent the future experience of the existing members. Though theoretically such adjustment should be applied to both mortality and withdrawal rates, it is of greater importance with regard to the latter, since withdrawal rates are influenced far more by duration than by age. At the younger ages the rates brought out by the experience will very closely approximate to select rates, for they will be based almost entirely on new entrants. In these circumstances the error is insignificant in view of the fact that a member who withdraws soon after entry can have but little effect on the finances of the fund. With regard to the experience at the older ages, the data will depart more and more from the select form, since the exposed to risk will be composed of members with widely different durations of service. Thus it will be seen that the rates shown are too high as regards the members who entered many years previously, and too low as regards members of middle age who have entered recently.

The error involved can easily be seen by means of an example: Suppose that at the age of 39 there is a fairly large proportion of entrants, then there will as a consequence be an unduly high rate of withdrawal at age 40. A member who entered at 20 will certainly not be subject to such high rates at age 40 as a member who entered at age 39, but by using an aggregate table without adjustment the present value of his future benefits and contributions will be based upon this entirely false assumption. If the proportionate number of late entrants into the employer's service continues, it may very possibly be found that the rates of withdrawal obtained at successive investigations are similar, but this will be no justification for using an unadjusted aggregate table for the purpose of valuing by the prospective method. It must be remembered that where the full reserve value is not

returned at withdrawal the effect of high withdrawal rates is greatly to decrease the cost of the pensions, and the greater the difference between the reserve value and the withdrawal value allowed, the greater is the relief to the fund. If there should be a large number of entrants at older ages whose average duration of service is short, and consequently whose reserve values are very small, the financial relief to the fund would be small. In such circumstances the effect of using the full rates of withdrawal in an aggregate table would be to show too small a liability; in fact, the liability would represent the case where each withdrawal resulted in the correct average benefit to a fund not complicated by late entrants. It would seem that, theoretically, the only correct method of valuation is by means of elaborate select tables in which due allowance is made for varying wages and salaries, but such a method is for many reasons seldom, if ever, possible. In practice we have found that the adjustment of the withdrawal rates for the purpose of constructing an aggregate table suitable for valuation purposes is a matter which requires the most careful consideration. There can be no fixed rule in the matter, but speaking quite generally, the higher the age at withdrawal the greater should be the reduction in the withdrawal rates. Graphically expressed, the reductions would take something like the form of a wedge, the point of which would apply to the youngest age and the broadest portion to some older age. If the proportion of entrants at the older ages is small the wedge will be narrower than will be the case where there are many entrants at the older ages. This attempted explanation, we are quite aware, must appear curiously complicated to anyone who has not carefully considered the difficult problem involved, and our only reason for introducing it is in order to call attention to what we feel to be a most important practical point in the actuarial investigation of Pension Funds.

We should here like to mention another case where the use of the withdrawal rate is of outstanding importance, namely, in the valuation of Widows' Funds.

It has been suggested that the use of a standard table is sufficient for the valuation of Pension Funds and benefit societies generally, without any allowance being made for withdrawals. The mortality rates shown in the standard table are, of course, much higher than the actual mortality rates experienced, and were adopted as a sufficient approximation to the correct

mortality and withdrawal rates combined. When the incidence of the withdrawal rates is considered, it is difficult to see how it could in any case be approximately represented by such excess mortality, although it is true that with regard to a Pension Fund, mortality and withdrawal rates operate in the same direction and therefore may to a certain limited extent be interchangeable. In the case of Widows' Funds, however, the mortality and withdrawal rates operate in the opposite directions, that is to say, the death of the member must cause an immediate claim if he leaves a widow, whereas his withdrawal causes no claim but relieves the fund of a liability. The effect of using an incorrect method was plainly shown in a Widows' and Orphans' Fund with which we had to deal.

The previous valuation as at 31 December 1905 had been made on the basis of the Manchester Unity Mortality without allowance for withdrawals, and a very large deficit was shown. At the valuation made as at 31 December 1910 the Society's own experience with regard to mortality and withdrawals was used with the result that the deficit was reduced by over 90 per-cent. Some portion of this reduction was due to the fact that in the 1905 valuation all members were assumed to have been married, whereas the particulars as to marriage showed that the proportion of married men was about 80 per-cent. This only referred to the marital state of the members at that date, and in calculating the liabilities it was necessary to provide for the future marriage of many of the 20 per-cent unmarried members.

In close connection with the subject of withdrawals is that of negative values. It has been often argued that such values should be rigidly excluded in all circumstances. Such a course may be desirable where the fund is in a strong position and able to stand a valuation upon a stringent basis. Where, however, the question is one of rearrangement consequent on a large deficiency having been disclosed, the question as to how far negative values may be treated as an asset becomes important. Where the withdrawal rate is taken into account and the withdrawal benefit is valued as a liability, it is evident that at least some portion of the asset represented by such negative values is well secured. If the future withdrawal experience is in exact accordance with that anticipated, the whole of the negative values will fall into possession. If the future withdrawals are in excess of the expected, then some portion of the negative values

will be lost, and if the withdrawals fall short of the expected there will be a corresponding gain. The negative values will therefore be an unsatisfactory asset when the graduated rate at those ages where such values exist is appreciably less than that shown by the experience. On the other hand, the negative values will be well secured when an exaggerated view of the future withdrawals is taken. Negative values only occur at the younger ages, and in graduating the withdrawal rates it has been our practice closely to adhere to the full withdrawal rates at those ages where negative values may be expected. Having in this way taken precautions against any reasonable possibility of loss, we have been enabled to take credit for some portion of the negative values disclosed, with benefit to the fund and with absolute confidence that the asset will be realized.

Many and varied are the difficulties with which the actuary has to contend when valuing Pension Funds which have been some time in existence, more particularly when, as is too often the case, a considerable deficiency is disclosed. When it is remembered how vital are the interests involved and how many sad examples there have been of Pension Funds which after many years have been found to be in a hopeless state of insolvency, it is extraordinary that there should still be so many where no provision is made in the rules for periodical actuarial investigations. Still more remarkable is it that even at the present time there should be business men who will venture to initiate Pension Funds without first seeking expert advice. When members have for years been paying contributions without doubting the certainty of eventually obtaining the promised pension, the duty of having to disclose a state of insolvency is often extremely painful. The sympathy naturally excited by the disappointed hopes of elderly men who have been looking forward to a suitable provision for their old age, is such as to cause the actuary to put forth his utmost endeavours to find the most suitable scheme of reconstruction. To reduce the amount payable as pension may be to inflict almost intolerable hardship, and yet to increase the contributions to a sufficient extent to make good the deficiency may be altogether impracticable.

In compulsory schemes we have found that, generally speaking, the only practicable course was for the employer to shoulder the greater part of the accumulated deficiency. When this can be done the difficulties of the actuary are lessened, although they may still be considerable. In such a

case an entirely new scheme of contributions will be necessary, and probably the employer will not consider it advisable that the employees should merely continue to contribute at the old rates, whilst he is called upon to pay much larger contributions both for existing and future members. If, as is often the case, the employer has paid a fixed proportion of the total contribution, he may agree to do the same under the readjusted scheme, and then on the assumption that the members agree to pay increased contributions, he may undertake to make up the remaining deficiency. It will probably still be necessary to make special arrangements for members who will not (possibly cannot) pay any greater contributions than in the past.

The employer, however, will certainly not want to make good the deficiency by the payment of a lump sum; to do so would seriously affect the profits of that particular year, and for many reasons it would be inadvisable. There is, however, generally no reason why an annuity-certain should not be substituted for the lump sum, and this course is almost invariably adopted. Another source from which increased revenue may sometimes be obtained is the interest obtainable on the invested funds. If only a comparatively low rate has previously been earned, the employer may undertake to guarantee some higher rate on all future investments. If such guarantee includes the payment of all income tax, the advantage is considerably increased. With income tax payable at the rate of 2s. 6d. in the £1, it must be remembered that a gross rate of 4 per-cent means an effective rate of $3\frac{1}{2}$ per-cent, and although we trust that such a high rate of tax will be only temporary, it is well to remember that any future fluctuation must be a matter of the most careful consideration in estimating the liabilities of Pension Funds, where there is seldom any possibility of surplus.

It is in funds where the responsibility rests with the member that the principal difficulties are met with, and we should like to use as illustrations the benefit societies of the Great Western Railway Company, most of which include pensions amongst their benefits. These societies consist of the Pension Society, the Provident Society, the Engine Drivers and Firemen's Society and the Locomotive and Carriage Department Society. There are also other societies connected with the Company, the principal one being the Superannuation Scheme for the salaried officers, which was remodelled in 1908 subsequent to a valuation by Mr. F. Schooling.

The Pension and the Provident Societies were originally one society, but now form separate organizations, although the members are practically the same in each case. The membership at the present time is approximately 20,000.

The Pension Society had been warned that it was financially unsound, and in 1904 altered its rules. Prior to that date the contributions and benefits were as follows :

Contributions—3d. per week by the member and an equal amount by the Company. In certain cases the members and the Company paid double contributions for a limited period.

Pensions—After 30 years' membership and the attainment of age 55, a pension of 10s. per week, with an additional allowance of 1s. per week for every completed term of 5 years' membership beyond the first 30 years. On disablement before age 55 a pension not exceeding 4s. per week, which, however, was to be increased to 10s. on the attainment of age 55.

Death Benefit—Return of member's own contributions.

Withdrawal Benefit—Under 5 years' membership, nil. 5 years and less than 10 years, one-half of member's own contributions returned. 10 years and upwards, member's own contributions returned in full.

In March 1904 drastic alterations were made and a new class of member was instituted. These were termed "New Members", and the then existing members "Old Members." The rules were modified to read as follows :

Pensions—

(a) New members. A pension of 5s. per week at age 65.

(b) Old members. A pension of 5s. per week at age 60.

Disablement pension. 2s. 6d. per week, to be increased to 5s. on the attainment of age 60 (old members) or 65 (new members).

Contributions—

(a) New members. A graduated scale increasing from 4d. at age 18 at entry to 7d. at age 30 at entry.

(b) Old members. 5d. per week.

No contributions were made by the Company.

Death or withdrawal benefit—The return of the whole of the member's contributions on death or withdrawal at any time before pension age.

It will be seen that the contributions of the Old members were increased by 2d. per week, the pension age altered from 55 to 60,

and the amount of pension reduced to 5s. per week. The Company withdrew its contributions at this time but instituted a scheme of supplemental pensions as follows :

A pension equal to one-quarter of the member's weekly wage provided he had served 40 years, with proportionate reduction for shorter service. The maximum pension under this scheme is limited to 15s. per week, and the Company guarantees that the supplemental pension together with the pension given by the Society shall not be less than the pension received under the old rules.

In addition to this the Company undertook the payment of one-half of the pensions which had emerged at the date of the alteration. Further, they undertook to make a grant of £2,000 per annum during the first quinquennium.

A valuation was made as at 31 December 1909, and, whilst showing a small surplus in respect of the New members, brought out a very large deficiency in respect of the Old members. To cover the deficiency by means of a flat addition to the contributions would have required an increase of 150 per-cent to the rates paid. This would have resulted in ridiculously high contributions (that is, in proportion to the benefits) at the younger ages, and, moreover, would have produced very large negative values which, as membership was not compulsory, would probably not have been realized. On the other hand, a graduated scale would have produced prohibitive rates at the older ages.

It will be recognized that the difficulties presented were very great indeed, and had it not been for the Company's generosity it is impossible to imagine what would have been the outcome of the extended negotiations resulting from the valuation report. The members eventually agreed that the pension to be received from the Society should be reduced by an amount equivalent to the pension that could be obtained under the Government Old Age Pension Act. This, it was anticipated, would be a considerable relief to the Society. For the rest the Company agreed that for the next quinquennium it would pay one-half of the pensions that had emerged since 1904, this proportion to be reconsidered if the next actuarial investigation showed it to be necessary. The Company continued to pay one-half of the pensions which were in force prior to 1904.

TABLE A.

Great Western Railway Pension Society.
Pensioners' Mortality Experience.

Age	Rate of Mortality per-cent	Age	Rate of Mortality per-cent
20½	6·3	45½	6·3
21½	6·3	46½	6·3
22½	6·3	47½	6·3
23½	6·3	48½	6·3
24½	6·3	49½	6·3
25½	6·3	50½	6·3
26½	6·3	51½	6·3
27½	6·3	52½	6·3
28½	6·3	53½	6·25
29½	6·3	54½	6·15
30½	6·3	55½	6·05
31½	6·3	56½	5·95
32½	6·3	57½	5·85
33½	6·3	58½	5·75
34½	6·3	59½	5·6
35½	6·3	60½	5·3
36½	6·3	61½	4·85
37½	6·3	62½	4·35
38½	6·3	63½	3·95
39½	6·3	64½	3·7
40½	6·3	65½	3·6
41½	6·3	66½	3·7
42½	6·3	67½	3·95
43½	6·3	68½	4·3
44½	6·3	69½	4·8
—	—	70½	5·45

We have already referred to the rates of retirement and mortality experienced by the active members of this Fund (See diagrams 1 and 4). In Table A we give the graduated rates of mortality amongst the pensioners. It will be noticed that up to age 52 the rate is constant. This is quite a usual feature with ill-health pensioners' mortality, but in this case the rate is somewhat lower than usual. The following extract from the valuation report may be of some interest :

"It was necessary to form a special table of mortality applicable to those pensioners at older ages. These mortality rates which I have used for New as well as Old members were obtained from the experience of the quinquennial period (31 December 1904 to 31 December 1909).

“It will be noticed that from age 20 to age 52 the rate is constant ; it then decreases until about age 65, after which it rises, until at about age 75 the data, which are somewhat scanty, seem to indicate rates about equal to the Government Annuitants Experience (1883). I must here explain that the rates of mortality as originally obtained showed a more rapid fall at age 55, reaching a minimum at about age 60. Upon examination I found that until 1904 there had been a rule permitting members of the Pension Fund to retire at age 55 instead of age 60. Undoubtedly the addition of a number of pensioners at age 55 who were not in ill-health, must have resulted in a considerable improvement in the mortality rates, and as in future this influx of pensioners will be deferred until age 60, I have thought it advisable to adjust the experience accordingly. At some future time a further adjustment will probably be necessary for the New members, since their pension age is 65 instead of 60, but at the present time I do not consider that such adjustment is necessary.”

In the Provident Society where, as already mentioned, the members consisted of the same persons as in the Pension Society, no superannuation benefit was allowed, but we should like to refer to the experience of the Society, as it is one of the few examples of an insolvent society taking advantage of Section 72 of the National Insurance Act, and thereby placing itself on a sound financial footing.

The benefits given by the Society consisted of:

Sickness—First 26 weeks' illness 12s. per week. Second 26 weeks' illness 6s. per week. Remainder of illness 4s. per week.

Funeral Benefit—Member, £10. Wife, £5.

Medical Benefit—The members had the right to receive medical attendance, the Society paying the surgeons at the rate of 4s. per annum.

No surrender-values were allowed to members on withdrawal.

Contributions—Age at entry 30 or over, 9d. Age at entry over 25 but under 30, 8d. All other members, 7d.

In addition, the Company contributed £1,000 per annum.

A valuation was made for the purpose of the National Insurance Act, and disclosed a deficiency of close on £200,000.

Various schemes were suggested in order to reduce this amount, and in all 12 different schemes were propounded. The

consideration and discussion of these schemes involved many meetings with the Committee of Management and Delegates, and we found that they had a very clear idea of the actuarial difficulties involved in the rearrangement, and also of the necessity for its being undertaken.

One meets on occasion members of Friendly Societies who have but little idea of the meaning of a deficiency, referring to it in terms of pity as merely an 'actuarial' deficiency, and it is pleasant to be able to show that this was very far from being the attitude of the Committee of the Great Western Railway Provident Society. Our meetings with the men took place in association with Mr. A. E. Bolter, the Secretary of the Railway Company, whose many valuable suggestions were of the utmost assistance.

The effect of eliminating the benefits which were duplicated by the National Insurance Act, and reducing the contributions by 4*d.* until age 70, would have been to reduce the deficiency to less than £100,000, or about one-half of the previous amount.

Eventually the scheme adopted was as follows :

Sickness—For existing members who were employed contributors under the National Insurance Act :

First three days of illness, 12*s.* per week. 26 weeks of illness from the fourth day, 2*s.* per week. Second 26 weeks of illness, 2*s.* per week. Remainder of illness, Nil.

For existing members who were pensioners of the Great Western Railway Pension Society at the date of the rearrangement :

First 26 weeks of illness, 5*s.* per week. Second 26 weeks of illness, 2*s.* 6*d.* per week. Remainder of illness, 1*s.* 6*d.* per week, and medical benefit throughout life.

For all members on attainment of age 70 :

First 26 weeks of illness, 5*s.* per week. Second 26 weeks of illness, 2*s.* 6*d.* per week. Remainder of illness, 1*s.* 6*d.* per week.

The death benefit remained as before, but the medical benefit was abolished in the case of all members who were employed contributors under the Act.

One of the difficulties in the rearrangement was how to deal with younger members. These were paying contributions that were adequate to provide the benefits given, the deficiency being the result of insufficient contributions in the past.

Eventually the contributions were fixed as follows :

In active service. Pensioners.

Those who joined the Society when over

30 years of age	5 <i>d.</i>	6 <i>d.</i>
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Those who joined the Society when over

25 years of age	4 <i>d.</i>	5 <i>d.</i>
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The remainder	3 <i>d.</i>	4 <i>d.</i>
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In the case of those who were on the active list, the above rates secured the full reduction of 4*d.*

The arrangement was rendered easier owing to the fact that the Company agreed to pay 1*d.* of the weekly contribution of the members who joined the Approved Society formed in connection with the Provident Society. This payment, which was in substitution for the £1,000 a year formerly granted to the Provident, amounted to over £4,000 a year in respect of existing members, and is, of course, growing with the accretion of new members.

We think that the case of the Great Western Railway Provident Society is a good illustration of what can be accomplished for an insolvent society by the aid of Section 72 of the Act.

Although we have not had the opportunity of analysing the sickness experience of the Society since the National Insurance Act came into force, we understand that no increase in the sickness rates is observable.

The experience of the above Society is in marked contrast with that of another similarly constituted society, numbering some 12,000 members, with which we have recently been concerned. The members refused to take advantage of Section 72, and insisted upon having the full benefits under the National Insurance Act in addition to those promised by their own Society.

The experience for the quinquennium 1 January, 1908 to 31 December, 1912, was analysed, and afterwards the experience for the year 1913. The variation in the latter year is so remarkable that we reproduce it below.

TABLE B.—*Sickness Experience 1913.*

Age	1ST 26 WEEKS' SICKNESS		2ND 26 WEEKS' SICKNESS		REMAINDER	
	Expected Sickness, 1908-1912 Experience	Actual Sickness	Expected Sickness, 1908-1912 Experience	Actual Sickness	Expected Sickness, 1908-1912 Experience	Actual Sickness
	Weeks	Weeks	Weeks	Weeks	Weeks	Weeks
15½	330	288	7	18	—	—
16½	357	456	12	25	3	—
17½	326	651	14	24	6	—
18½	321	625	18	27	10	52
19½	340	549	22	16	14	—
20½	384	484	28	16	20	—
21½	305	512	27	26	19	74
22½	230	457	23	33	19	20
23½	254	414	29	45	24	1
24½	207	431	27	28	22	52
25½	207	243	27	59	25	—
26½	192	237	25	28	25	—
27½	278	241	36	9	39	52
28½	242	436	31	37	36	69
29½	262	326	33	69	41	38
30½	272	367	31	62	45	17
31½	281	366	32	29	49	52
32½	265	384	29	75	48	—
33½	280	350	28	73	56	21
34½	295	426	29	38	64	17
35½	321	427	32	9	76	52
36½	329	543	32	64	83	104
37½	277	460	27	7	75	5
38½	295	387	28	76	84	80
39½	280	483	26	4	84	104
40½	297	441	27	45	93	156
41½	278	371	25	6	90	21
42½	325	317	32	47	109	78
43½	297	352	28	21	106	159
44½	307	494	31	47	117	19
45½	326	470	35	21	132	—
46½	303	365	34	—	132	268
47½	266	441	33	48	122	156
48½	313	312	43	6	159	129
49½	317	298	46	—	176	178
50½	290	375	45	29	205	105
51½	272	366	45	5	250	52
52½	311	417	55	19	285	82
53½	308	253	58	6	282	207
54½	291	385	57	69	262	342
55½	255	351	52	74	227	245
56½	266	248	56	8	230	461
57½	272	285	59	33	228	376
58½	291	273	65	14	237	208
59½	258	288	59	31	204	373
60½	210	388	49	89	160	229
61½	211	188	49	77	154	16
62½	218	190	51	7	153	40
63½	172	324	41	31	116	131
64½	176	183	42	44	115	7
65½	97	195	24	47	86	145
66½	132	86	32	19	86	260
67½	116	152	32	63	80	12
68½	58	118	18	40	45	1
69½	53	61	20	33	47	66
70½	47	68	26	7	47	52
	14,463	19,598	1,922	1,883	5,702	5,384

It will be seen that the second 26 weeks' sickness and the remainder sickness (where duplication of benefit was practically impossible) were, on the whole, lower in 1913. The first 26 weeks' sickness, that is to say, the sickness for the period during which double benefits were being received, was, however, nearly 50 per-cent higher under age 30, 35 per-cent higher from 30-50, and 20 per-cent higher at ages over 50. As we were unable to trace any exceptional general conditions to account for this striking increase, we were forced to the conclusion that it was due to over-insurance, and the new definition of incapacity that seems to have sprung up owing to the introduction of the Act.

Group Assurance.

A well-organized Pension Fund liberally assisted by the employer is undoubtedly a powerful agent in fostering a friendly relationship between employer and employee. For this and several other reasons we are convinced that pension schemes will become more and more general in the future. In the United States the many attractive features of pension schemes have long been recognized, but some cheaper method of attaining similar results has been constantly sought.

At the present time there is a very wide movement in favour of Group Assurance. As it is quite possible that the movement may extend to this country, we feel that it may not be out of place to make some few remarks in reference thereto.

It is claimed for Group Assurance that it is not subject to many of the restrictions which limit the application of assurance in its more usual form, and further, that it reduces the cost.

There is no actuarial reason why the whole population should not be insured irrespective of the state of health of individuals, and if there were but one assurance office and all persons were compelled to assure therein, no difficulty would present itself in assessing the premium required. Moreover, in a stationary population unaffected by emigration and immigration, the premium would remain constant from year to year, and if the population were an increasing one the flat rate per unit of population would (except in certain cases of exaggerated birth rates) actually decrease. The flat rate of premium would also be reduced from year to year either in the case of an increasing or a decreasing population, provided that there was a sufficient proportion of young immigrants and old emigrants. In such circumstances the benefits of assessmentism would be secured

and the usual disadvantages obviated. The opportunity of assuring the whole population has not arisen, but it is possible to find what may be termed sample populations to which the principle may be applied with safety.

It is on the principle enunciated above that Group Assurance proceeds, and the sample populations to which it is being applied are the staffs of large manufacturing works or stores.

In the United States the necessity for creating some bond between employer and employee, other than that of salary, has been recognized as fully as in this country, and many efforts have been made to increase the attractiveness of the service. Group Assurance has been adopted as a means to this end, partly, no doubt, on the ground that it is less costly than a pension scheme.

Briefly the plan is as follows :

The employer supplies the assurance company with a list of his staff, giving the age of each employee. The company, after making certain enquiries, issues a policy covering the whole staff at an inclusive premium under which it agrees to pay either a fixed sum or a sum depending on the man's wage, on the death of each employee during the currency of the policy, provided death occurs whilst in the service. The contract is for one year and the assurance company has the right to adjust the premium at the beginning of each period. There is no medical examination and the whole staff is assured without exception.

The enquiries made relate mainly to the conditions of employment, healthiness of the environment, and fluctuations in the constitution of the staff. The premium charged varies from 1 to $1\frac{1}{2}$ per-cent of the sum assured. Sometimes the sum assured is a fixed amount per person, or as an alternative the pay roll is assured, in which case the sum payable on death is one year's wage. At first sight it would appear that the premium must increase year by year in accordance with the ordinary increase in the rates of mortality, and were there no withdrawals from the service this would be the case. In the normal case, however, there is considerable fluctuation in a staff of this description, and there is every reason to suppose that the new entrants are on the average younger than those who leave. Where the staff is expanding, the new additional staff is almost certain to be younger than the average.

In these circumstances it is quite possible that the premium

may remain constant over a long term of years, or may even decrease, for the staff forms what may be termed a population such as that contemplated above when we referred to the possibility of assuring the whole community.

The plan is attractive to those who desire to see the basis of assurance broadened and freed from restrictive conditions, for it certainly reduces assurance to its simplest terms.

The bases for construction of premiums are of some interest.

The assured will be a homogeneous body similar in constitution to the members of a Pension Fund, and this will lead to the expectation of extremely light rates of mortality. The light mortality experience of a Pension Fund is due to members in impaired health withdrawing or taking ill-health pensions (which is indeed another form of withdrawal).

If the light rates of mortality of the average Pension Fund are to be reproduced under Group Assurance, the conditions of the policy must be very carefully drawn to exclude from benefit deaths which occur off the active list.

The rate of interest to be employed is of little importance as it can hardly affect the premium.

With regard to the loading, as already seen, the expenses are much less than under an ordinary assurance. Commission would be paid on the term assurance basis, which does not usually allow for any difference between the new and renewal commission.

We have worked out the premium required for an assurance of £100 on each of 1,000 workmen, whose limits of age lie between 20 and 65 and whose age distribution is the same as that of the general population. We have used the seventh English Mortality Table, and for comparison the mortality as experienced by the Great Western Railway Pension Society. The net premiums are as follows, interest being taken at 3 per-cent in each case :

Seventh English Life Mortality, £1,266.

Great Western Railway Pension Society Mortality, £718.

These rates are equal to a flat rate of £1 5s. 4d. and 14s. 4½d. per-cent respectively.

One of the most interesting practical points is the question of the form of policy. We are informed that it would be illegal in this country for a Life Assurance Company to issue a policy of this kind without stating the name of each individual assured under the contract. This renders the question of providing for

entrants during the year a matter of some difficulty. Failing any other solution it might be arranged for by making the premium payable half-yearly and issuing a new policy with the names of the employees on 1 January to 1 July in each year. This method would automatically provide a waiting period for new entrants which would certainly be required, and if it were desired to have a fixed waiting period a provision could be inserted, restricting claims to those who had been at least 6 months in the service.

The necessity for restricting the claims to those who die whilst actually on the active list has been referred to. A very careful definition as to when service is to be assumed to cease, will be required, and it would seem advisable to make it a condition of claim that the deceased was in receipt of full wages at the time of death.

A further legal point is the question of insurable interest. It is clear that the relation of employer and employee does not constitute such an interest, and the former would have no control over the policy money. It is doubtful, however, even if this point were satisfactorily secured, whether the employer could legally take out such a policy on the lives of his workmen.

It is stated that Group Assurances aggregating a very large amount have been written in the United States and are growing rapidly, but whether they will ever become popular here is open to question. The benefits offered are in no way comparable to those generally provided in pension schemes. It is quite evident that the benefit afforded by the suggested assurance is extremely limited; indeed, the low cost is only to be accounted for by the fact that comparatively few claims are paid. As we have already explained, the low rate of mortality usually shown amongst active members is largely due to the fact that in many instances members who are in an unsatisfactory state of health are not retained on the active list.

It is hardly necessary to point out that sooner or later there is almost certain to be a considerable amount of dissatisfaction, when it is found that in many cases of death there is no insurance money payable.

The whole scheme is open to the criticism that it appears to afford better protection than is actually the case. We are strongly of opinion that there is a great national need for *bonâ fide* life assurance which has not yet been fully recognized by the general public, and that any scheme which might tend to substitute an incomplete and unsatisfactory policy should not be encouraged.

In conclusion we should like to acknowledge the very valuable help which we have received from Mr. P. C. Crump, F.I.A., whose practical experience of the working involved in valuations has been of the utmost assistance.

ABSTRACT OF THE DISCUSSION.

Mr. THOMAS TINNER said that the establishment of a pension fund without any contribution from the staff was, for many reasons, attractive to the employers—in view of possible wages disputes, for instance. He believed that the employees' interest in the pension fund had often tied the hands of the employers, because of the contention that the employer had no right to deprive his servants of the benefits for which they had contributed for many years. Even if the fund were maintained entirely by the employer with the intention of enabling him to grant retiring allowances as a reward for faithful service, the men's leaders might say that the fund was really an accumulation of deferred pay which morally belonged to the employees.

How far could the needs of members of the staff be met by deferred annuity policies? The defect of all schemes involving the taking out of deferred annuity policies was that the provision in case of breakdown before reaching the pension age was inadequate, and had to be supplemented by the employer, as in the cases of the scheme set up by the Government for elementary school teachers and of that recently proposed by the Departmental Committee for the superannuation of teachers in secondary schools. The need of providing for the man who became incapacitated afforded the strongest argument for the establishment of a pension fund, because the cases of death in the service or survival to the normal pension age could well be met by insurance. It was often taken for granted that if the scheme were contributory the members must take part in the management, but in cases where the employer was undeniably able to fulfil his engagements it was doubtful whether it was advisable for them to do so, because sharing in the management involved sharing in the responsibility for any deficiencies that might arise. As a rule the amount and kind of benefit in any particular case were fixed by the rules, the management having no discretion, and therefore the only matter over which the members could exercise any control would be the investment of the funds, a matter for which they had very little qualification, if any. If they left the management to the employer they were in a much stronger position with regard to deficiencies, the likelihood of friction was lessened, and the machinery of management would work more smoothly.

The authors raised an interesting question in their remarks on the use of salary scales, and it would be useful if they would give their reasons for thinking that better results could be obtained by making separate valuations in respect of existing salaries and future increments. It had always seemed to him to be axiomatic that increments

of salary were approximately proportionate to the salary itself and that to assume that future increases of salary would depend upon the number of members without regard to their present salaries was unsound.

The scheme described in the paper suggested a difficulty which arose when an employer started a fund by paying in a large sum and undertaking to contribute a certain proportion of the members' salaries. Usually the initial payment was intended partly to cover past services of existing members, and partly to form a perpetual endowment, and when a valuation was made the question arose as to how much of the total should be appropriated to existing members. He had found it advisable to avoid apportioning the amount and to value the fund in perpetuity instead, making for that purpose assumptions, approved by the authorities of the fund, as to the future membership.

The great difference between the original scheme for the Union Castle Service and that actually adopted afforded a very good illustration of the volume of work to be done between the first consideration of a proposal to establish a pension fund and the putting into operation of a scheme; and in that connection it would be useful if some method of calculation could be devised which would save the necessity of doing the greater part of the work over again on each modification of the scheme. The convenience of such a method was brought before him in the case of a fund with several thousand contributors, when an improvement of the scheme was to be made consequent on the obtaining of a permanent constant addition to the annual income, the nature of the improvement being dependent on the amount of the additional income. The benefits had up to then depended on the amount of deferred annuities which could be purchased with the members' contributions, and it was desired to substitute pensions based upon average or final salary at the rate of $1/n$ th for each year of service, with a certain maximum. Before a satisfactory scheme could be produced which would utilise the whole of the income and no more, experimental valuations had to be made. For that purpose he arranged the particulars and calculations according to age at entry and duration so as to show the values of pensions equal to final salary which would emerge after ten years, eleven years, and so on. The results were then summarised so as to combine the figures for all ages at entry, and from the summary it was possible in half-an-hour to calculate the value of pensions according to any scale based on final salary. It was also possible without undue labour to arrive at the approximate cost of pensions based on average salary, with the same facility of varying the proportion allowed for each year of service and the maximum number of years counting for pension, &c. The drawback to the method was that it could be used with advantage only when the number of members or prospective members was large, because the work was practically as great for a small as for a large number. On the other hand, it was so simple in application that although it might involve more labour than the usual methods that labour

might be unskilled to a much larger extent. Anyone who would devise a method which would enable the cost of alternative schemes to be calculated without undue labour and which would be equally suitable for large and small staffs, would have earned the thanks of the profession.

With reference to the subject of negative values, he understood the authors to state that negative values were reduced by increasing the rate of withdrawal at the ages at which these values appeared. That was a somewhat sweeping assertion, as would appear from the following considerations. As the requisite rate of contribution increased with the age it followed that the period of negative values was prior to the age at which contributions and benefits in respect of subsequent services were equal. The negative value was made up of the excess of contributions over benefits in the case of those who withdrew or died, plus the algebraic excess of contributions over the reserve required at the end of the negative value period, for benefits in respect thereof, for those who survived it. As a rule, the gain to the fund from members leaving increased with their length of membership on exit, and any increase of the rates of withdrawal had the double effect of altering the number assumed to leave during the negative value period and varying their average length of service. It seemed, therefore, that an increase in the withdrawal rate if confined to the early part of the negative value period would increase the number leaving in that period but decrease their average period of service, and might or might not decrease the negative values, although it would usually have that effect; while an increase in the rates over the whole of the negative value period would probably increase the negative values, and an increase in the later part of the period would certainly do so. The point was not perhaps of great importance, because it usually happened that when a member left a large service he was replaced by a younger man, who brought in at least as large a negative value as the valuation would attribute to his predecessor. There was a danger, however, in raising the withdrawal rates, because any increase in those rates caused a lower rate of contribution to be brought out. On the whole he thought the suggestion of the authors should only be adopted in very exceptional circumstances.

The mortality of pensioners was, of course, one of the most important subjects for consideration in connection with the formation or valuation of pension funds, and he should like to enter a plea for the use of separate annuity tables for valuing pensions granted owing to ill-health and those taken on ordinary retirement. The mortality experience of those two classes exhibited widely different characteristics, and the use of a single table for both was fraught with considerable risk of error. So far as his experience went, the mortality rates of invalid pensioners ran on smoothly from age to age, as might be expected of data derived from homogeneous material, but if the data relating to that class were mixed up with those relating to ordinary pensioners, who seemed to experience astonishingly light mortality, the results could not be regarded

with much confidence. The pensions of invalids were much smaller on the whole than those of ordinary pensioners, and that difference should be allowed for if the combined rates were to form the basis of monetary calculations; but he had never heard of such an allowance being made. The effect of using combined rates was to increase the value of the invalid pensioners' annuities, owing to the decrease of the rates due to the introduction of the healthy pensioners. That decrease in rates, however, did not appear until the invalid pensions had been in force for some years, so that the effect on the value of the yearly pensions might not be very large. When the effect on the ordinary pensions was considered it could be seen that the under-statement of the mortality due to the mingling of data resulted in under-estimating the cost of the pensions to that class and that the under-estimate ran from the commencement of the annuity. The result of combining the data of the two classes was, therefore, to cause an under-estimate of the payments in respect of the larger pensions throughout their period of payment, and the only set-off against that error was an over-estimate of the payments to early pensioners during the latter part of their pension life, and thus it would be seen that the use of combined mortality tables resulted in a persistent error on the wrong side.

The illustrations given by the authors of their treatment of the problems arising in the case of the Great Western Railway were very interesting, and would be helpful to other actuaries who might have to re-adjust contributions and benefits. Table B was very useful, as showing more conclusively than the most exhaustive arguments that over-insurance was a very real danger to the finance of friendly societies, and that, as some humorist had said, "There's a lot of human nature in man."

With regard to group insurance, he had to confess that his acquaintance with it was purely theoretical, but he agreed entirely with the authors' arguments. He was afraid that considerable disappointment would arise if the system were largely adopted in this country, because people would find that they were not getting the article they thought they were purchasing and would find they were disappointed in their old age, when perhaps it was impossible for them to make other provision.

Mr. E. C. THOMAS congratulated the authors on their having been able to give particulars of an actual scheme with which they had dealt, and to describe how the difficulties had been overcome. It was not often that actuaries were in the happy position of being able to give full details of their problems without awkward reservations, which robbed the description of very much of its value and concealed the lessons which might be learned. At the commencement of their paper the authors referred to the question of the desirability of an assurance company guaranteeing the benefits of the fund. Some years ago he had occasion to study rather closely the exhaustive and able report of Mr. George King upon the Elementary School Teachers' Fund, and although he had not been able to refresh his

memory by reference to the report, he remembered that Mr. King found it necessary to recommend a drastic reduction in the benefits, which were already sufficiently low. The reasons for the unsatisfactory state of the fund were (1) that for some years the fund had been compulsorily invested in Consols, which had only given a little over $2\frac{1}{2}$ per-cent interest with a constant depreciation of capital, and (2) that the mortality of the members had been much lower than was represented in any table published up to that time. The result was that, on examining the benefits granted by an ordinary insurance company to the public, he found it would have paid the members very much better to have taken their contributions to the company and bought deferred annuities at the ordinary prospectus rates. In some cases the amount they could have obtained from the insurance company for their own contribution was more than they received for their own contribution in the Government scheme, plus the additional pension given by Government. Consequently, an insurance company could not have undertaken the scheme, even on its ordinary prospectus terms, without heavy loss. He thought that the same consideration would apply in the case of small pension funds of private firms. In the particular case he had just cited there was no death benefit, so that an unusually low mortality was a deadweight loss. Of course, in an ordinary pension fund there were generally returns on death which would mitigate the effect of a very low mortality, but even then there was left an excess vitality after the pension age. If the breakdown cases were set aside and treated as surrenders, which he assumed would be the only practical way for an insurance company to deal with them, the result would be that the fund would be left with a very select class of those who reached the pension age, a much more select class probably than any body which had been the subject of investigation up to the present time. Those who were pensioned in the early ages, owing to ill-health, were those who kept up the average of mortality at the pension ages and after. Mr. Tinner had made some reference to the point and had suggested that it was desirable to have two separate mortality tables, one for valuing those who retired in the early years of life and the other for valuing what might be called the normal retirements at the pension age. In the case of a large fund which he had had before him some time ago, he found that the mortality of the survivors of those who were pensioned in early life was rather more than double that of those pensioned at 60 or over for the whole stretch of ages from 60 to 75. From that point the two curves began to approach one another, coalescing about age 85. In that particular case separate annuity tables were used for valuing the pensions of those retiring up to age 60 and those retiring after age 60. The difficulty of doing that in most cases would be the paucity of data; it required a very large fund indeed to provide sufficient material to get even one satisfactory table of mortality from the pensioners, and if two tables were required it was only one fund here and there that would be large enough for the purpose.

For the reason he had mentioned, he considered that a guarantee by an insurance company would seldom be a satisfactory arrangement for either side. Apart from other considerations, an insurance company could hardly ever give exactly the benefits that the pension fund would require, benefits on withdrawal and breakdown in health and so on. The matter was a very urgent one, because there were probably thousands of small firms throughout the country very anxious to start some such scheme who could not see their way to doing so, and could not obtain what they required from insurance companies, and it had occurred to him that there was an opening for a special company, which without competing in any way with the ordinary business of insurance companies might adapt its organisation and methods to catering for pension fund benefits. The details, of course, would be left to the firms themselves; the payment of benefits and collection of contributions would be made by them, and the accounts adjusted with the guaranteeing company at, say, quarterly intervals. The company would, of course, guarantee the benefits, but would reserve the right to introduce variations with regard to new entrants if it were found necessary. He threw that out as a suggestion for what it was worth, as he could not see that there was any other satisfactory way of dealing with the matter. He had been often approached on the subject by different employers and had never been able to give them any hope of obtaining for them what they required through an ordinary insurance company.

The authors had made some novel suggestions with regard to the question of salaries. Mr. Tinner had already dealt with that point; and, like Mr. Tinner, he was quite unable, with all respect to the authors, to see where the improvement lay in treating the future increases as the arithmetical first differences of the salary scale. He was not sure that he correctly understood the authors. He thought at first that the meaning of the paragraph in the paper was that the first differences of the salary scale were to be made proportionate to the actual salaries, but of course if that were so there would be no difference between that method and the one that had usually been followed by Mr. King and the late Mr. Manly and others in the past. Therefore he assumed the authors meant the actual first differences. It seemed to him that just as it was desirable to bring in the ascertained facts with regard to the past service and the present status of the member so the ascertained fact of the present salary was the best guide one could have to the probable future increases of salary, and without some further evidence of the advantage of the method proposed he personally preferred the method formerly used.

The authors had given a very ingenious solution of the difficulties they encountered in connection with the Union Castle scheme. The particular scheme that they approved depended on two assumptions, first, that the retirements took place at or around a fixed age, and, secondly, that there was a clear distinction between the retirements from ill-health and those from old age. In the course of a

fairly extensive experience it had never been his good fortune to find that those assumptions fitted the facts. He had always found that, where there was an optional age of retirement and a compulsory one, the retirements commenced before the optional age and continued after the compulsory—that was to say, they did not all occur at the compulsory age, but there was a regular progressive rate of retirement which continued after the so-called compulsory age. Nor had he ever found a clear dividing line between the retirements from ill-health and those from old-age; they insensibly merged into one another somewhere about age 60, and it was very difficult to distinguish between those who retired through a genuine breakdown in health and those who retired either from laziness or from the employers preferring their room to their company. In that connection he should like to refer again to the question of the influence of the mortality of the ill-health pensioners on the pension mortality after the pension age was reached. The authors, in the particular case they were dealing with, were able to set aside the question of ill-health retirements and treat such retirements practically as surrenders, giving them reserve values on the assumption that they were normal lives. That method of treatment was novel to him, and he should be very interested if the authors would state whether that method resulted in giving pensioners more or less than would be secured if they took pension according to scale. He also wished to ask the authors whether they had considered in that connection the effect, on the mortality of those who survived to the pension age, of eliminating in their calculations the breakdowns in health.

Mr. VYVYAN MARR said that there was a very general belief among employers that adequate pensions could be obtained by a contribution averaging from 5 to 6 per-cent of the salaries, and that had to some extent been fostered by the Railway Pension Schemes. Those schemes were started at a time when the conditions of retirement were very different from what they were now. While a contribution of 5 or 6 per-cent of the salary would give an adequate pension for a young entrant it did not enable an actuary to draw up a scheme giving satisfactory pensions to the existing staff. In that connection he had found the paper of Mr. Manly and Mr. Ackland on the Metropolitan schemes extremely useful (*J.I.A.*, vol. xlv, p. 327), and had referred to their tables in many instances. There was one aspect of the cost to the employer which he thought had not received the attention it deserved. In an established scheme the employer's contribution should be taken into account in fixing his assessments for local taxation, just in the same way that life assurance premiums were taken into account in assessing incomes for income tax. That idea was put before him by the Managing Director of a large concern which he was advising. The company in question was employing local labour and had created an industry in a hitherto barren district. It was heavily assessed for local rates and there was no doubt that the establishment of a pension scheme by them for their employees would in time have a consider-

able effect on the cost of poor law relief in the district. That was a point which he thought should be borne in mind in connection with local taxation.

He would like to know the authors' opinion as to how far an adverse selection influenced mortality rates when existing employees were given the option of joining a scheme. He had in view more especially the establishment of a widows' fund, rather than a pension scheme. If the option were given to those on the active list only he did not think the selection would amount to much in an efficiently-managed business. He had before him at the present time the case of a widows' fund established 17 years ago, where an option was given to the bulk of the then existing staff to join the scheme without medical examination. 1,010 members joined, and the membership, which was now compulsory on new entrants, at present amounted to 1,900. They were recruited from selected lives about 20 years of age at entry. The mortality experience of the original entrants showed 14,900 years of life exposed; there were 86 deaths; and the expected deaths according to the O^M Tables for ages three years younger than the true ages were 133; showing a ratio of actual to expected of less than 65 per-cent. Taking the experience of the new entrants on the same basis the ratio of actual deaths to expected was just about 48 per cent. Seeing that the original entrants showed less than 65 per-cent of modified O^M mortality rates he did not think that selection influenced that fund very much.

Mr. Tinner had referred to the question of what proportion of an initial grant should be reserved for future entrants. That was a point which arose in almost all the widows' funds connected with the various Scottish churches and law societies in Scotland. Owing to endowments, favourable experience, and careful management, all those funds were able to grant annuities very much in excess of the amounts provided by members' contributions and interest thereon. Mr. Tinner would find, if he referred to the reports, that the question was usually dealt with by a method similar to the one which he adopted.

With reference to Mr. Thomas's suggestion that a new insurance company might be formed to transact pension fund business, he thought that existing offices were well able to deal with the matter. It was not a question of what people wanted. What people wanted was not always the best for them. Existing offices could draw up, and had drawn up, suitable schemes of insurance to meet cases of small concerns in which it was impracticable to form a separate pension scheme confined to their members.

Mr. JAMES BACON said that on several occasions Mr. Burn had put in a plea for students, and he himself wished to raise a similar plea that evening. The authors said that each fund had characteristics peculiar to itself, and as each fund was a law to itself, it was perhaps due to the students that the particular characteristics of any fund brought before their notice, which decided the methods adopted in the valuation or in obtaining the experience, should be clearly stated. For example, the authors gave a method of dealing

with the rate of withdrawal by duration alone, and the method was said to be the best available for adoption in the particular case, but he thought the student who read that without knowing the peculiarities of the experience might proceed to value his first fund on similar lines and go very far wrong. Before expressing an opinion on the suitability of such a procedure they would want to know the range of the entry ages. Presumably in the particular case that had been mentioned the entry ages must have covered only a very short period of life, say two or three years, and the average age of 26 could be assumed for all purposes. Otherwise the student would probably be considerably puzzled as to how cases coming in below age 26 were to be treated.

With regard to the question of light mortality, which had been touched upon by one or two of the speakers, he had recently taken out the mortality experience of a pension fund for clergymen of a certain religious denomination, and he found that the members of that fund appeared to have a mortality experience represented by the O^(am) Table rated down three years. That was after excluding the experience of new entrants during the last ten years, and the inclusion of such selected new entrants might account to a certain extent for the very light mortality referred to by previous speakers. It was obvious that to give full effect to an apparent experience of that kind, unless it were clearly not due to special circumstances, might lead to very erroneous results and a very serious over-estimate of the liability for pensions, and for that reason he was glad the authors had touched on the necessity of seeing that some allowance was made for withdrawals. In his particular case there was a double selection going on, and that would probably apply to all such funds. Some sort of physical selection was applied to applicants for appointments, even if there were no medical examination, and this continued to some extent throughout the whole of the time they were in business. The man who was in bad health was likely to leave, either with or without pension, or to fall below the general standard of efficiency and be compelled to leave. He either went out as an early retirement, or went out as a man whose services were not required. This selection would probably be fairly severe in a trying occupation like that of marine engineer. Because of this continuing selection throughout the whole of an employee's service he agreed with Mr. Marr that when a fund of this kind was started there would, except in special cases, probably be no very serious effect as a result of allowing all the employees to come in. He did not know that everyone would agree with regard to making the full allowance for withdrawals—some would like to keep a little margin of safety—but some allowance at any rate should be made to bring the total combined mortality and withdrawal (or early retirement) rate, up to at least what would be expected to be the normal mortality rate of members of that particular fund. In the case of a pure pension fund it was obvious that, eliminating the effect of cases going out early through ill-health and assuming that everybody was subject only to the very light mortality exhibited

by members remaining in the experience, there would be a very considerable over-estimate of the cost of pensions. Whilst on the question of the rate of withdrawal he could not at the moment see the advantage of substituting an apparently arbitrary reduction in the rate for the usual method, when Select Tables were not used, of eliminating entrants during the last ten years or more in obtaining the experience. This produced the same type of adjustment as the authors had in mind, but did not leave it to the skill or whim of the operator.

With regard to the salary scale, also, he wanted to put in a plea for the student. It was said that a certain method of using the salary scale was more correct than other methods that had been put forward, but nothing was said as to how the salary scale was got out. They were not told whether the scale was based on average salaries or built up from a consideration of past increases of salary. Information was wanted as to whether the scale had been constructed in one of those ways, or had been arrived at after consultation with officials as to the probable future progress of increase of salary. The method of construction adopted must necessarily have very great weight in helping them to determine whether the salary scale finally arrived at could be correctly used in the way suggested by the authors. He should also be glad if they, in their reply, would say exactly what standard of correctness they applied.

Further on in the paper the authors gave a formula for obtaining rates of withdrawal, mortality and retirement. For the exposed to risk the method of obtaining the ages at entry and exit was such, that a case actually going out at any age between age $x - 1$ and $x + 1$ was assumed to go out at age x . He did not think that was very material when one was dealing with the rates of withdrawal or mortality, but the financial effects might be very great in respect of rates of retirement, particularly when these were very heavy at a special point, such as an optional retiring age. If one could get the year of birth one could probably get the date of birth, at any rate approximately, and so obtain rates of retirement allocated to the proper year of age. But, apart from that, the rates the authors got out, so far as he could see, were not what actuaries commonly termed rates of mortality or retirement at all. He believed that the Institute had under consideration a notation for pension funds, and this question might perhaps be considered. There was the ordinary rate of mortality obtained by giving the deaths a full year's exposure and the withdrawals and retirements only half-a-year, and there was what he had been accustomed to call the ratio of cases going out to cases entering on the year of age, in which all cases going out had a full year's exposure. If one had the ratio of cases going out to cases commencing the year, one could obtain the combined rate of exit by adding the rates of retirement, withdrawal and mortality; but if one had a true rate of mortality and a true rate of withdrawal one had to use Dr. Sprague's formula,

$$\frac{q_x + q_x'' - q_x q_x''}{1 - \frac{1}{4} q_x q_x''}$$

and a similar but more formidable expression where more than two rates of decrement were involved.

The authors gave an example towards the end of the paper which was liable to a little misunderstanding. They had mentioned how a particular society had been made solvent by the adoption of a scheme under Section 72 of the National Insurance Act, but it would appear from the facts given that the scheme had not been made solvent in that way at all, but by adopting a very drastic reduction of benefits after age 70, which could not be brought into the scheme under Section 72 of the National Insurance Act. Perhaps they might be inclined on consideration to modify the statement that that was a good illustration of what could be accomplished for an insolvent society by means of Section 72 of the Act.

Mr. H. H. AUSTIN expressed satisfaction with the emphatic way in which the authors endorsed the basing of pensions on total or average rather than on final salaries. It was probable that the fixing of pensions on the basis of final salaries had arisen from private practice. If an employer wished to confer a pension on an employee the natural standard of measurement which would suggest itself to him would be the salary the employee was receiving. If the contribution were entirely voluntary on the part of the employer no exception could be taken to such a basis, but when dealing with an organised contributory pension fund the case was entirely altered, even if the contributions were partly paid by the employee and partly by the employer; and especially so when the employer's contributions were expressed as a percentage of the employee's salary. It has been argued that the employer could give his contributions to whom and in what proportion he liked, but that was an argument which was often used to make the best of a rather poor case. If in an organised pension fund an employer simply gave a lump sum for the general benefit of the fund perhaps there was some weight in the argument, but if he stated that he would increase the percentage contribution of the employee by a fixed amount, as was very common, there was an implied promise to the employee of a definite benefit—a benefit which was certainly not realised in many cases when the scheme was based on final salaries, and was not even realised in many cases when it was based on average salaries. He thought a better plan would be to take the accumulated salary (with interest) as a basis for measuring the rate of pension which could be given. Even with the average salary method, when individuals received large increments of salary during the latter part of service, they took considerably more than the equivalent of the contributions. If one considered the case of a fund in which on the death, withdrawal or retirement of an employee the whole of the contributions paid by him, or on his behalf, were returnable with compound interest, one had only to take account of the salary scale and the annuity value at the pension age; the rest was a simple matter of compound interest. In such a case the pension would be measured by the accumulated contributions, or, as the contributions were a percentage of the salary, by the accumulated salary. In the

ordinary type of fund where there was joint contribution, the employer's contribution was not an absolute gift to the employee, but was usually conditional on his serving in the company to the pension age. On that account there was a considerable gain to the fund from the contributions forfeited on death or withdrawal, but unless the employer distinctly and emphatically stated that this gain, arising mainly from his own contributions, should be applied for the benefit of the more fortunate or more able members of the staff who attained to the higher salaries, he thought the argument for basing the rate of pension on the accumulated rather than on the total salary was strengthened.

It might be asked whether the suggested basis would make sufficient difference to warrant the extra work involved in the calculations. If one compared pensions based on total salary with those based on accumulated salary, taking the latter as the standard, it could be readily shown that a variation in defect or in excess of upwards of 6 per-cent was quite possible. The significant effect of that was not, perhaps, so apparent from the percentage as from the fact that usually it would be the employee with the higher rate of salary, and consequently the higher rate of pension, who would get the excess. Taking for example, the case of a man retiring with a pension of £420, when according to the standard his true pension should be £400, that might result in four other men having a pension of £100 when they should have £105—that was, an increase of 5 per-cent in one case might mean a reduction of $4\frac{3}{4}$ per-cent approximately in four others. The comparison with pensions based on final salaries was much more striking and one could easily have variations of 20, 25 or even 30 per-cent from the suggested standard. In such cases it should be remembered that the employee, so far as his own contributions were concerned, paid for his particular share of the pension, so that any defect in pension should be measured by the employer's contribution. If the defect from the standard were upwards of 25 per-cent in a fund with equal joint contributions, the case arose that the authors mentioned, where the employee not only did not benefit from his employer's contribution but actually did not receive as much as he paid for himself.

Mr. C. R. V. COUTTS said that to those who had not had much to do with the practical working of pension funds not the least interesting section of the paper was that dealing with group insurance. He was glad that the authors condemned this new form of insurance, though he wished they had condemned it in still more forcible language. A policy which went automatically out of action just when the benefit of assurance was most necessary, namely, when the assured retired owing to ill-health, was nothing more than a trap to the unwary. He was glad to hear there were legal difficulties in regard to starting that form of assurance in Great Britain, and he hoped those difficulties would be increased. After all, it was quite possible to supplement pension schemes by staff insurance schemes, either compulsory or optional, and it was now done to a large extent on sound lines. The ordinary form of endowment insurance or whole-

life policy was either offered to the member of the staff when he joined or he was obliged to take it up, and if the scheme was compulsory there was no sort of selection against the office. Such policies surely formed a much better basis than the so-called group assurance, which was very little more than a mere accident insurance.

Mr. T. G. ACKLAND thought it would be generally agreed that the construction and valuation of pension funds were among the most difficult problems with which the actuary had to deal, and that there was no class of work in which his attempts to look forward into the future were more likely to be frustrated by the actual facts as brought out five or ten years later. In his remarks on the paper he would refer to two or three features which appeared to him to be rather outstanding. The authors discussed in an interesting manner the question of final salaries (by which he understood them to mean the average salaries for the last five or seven years) and average salaries over the entire period of service. It would be remembered that, in the Report of the Departmental Committee on Railway Superannuation Funds (*J. I. A.*, vol. xlv, p. 27), this matter was very fully discussed, and the Committee came to the conclusion that, on the whole, the pension based upon the average salaries over the entire period of service most fully secured the purposes in view. The authors had referred to the possibility of inequity as between higher salaried members and lower salaried members; but it had been pointed out in that report, and had been shown by Mr. Manly, that that was very largely, if not fully, compensated, at least where the pension was based upon total average salary, by the fact that the higher salaried official tended to hold on longer, and therefore paid his contribution for a longer period than the average. The higher salaried man found that a pension based upon total average salary was so inadequate, and would mean so considerable a change in his mode of living, that he postponed as long as possible the evil day when he must retire. Moreover, the higher salaried man was a valuable official, whom the employer desired to retain as long as health permitted. Those two considerations both operated in the direction of maintaining equity between the higher and lower salaried employees. He had found in a particular case that Mr. Manly's results were entirely confirmed—that, while the average retiring age for the general body, excluding the higher salaries, was 63·61, the average age for the higher salaried officials was 65·79, or more than two years later, and that that in effect restored the balance of equity. The ingenious suggestion of the authors that the pension should be a combination of final and average salaries was also, he thought, referred to by the Departmental Committee, as in the case of the Great Western Railway, which was the one he supposed the authors had under consideration.

With regard to the construction of salary-scales, it was a very singular thing that no writer on the subject appeared to give a clear statement as to the method he had employed. There was, he thought, no such statement on record in the Journal, and he was afraid the authors had not added to their knowledge in that respect.

He would very much like to know what process they had adopted in getting out the salary scale, because that was perhaps the most difficult thing of all in a difficult subject. He did not know whether they had taken average salaries as at the date of valuation or some other date, or ratios of increase of the same individual employees. In his opinion the latter basis was the more correct.

That led him to the question of the employment of the salary scale in the valuation, and he had been much surprised, in common with other speakers, by the authors' suggestion, which he understood to be that, while the actual salary was to be brought into account for valuing that portion of the pension arising in the past, the future increment of salary was to be strictly based on the salary scale. It appeared to him that the question as to the correctness of this method—and with Mr. Bacon he would ask what was the criterion of correctness?—had to be considered in connection with the method of constructing the salary scale. If the salary scale had been got out by what he thought was the less preferable method, *i.e.*, by taking average salaries at the date of valuation, the salary scale and the valuation data would practically coincide, subject only to the qualification that in the salary scale the data had been graduated; and, in that case, he supposed the authors' suggested plan might be useful and consistent, but he could not conceive of any other circumstances in which it would be more correct than the usual method, and he would rather question whether it could be of general application. It would be interesting to know what was really in the minds of the authors, and in particular upon what basis they concluded that one method was more correct than the other. It was unnecessary for him to stop to illustrate the anomalies that might arise; they would be quite evident to every member present who had dealt with the matter in a practical way.

With regard to the formula for deducing the number exposed to risk, that was an ingenious and, so far as he could tell, a new formula, applicable in the case assumed by the authors where only the year of birth and the year of entry were known. It proceeded on the assumption, as Mr. Bacon had pointed out, that the different decrements passed out at the end of the year of experience, so that the exposed to risk were continued to the end of the year, and the separate decrements for deaths, withdrawals and retirements were divided by a common denominator. That was a convenient method, and useful in the practical deduction of the Service Table, because the decrements so constructed could be simply added together, exactly as if they were forces of mortality, withdrawal or retirement. He did not think it was, perhaps, sufficiently appreciated by students—it certainly was not understood by himself until lately—that the device of extending the exposure to the end of the year of risk provided against the overlapping of the three rates of decrement. It had been employed, as Mr. Bacon had pointed out, by Dr. Sprague in his useful formulas (*J. I. A.*, vol. xxi. pp. 416-7); and an alternative method had been suggested by Mr. Chatham (*T. F. A.*, vol. ii, pp. 338-9) of obtaining separate mortality tables, one in

respect of each class of decrement, and then deducing the product of the numbers living at each age. He had ascertained by direct computation that both methods gave practically the same result as that obtained by Mr. King and Mr. Manly, whose method had been followed by the authors.

With reference to the graduation of rates of decrement by the summation method, Mr. King certainly appeared to have advocated that method, but it seemed clear that he could not have intended it to be applied to rates of retirement, as his own table of retirements was not graduated by the summation method. When one considered the special characteristics of the retirement table the reason for the unsuitability of the summation method was plain. With regard to the withdrawals and deaths, the method was quite applicable, and he should personally prefer Mr. Spencer's 21-term formula, which he thought would compete also with the graphic method as regarded convenience and accuracy.

In their remarks on deficiencies arising in the valuation of pension funds, the authors very rightly said that these would usually fall upon the employers. Such deficiencies could be made good by the employers guaranteeing the whole pension, as was the case with some of the railway companies, or by their guaranteeing a higher rate of interest, in order that the solvency of the fund might be secured, or by their paying into the fund an annuity-certain equivalent to the estimated deficiency. Another plan which had come under his notice was interesting, and he was entitled to speak about it because the final solution of it was obtained by a private Act of Parliament, which was available for public reference. In a large Railway Fund a considerable deficiency had arisen, almost entirely owing to the superior vitality of the pensioners, and it was a grave problem as to how such deficiency was to be dealt with. After prolonged deliberation the railway companies agreed to pay to the Fund the pension of anyone who retired between 60 and a certain age which was styled, not very happily, the "solvent age" and also to pay his periodical contributions up to the solvent age, just as if he had remained in active service. That fully met the case, and was duly carried into effect. The so-called solvent age, actuarially calculated, was 68, and that meant that, with regard to anyone who retired between 60 and 68, the railway companies would pay the pensions until the attainment of age 68, and would also pay into the Fund the contributions which would have been paid if the man had remained on active service. It was not quite the same as taking up the pension at age 68, because the pension that was taken up at that age was really the pension secured at the earlier age of actual retirement. For instance, a man might retire at 63, and then at age 68 the Fund was charged, not with the pension at age 68 but with the continuing pension calculated as at age 63. For that reason he demurred to the term "solvent age" which seemed to imply that it was the age at which, if everybody retired, the Fund would be solvent, which was of course not the case.

With regard to the mortality of pensioners, the table given by

the authors was a very interesting one and showed low rates, as compared with those given by Mr. King and Mr. Manly. He thought it would be found that as time went on those rates of mortality would progressively diminish. In some of the earlier funds the mortality was so graduated by Mr. King and Mr. Manly as to fall into the English Life Table No. 3, an almost incredible proposition now-a-days, although no doubt it was perfectly reasonable at the time. The authors suggested what they called the Government Annuity Table 1883, but which he preferred to call the Government Annuity Table of 1875, published in 1883. That was rather an ancient experience now, and he had found, in funds recently dealt with, that the mortality of the pensioners approximated to the *O^{ann}* Table, or to the new Government experience up to 31st December 1903.

In conclusion, he was asked to say that a note had been handed to the President by an esteemed member who was unable to be present that evening but who wished to call attention to one or two points. The writer wished to refer particularly to Table B, giving the figures referring to the sickness in 1913, as compared with 1912, and suggested that it would be useful if the number of cases of sickness, actual and expected, were shown, so that it could be seen whether the excess sickness was due to an abnormal number of cases, or to an abnormal duration of claim, and if the authors were to contract the table into quinary age-groups, thus rendering it much more useful than in its present shape. He also suggested that, as "over-insurance" was clearly indicated in the figures given, actuaries should be careful not to accept present experiences as a basis for estimates of the future; he felt strongly that, whilst schemes might be drawn up and recommended under Section 72, new evils required new methods of administration, and every society ought to be advised to overhaul its methods of supervising claims, sick visiting, &c.

The PRESIDENT, in proposing a vote of thanks to the authors, thought it would be of interest to the members to know that a communication had been received from the Institut des Actuaire Français, which was not quite in the same happy position as the Institute in England. Their French colleagues had not been able to hold their annual or other meetings and had suspended all their proceedings. Their Honorary Secretary, M. Emile Fleury, was a prisoner. The communication mentioned certain Hon. Correspondents of the English Institute, and stated that MM. Lepreux and Hankar were at their official posts in Brussels, but that the whereabouts of M. Begault was unknown. It also mentioned, that an Austrian Hon. Member of the Institute, Herr Klang, was dead.

Mr. Ackland had referred to a letter from an esteemed member as to a point about which he (the President) felt very strongly, namely, the question of the increased cost of sickness where there was over-insurance. He could not but think that if over-insurance were not checked the existing valuation tables would not be very reliable, as the rate of sickness must inevitably be much heavier. All actuaries ought to bear in mind the importance of impressing on

societies the absolute necessity of preventing over-insurance. The Report of the Departmental Committee, recently published, dealt very seriously with the question. It pointed out, however, that possibly the increased sickness was to a certain extent due to the doctors being more lenient and to want of experience in administration, and it had also been given in evidence that it was not so much over-insurance that was in fault as that in the past there had been under-insurance. In his (the President's) opinion the evil of over-insurance was clearly shown in the two cases mentioned in the paper, *i.e.*, the one where reduced benefits had been accepted and no increase in the sickness rates was observable and the other where the sickness for the periods during which double benefits were being received was from 20 to 50 per-cent higher according to age. Evidence given before the Committee corroborated this, for it showed that in one society where the contributions and benefits had been compulsorily reduced the experience had been eminently favourable and in another a lighter rate of sickness on its private side had been experienced in 1913 than in 1912. It was a subject which ought to be very seriously brought before the nation and especially the working classes. He was speaking more with regard to Men's Societies, because the question of the rate of sickness in Women's Societies was a totally different one.

The vote of thanks was passed unanimously.

Mr. BURN, in replying, thanked the members for their vote, and said that although he had taken very full notes he would have to be brief in his reply, and deal more fully with the discussion in a written contribution to the *Journal*. With regard to Mr. Tinner's observations on the use of existing and future salaries for the purpose of valuing the contributions and future benefits, it might be well, especially as other speakers had referred to the subject, to give some explanation of what was intended. There was no doubt as to the desirability of using the present salaries for the estimate of both future contributions and future benefits because those salaries were known, and therefore in using them one was on firm ground. The uncertain question was that of the future increases of the salaries, and with regard to that the authors did not say that they disagreed with Mr. King's method or that his method was not so good as the one they had referred to in the paper; they merely wished to explain that in the cases with which they had had to deal they had found this particular method more suitable. He could quite believe that in some cases Mr. King's method might be more appropriate. Both methods were based on the existing salaries, but there was a difference with regard to the treatment of future increases of salaries. Mr. King adopted the increases shown in the salary scale as proportionate increases of the existing salaries. They, on the other hand, found that in the particular fund referred to in the paper better results were obtained by assuming the future increases to be the actual increases shown by the scale. As an example, take the case of a man whose actual salary was £150, but at whose age the average salary according to the scale was

£100; if the final salary in the salary scale were £250, the authors' valuation would come out as follows:—First, the existing salary of £150 would form the basis of one set of calculations, and, secondly, the future increases would correspond to the increases in the salary scale, *i.e.*, from £100 to £250; this would give a final salary of £300. Mr. King, on the other hand, would say that the present salary was £150, and there would be future increases of 150 per cent. of the present salary, and would thus arrive at a final salary of £375. One or other of those two methods might be more suitable, according to circumstances.

Mr. Tinner had suggested that a method of arranging the calculations might be devised, that would facilitate the preparation of estimates of cost of different kinds of schemes and obviate the necessity of repeating the greater proportion of the work with each modification required. He (Mr. Burn) had had a good deal of experience in those matters, but confessed that he knew of no such method for overcoming the various difficulties which arose. Nothing, however, would please him so much as to discover one.

EXTRACTS FROM SUPPLEMENTARY WRITTEN COMMUNICATIONS.

Mr. C. S. KELHAM writes, with reference to the authors' method of dealing with the valuation of future salaries:—

Mr. King makes the increases of salary proportional to those of the salary scale—in other words, if a group is in receipt of higher salaries than the average, the future increases of that group will be higher than the average. Whether this is so or not must depend on the causes which give rise to the variations from the scale salaries. If there is reason to suppose that those in receipt of higher salaries will in the future obtain higher increases, then the proportional method would probably give satisfactory results.

Is it not possible, however, for a group to show salaries higher than the scale for the reason that it contains a large number of men who entered at young ages and consequently have received a large number of average increases? Or, conversely, for a group to show lower salaries than scale, on account of some of the members having entered recently at normal commencing salaries, and so having received but few normal increases? If this be so, there does not appear to be any reason for assuming that their future increases will not be according to scale, particularly if, as in the case dealt with by the authors, highly paid officials and their successors have been excluded and treated separately. . .

. . . The authors' method requires a little extra work, but I think there is considerable advantage in having the results brought out in two parts, *i.e.*, the value of the *known* present salaries and the value of the *problematical* future increases.

Mr. C. W. KENCHINGTON, after referring to the great value, from the point of view of students, of a concrete example of Pension Fund work such as that afforded by the authors' paper, continues:—

I was much interested in the suggestion for minimizing the inequity as between the higher and lower salaried officials of the constant ratio of contributions to salary, when the pension is a fixed percentage of the final salary, by substituting the average salary throughout service as the basis for determining the pensions resulting from the members' own contributions, while retaining the final salary as the basis for that part of the pension which accrues from the employer's contribution. In this connection it occurs to me that in order to give full effect to this equitable distribution it might be necessary to deal separately with the two sections, for purposes of valuation, in regard to rates of withdrawal, as it would probably be found that the higher paid officials experienced quite a different rate from that applicable to the majority of the staff. Although, in fact, the higher and lower salaried officials are involved in both sections of the fund in the same manner, yet the incidence of the withdrawals as affecting the finance may not be identical. Thus, supposing that the rate of withdrawal is almost negligible among the highest salaried officials, while a normal rate is experienced by those earning lower salaries, the saving effected from the withdrawals will have much less effect in increasing the pensions than if the withdrawal rate was the same in both classes. It would seem to be indicated, therefore, that in investigating, in such circumstances, the rate of withdrawal the rate should be deduced on the basis of salary rather than on the basis of the number of members. The idea is analogous to the investigation of the rate of mortality on the basis of amounts assured, as contrasted with the basis of lives. The other suggestion which is put forward by the authors for dealing with the pensions of officials in receipt of salaries of large amount, namely, that salary in excess of £1,000 should not count for either pension or contribution, appears to me to be eminently fair to the general body of the members of the fund. It is probably true to say that officials in receipt of the largest salaries are generally in a position to obtain from their employers, out of funds entirely distinct from those that are ear-marked for pensions, the guarantee of such addition to the amount to which they are entitled from the pension fund, as would make the total sum adequate in the particular circumstances.

Dealing with the general question of contributions to Pension Funds the authors state "as the employer's contribution is in effect a portion of the salary, it is desirable that it should bear a *constant* ratio thereto." It does not seem to me that the argument here used is conclusive, and bearing in mind that the contributions made in early life purchase more pension than contributions of equal amount in later life, I think there is a good deal to be said for basing the contribution on a sliding scale. To suggest a fixed annual subsidy per member would probably be going too far in the opposite direction, but the employer's contribution might be fixed at say. £5 a year for all salaries not exceeding £100, £9 a year for all salaries exceeding £100 and not exceeding £200, £12 a year for all salaries exceeding £200 and not exceeding £300, and so on.

Mr. BURN writes :—

With regard to Mr. Tinner's remarks as to the members sharing in the management, there is much truth in what he says as to whether it is well for the members in their own interests to do so. The question is, of course, one for the members of each fund to decide, having in mind the particular circumstances of the service to which they belong. In this connection I may mention that there has recently been considerable agitation amongst the members of one of the large Railway Pension Funds for a voice in the management, and I think, generally speaking, there is a strong feeling that where contributions are paid the members should have some control over these contributions. It is, indeed, analogous to the "no taxation without representation" principle which governs our parliamentary practice.

The question of the appropriation of a fund between existing members and future members is always a difficult one, but it seems to us to be better to obtain the consent of the promoters of the fund to apportion certain definite amounts to the two sections. This was done in the Union Castle scheme.

Mr. Tinner's remarks on the subject of negative values hardly represent what we have said, and I think he has only confused the issue by introducing the negative value period. On pp. 219 and 220, we say "If the future withdrawals are in excess of the expected, then some portion of the negative values will be lost, and if the withdrawals fall short of the expected there will be a corresponding gain." When we wrote this sentence we were both satisfied that it could not be misunderstood, but apparently our hopes have not been fulfilled. Even at the risk of again failing to speak clearly upon this complicated matter I will risk a further explanation :—

In any case where a negative reserve is shown a withdrawal will mean the removal of such negative value and consequently the loss of an apparent asset. An excess of withdrawals over the expected will in all cases of negative values result in a loss to the fund, and *vice versa*. If, therefore, all negative values are retained and the withdrawal rate is underestimated, then at those years when such negative values are shown the result will be an underestimate of the liabilities, whereas if the withdrawal rate is overestimated at such ages the result will be an overestimate of the liabilities. It follows from this that if at such ages the withdrawal rate is overestimated, then it is unnecessary to write off the whole of the negative values. The statement of course applies to the existing members who are under observation at the valuation, and the basis would obviously require modification when used for scales of contribution.

Mr. Tinner referred to the mingling of the data for ill-health and ordinary pensioners in fixing the value of the pensioners' annuities. Theoretically, the objection he raises applies to all aggregate tables, and the paucity of data is a practical bar to the construction of separate mortality tables.

It has been our experience that those who are pensioned a few years before the normal age do not differ materially from normal

pensioners, while the others are proportionately few in number. The use of the aggregate rate would seem, therefore, to be safe from the mortality experience point of view provided the flow of new pensioners continues as in the past. The use of the aggregate table would also be safe from the financial point of view if each ill-health and each normal pension were approximately equal in amount. As, however, the former are smaller in amount a small margin is necessary, and I think every prudent actuary would keep such a margin.

Mr. Thomas also refers to the question of ill-health pensioners and instances an experience where the mortality was double that of the normal pensioners from ages 60 to 75. He does not state whether these were genuine ill-health pensioners or those who later in his speech he described as retiring "partly from laziness and partly from the employer's preferring to have their room rather than their company." Presumably the latter class was not included.

With regard to the method adopted in the Union Castle scheme for ill-health pensions, the annuities produced by crediting the retiring member with his reserve are less than the scale pensions, but steadily approximate thereto as the age at retirement approaches the normal retiring age. The method is useful where an ill-health pension is desired, but no data are available on which to found rates.

Mr. Marr's suggestion that an employer's contributions towards pensions should be taken into account for purposes of assessment for local taxation, is most interesting, and I should be glad to hear further on the matter. In one case which came under my observation such contributions by the members were considered as premiums for the purchase of deferred annuities, and rebate of Income Tax was allowed accordingly, whilst as regards the Company's contributions, since these were provided for under a trust deed, they were allowed as expenses for the purposes of the income tax assessment on profits.

Mr. Bacon certainly hit the nail on the head when he stated that each pension fund is a law to itself, and that any student who would proceed to value his first fund by getting out a rate of withdrawal simply having regard to duration was likely to go very far wrong. I am bound to say that anyone who values a pension fund without practical knowledge and the exercise of a considerable amount of discretion will probably fail very badly.

Our reference to the Great Western Railway Provident Fund taking advantage of Section 72 of the National Insurance Act and thereby placing itself on a sound financial footing, taken by itself, may be misread. The scheme adopted, however, shows that the elimination of one-half the deficiency was due entirely to the Act, and we say in concluding the description of the scheme that the case is "a good illustration of what can be accomplished by the aid of Section 72" (p. 227). Mr. Austin's suggestion that the pensions should be based on the total salaries accumulated at interest is interesting and new to me. Its effect is evidently in the direction of equalizing the pensions.

Mr. Ackland pointed out that the high salaried members generally served for a longer period, and thus by paying contributions until a later age to a large extent made up for the more liberal treatment received as regards pensions. Whilst quite agreeing with what Mr. Ackland said, I believe that there is a growing tendency to replace the older and higher paid officials at earlier ages and not to allow them to remain in harness after they have ceased "to pull their full weight."

Probably no one knows better than Mr. Ackland why it is that authors do not give explanations as to how they formed their salary scales. I personally have only met with one man who was quite willing to give such an explanation and that was before he had valued his first Pension Fund, and therefore before he had had any experience of the practical difficulties.

Two Notes on Questions of Office-Practice. By RALPH TODHUNTER, M.A., F.I.A., Actuary and Secretary of the University Life Assurance Society.

[Submitted to the Institute, 29 March 1915.]

I.—LIMITED PAYMENT POLICIES.

IN this note I submit for discussion two questions in connection with limited-payment policies: (1) by what method should the office premiums be calculated, and (2) how should the policies be valued. Neither of these questions appears to have been brought—except incidentally—before the Institute; both are of some practical importance in view of the large amount of limited-payment business transacted in the United Kingdom; and in regard to both there seem to be greater differences in practice than would be justified by theory. The following remarks refer more particularly to participating policies of the kind issued by the majority of British companies—that is to say, policies entitled throughout their currency to the same reversionary bonuses as similar ordinary whole-life policies—but similar considerations would apply to non-participating policies.

The only recent reference in the *Journal* to the method of calculating office premiums for limited-payment policies appears to be one by Mr. Rietschel in his paper on the "Analysis and Apportionment of the Expenses of Management of a Life Office with a view to ascertaining the Office Premium Loadings" (*J.I.A.*, vol. xlv, p. 415), in which he gave—without calling forth any comment in the subsequent discussion—a without-profit premium formula based on the commutation

of the whole-life premium by the O[M] $3\frac{1}{2}$ per-cent table, subject to an adjustment, in the case of a single premium, for the saving (if any) on commission. The subject has been more fully dealt with by Mr. Moir in his well-known papers on the Calculation of Office Premiums. In the original paper (*Trans.A.S.E.* vol. iv, p. 255) the usual commutation-formula is given with a brief suggestion that the annuity-values (more particularly for single premiums) might be taken at " $3\frac{3}{4}$ per-cent, or even 4 per-cent, "should the Office be able to realize this latter rate, because the "expenses in collection of premiums, &c., are likely to be "lighter." In the revised version (*T.F.A.*, vol. ii, p. 207), prepared after publication of the British Offices' Mortality Experience, Mr. Moir emphasizes this suggestion by adding, with more particular but apparently not exclusive reference to with-profit premiums, that "the highest net rate of interest that the company can expect to realize should be adopted." I shall refer presently to the reasons which Mr. Moir gives in support of this view, but it will be convenient at this stage to compare the numerical values of the commutation-factors on certain bases, at a specimen age, with the actual ratios that exist in practice between limited-payment and ordinary whole-life premiums. For this purpose I have taken the published rates of 14 companies—selected merely as representative companies giving the same compound reversionary bonuses to ordinary whole-life and limited-payment policies—and the resulting comparison is shown in the following table :

t	VALUES OF $a_{30}/a_{30:t}$				VALUES OF tP'_{30}/P'_{30}		
	O[M] $3\frac{1}{2}\%$	O[M] 4%	H[M] 4%	H[M] 4%	Highest	Lowest	Average
1*	19.79	18.47	17.98	18.13	19.25	17.53	18.35
10	2.35	2.24	2.20	2.22	2.40	2.17	2.25
20	1.43	1.39	1.37	1.38	1.48	1.36	1.39
30*	1.15	1.13	1.13	1.13	1.15	1.11	1.13

* For these values of t the figures relate to those companies only that publish single and 30-payment premiums.

The differences between the several factors, as exhibited in this table, may not appear at first sight very great ; but they produce (except for $t=30$) substantial differences in rates of premium, when multiplied by a whole-life with-profit premium of about £2. 10s. per-cent, and would give rise to appreciable

differences in the rate of bonus. It will be seen that the average ratios are very close to the $O^{[M]}$ 4 per-cent commutation factors. Only one, however, of the 14 companies appears to employ this basis—at any rate without adjustment—while in several cases the rates seem to be still based on the $H^{[M]}$ or H^M 4 per-cent table.

It is perhaps hardly necessary to state that the variations found in the values of ${}_tP'_{30}/P'_{30}$ in the tables of the selected companies are not merely the result of a practical application of Mr. Moir's principle as to commuting at the highest net rate that the company can expect to realize. There is indeed, no sort of correspondence between the ratios for individual companies and the rates of interest earned; the lowest rates of interest are in some cases associated with the lowest ratios, and some of the companies that commute at 4 per-cent have not for many years—until, perhaps, quite recently—earned within several shillings of that rate. Another reason must be found for the low values of both the lowest and the average ratios, and that reason no doubt is the presumed superiority of limited-payment mortality to ordinary whole-life mortality. This reason obviously weighed with Mr. Moir (to whose discussion of the subject I now revert) although it is not expressly mentioned in his original paper, for in the revised version, after justifying the commutation of the whole-life premiums at the highest realizable rate on the grounds that expenses are lighter and that the possibility of a decrease in the rate earned is compensated by the chance of an increase, he proceeds:—"Superior mortality "might be used for limited-payment premiums in conjunction "with a moderate estimate of the interest rate. It is probably "better, however, to use a higher rate of interest than might "otherwise be considered right, on the ground that this course "is justified by the light mortality and saving in expense. "Moreover, this course would give the greater advantage to those "policies on which premiums run for short periods. This is "probably as it should be because the mortality in all likelihood "is more favourable on ten-payment and fifteen-payment "policies than on twenty-five-payment and thirty-payment "contracts. . . ." It is, however, open to question whether the supposed superiority of limited-payment mortality ought to be taken into account in calculating limited-payment rates, and in any case it seems undesirable to rely on several different reasons for adopting a given course without being

clear as to the precise amount of justification which each contributes. If none of the three reasons—the saving of expense, the possibility of investing the higher limited-payment premiums advantageously, the superiority of limited-payment mortality—is in itself sufficient, it does not necessarily follow that together they constitute a sufficient reason.

The saving of expense may, I think, be dismissed as negligible. Even in the case of a single premium the commission would, as a rule, be at least equivalent to that on an ordinary whole-life policy if allowance be made for the probability of the latter being terminated by lapse or surrender, and the saving in clerical work and postages on account of there being no renewal premiums to be collected would be a very small item.

As regards interest—considered solely as such and not as partly a method of giving effect to a difference in the rates of mortality—it will be generally agreed that if the limited premiums are obtained from the whole-life premiums by commutation, the rate of interest employed should be a future-experience rate. But this is not apparently what Mr. Moir means by “the highest net rate of interest the company can expect to realize”; it is rather the $3\frac{1}{2}$ per-cent rate which he himself employs for calculating whole-life premiums. For such purposes as the commutation of premiums, when the factor of competition does not enter, life offices invariably employ a rate materially lower than the current rate on investments; and rightly so, because it is certain that the rate of accumulation of capital must in the long run decline. The result of accumulating a pound for 1,000 years at 4 per-cent compound interest has often been quoted in popular lectures as an illustration of the power of compound interest, but it is more instructive financially as a reminder that a pound cannot be accumulated for any very long period at a uniform rate of compound interest. The retardation of the growth of capital is inevitable, whether it be brought about by an actual decline in the rate of interest such as was experienced in the latter part of last century, or by periodical destructions of capital such as that which is taking place at the present time, and life offices properly give effect to it by adopting, in calculations involving the accumulation of receipts during a long period, a lower rate of interest than that immediately obtainable. Precisely how much lower that rate should be is no doubt a matter upon which opinions may reasonably differ, but an assumption that the net rate will fall from 4 to 3 per-cent

in the course of 40 years would not, I think, err on the side of over-caution. What the rate will be subsequently is not of much importance for present purposes, and for simplicity it may be taken as remaining constant at 3 per-cent. If it be further assumed, for convenience in calculation that $1+i$ decreases during the 40 years in geometric progression, it will be found that the commutation-ratios by the $O^{[M]}$ table at age 30 would be as follows :

$O^{[M]}$ Values of $a_{[x]}/a_{[x]t}$.

Rate of Interest decreasing from 4 to 3 per-cent.

$t=1$	$t=10$	$t=20$	$t=30$
19.22	2.33	1.43	1.16

These ratios are about equal to the $O^{[M]}$ $3\frac{3}{4}$ per-cent—19.11—for $t=1$, and to the $O^{[M]}$ $3\frac{1}{2}$ per-cent for the remaining values of t .

The question of mortality presents more difficulty. If justification for the employment of low commutation-factors be sought in the British Offices' Experience it can no doubt be found. The superiority of the limited-payment mortality in that experience over the $O^{[M]}$ is well-known, and it can be measured approximately by reference to the limited-payment rates given by Mr. Chatham in his paper on "Premiums deduced from the Mortality (1864-1893) of the British Life Offices" (*T.F.A.*, vol. i, p. 109). It would not be correct for present purposes to take the ratios of Mr. Chatham's limited-payment premiums to the whole-life premium, because it may be supposed—and can be shown to be the case—that a superior class of lives such as that observed in the 1863-1893 limited-payment experience are, if assured by with-profit policies on the compound reversionary bonus plan, to some extent compensated for their superiority by increased bonuses; that is to say, the ratios of the premiums for compound increasing assurances will be greater than those of ${}_tP_x$ to P_x . Since, however, an average office premium at age 30 for a whole-life with-profit assurance can be represented by $A_{[30]}^{O^{[M]}2\frac{1}{2}}/a_{[30]}^{O^{[M]}3\frac{3}{4}}$ —that is, by the $3\frac{3}{4}$ per-cent net premium for a whole-life assurance with an annual compound reversionary bonus of nearly 30s. per-cent—the requisite ratios will be given by comparing with the value of

this expression the values of $A_{[30]}^{2\frac{1}{4}\%}/a_{[30]}^{3\frac{3}{4}\%}$ according to the limited-payment experience. The values of $A_{[30]}^{2\frac{1}{4}\%}/a_{[30]}^{3\frac{3}{4}\%}$ may be obtained with sufficient accuracy from Mr. Chatham's $3\frac{1}{2}$ per-cent rates by assuming that the limited-payment and $O^{[M]}$ values of $a_{30 \overline{5}|}$, $a_{30 \overline{10}|} - a_{30 \overline{5}|}$, $a_{30 \overline{20}|} - a_{30 \overline{10}|}$, and $a_{30} - a_{30 \overline{20}|}$, are in the same proportions at $2\frac{1}{4}$ and $3\frac{3}{4}$ per-cent as at $3\frac{1}{2}$ per-cent, and the resulting ratios are as follows:

$t = 1$	$t = 10$	$t = 20$	$t = 30$
18.29	2.18	1.33	1.07

These ratios represent practically the effect of using the limited-payment 1863-1893 mortality with an experience rate of interest, and might therefore be regarded as a suitable basis for obtaining the limited premiums from the whole-life (subject perhaps to a precautionary reduction of a quarter per-cent in the rate of interest) if the superior limited-payment mortality may be properly taken into account, and if that mortality may be supposed to be the same for all values of t . With regard to the second condition, Mr. Moir suggests that the superiority of the limited-payment mortality will be more correctly represented by using normal mortality with a higher rate of interest—the general effect of which is to give lower ratios for the shorter terms and higher ratios for the longer—on the ground that the 1863-1893 experience gives an average mortality for all values of t and may be supposed to show too low a mortality for the longer terms and too high a mortality for the shorter terms. This view has, I believe, been very generally accepted, but it seems doubtful whether it would be in accordance with the facts. Whatever may be the validity of the theory that mortality decreases as the premium increases, it is surely an extreme application of it to suppose that the mortality of whole-life policyholders progressively improves as t decreases from $\omega - x$ to 1, and that in the single-premium policyholder we arrive ultimately at the perfect life. Single-premium policies, and to some extent 5- and 10-payment policies, are taken out more as investments than as assurances, and while it is true that from an assurance point of view the better the life the greater will be the advantage of limiting the premiums it is equally true that from an investment point of view the worse the life the better

will the investment be. Self-selection is not of course the only factor conducive to favourable mortality, but so far as its influence extends it may quite possibly be most strongly operative in the cases in which the assurance element predominates, that is in the 25- and 30-payment cases.

There are, however, objections to using the British Offices' limited-payment experience in any way, whether directly or indirectly, for the purpose of calculating office premiums. Some of these objections are special to the particular experience and class of assurance; others are of a general nature. To take the former first:

(1) It has been pointed out by Mr. Chatham that the inclusion of duplicates in the limited-payment experience has the effect of somewhat under-representing the true mortality.

(2) Single-premium policies are not infrequently taken out concurrently with the purchase of an annuity solely for investment purposes—either without medical examination, if the annuity is purchased from the office granting the policy, or in any case without self-selection on the part of the assured.

(3) The selection of the limited-payment plan is probably much more influenced now than was the case in 1863-1893 by considerations as to the income-tax abatement and the proportionate paid-up policy option, and by the general tendency to restrict life assurance payments to the term of active professional or business life.

(4) Limited-payment assurances, especially on the 10- or 15-payment plan, are sometimes granted at ordinary rates in cases which would not be accepted without addition for ordinary whole-life assurances.

The objections of a general nature are that the self-selection that may have occurred in the early history of a particular kind of assurance when it was the "option of the few" cannot be relied upon as a permanent factor, and that differentiation as regards mortality in favour of a particular class will have the effect of attracting less select lives to that class. These objections have been pointed out on many occasions with more particular reference to endowment assurances—by Mr. Chatham (*T.F.A.*, vol. i, p. 125), Mr. Nightingale and Sir George Hardy (*J.I.A.*, vol. xxxvii, pp. 182 and 184) and Mr. Andras (*Congress Trans.*, VII, p. 772). It should be stated that both Mr. Nightingale and Sir George Hardy thought that allowance might perhaps be made for lighter mortality in the

loading of the premiums, but it is not clear how this would be done or that it is ever in fact done, and if class mortality is not taken into account in calculating endowment assurance premiums there is a certain inconsistency in using it for limited-payment premiums.

The conclusions to which the foregoing discussion of the subject seem to me to lead are that neither the saving of expenses nor the supposed superiority of limited-payment lives should be taken into account in calculating limited-payment premiums, that normal mortality and a future-experience rate of interest should be the basis of calculation, and that consequently (subject to such modification as may be caused by differences of opinion as to the probable future course of the rate of interest) the whole-life premiums might be commuted by the O^[M] table with $3\frac{3}{4}$ per-cent interest for single premiums and $3\frac{1}{2}$ per-cent for payments extending over 10 years or more.

It has been virtually assumed that the benefits assured by a limited-payment policy of the type under discussion are the same as those assured by a similar whole-life policy. This assumption, however, is not strictly correct, on account of the proportionate paid-up policy option which is usually given in connection with limited-payment policies. It may be said that a benefit of this nature is, or ought to be, entirely subsidiary, that the proper course is to charge the requisite premium for the assurance and to decide subsequently what paid-up policy option (if any) can be given, and that if the proportionate paid-up policy is too much it is the paid-up policy option and not the premium that should be altered. This is no doubt true, and it is perhaps unfortunate that when the practice of giving a paid-up policy option was introduced, the simple expedient of excluding the first year's premium from the calculation—that is, of making the paid-up policy $(n-1)/(t-1)$ -ths, instead of n/t -ths of the sum assured—was not generally adopted. But the proportionate paid-up policy is now firmly established in office-practice and in popularity, and in view of the probability of its being retained, it is worth while to consider in what way the objections to it are affected by the method employed in calculating the premiums. So far as participating policies are concerned, the practical objections (apart from considerations as to initial expenses) are not very serious unless the proportionate paid-up policy is (as is sometimes the case) admitted to participation in future profits in addition to having existing bonuses added to it. But in

the case of a non-participating policy it is well-known that in the tables of most companies $(t-n)_{t-n}P'_{x+n}$ is less, and sometimes materially less, than $t_tP'_x$ for many values of t and n ; and for reasons which it is unnecessary to state this is clearly undesirable. Now this feature becomes somewhat more pronounced as the rate of interest used in commuting the whole-life premiums is increased. This follows from the fact that the commutation-rate affects the shorter-term premiums more than those for the longer terms, but the precise extent to which a difference in the rate affects the two quantities in question may perhaps be usefully illustrated by the following examples based on actual office whole-life non-participating rates commuted by $O^{(M)} 3\frac{1}{2}$ per-cent and $O^{(M)} 4$ per-cent :

Values of $t_tP'_x$ and $(t-n)_{t-n}P'_{x+n}$.

x	t	n	P'_x COMMUTED BY $O^{(M)} 3\frac{1}{2}\%$		P'_x COMMUTED BY $O^{(M)} 4\%$	
			$t_tP'_x$	$(t-n)_{t-n}P'_{x+n}$	$t_tP'_x$	$(t-n)_{t-n}P'_{x+n}$
30	10	5	45.53	45.10	43.35	42.70
30	20	5	55.28	55.13	53.62	53.26
...	...	10	...	54.76	...	52.66
...	...	15	...	54.55	...	52.18
35	10	5	49.84	49.37	47.68	46.98

It will be seen, from these figures, that the $3\frac{1}{2}$ per-cent commutation, as compared with the 4 per-cent, lessens—although it does not remove—the inducement to a life still select to exercise the option, and that in any case it has the advantage of having secured a little more premium if the option is exercised. From a practical point of view both these considerations may be regarded as affording additional reasons in favour of the adoption of the lower rate.

Policy Values.—The second question submitted for consideration is not of the same importance as the first. If a sufficient premium is charged it is comparatively immaterial whether the method of valuation is quite consistent with that adopted for whole-life policies. The question is, however, of some interest, and not only academically, for in the possible event of a relatively large amount of limited-payment business being transacted in an inter-valuation period the usual method of valuation might

entail an inconvenient diminution of the surplus. The difficulty, of course, arises from the valuation rate of interest being lower than that used in commuting the whole-life premiums. If the valuation-rate be i and the limited-payment premiums have been obtained by commuting the whole-life premiums at rate j by the $O^{(M)}$ table, the value of the limited-payment policy after n years on the assumptions (1) that limited-payment and whole-life policies are to be placed on exactly the same footing as regards reserves and contribution to surplus, and (2) that the rates of mortality and interest are in accordance with the commutation basis, will be ${}_nV_x + [P'_x a_{[x]+n}^{(j)} - {}_tP'_x a_{[x]+n-t}^{(j)}] O^{(M)}.$ * (The value of the bonuses will be the same for both limited-payment and whole-life policies, and is omitted for present purposes.) This may be called the value by the commutation-basis method, and would be the true value if the second of the two assumptions made above were valid. A somewhat better estimate of the true value would be obtained by using a decreasing rate of interest instead of j in the valuation of the two premiums.

The value by the usual or Text-book method will be $A_{x+n} - {}_tP'_x a_{x+n-t} + (P'_x - P_x) a_{x+n}$. A deduction for renewal commission is sometimes made from ${}_tP'$ and P' , and a similar deduction might be made in the commutation basis formula. The adjustment would not, however, be suitable in all cases, and it does not make a very material difference in either the absolute or the relative values given by the two formulas.

Mr. H. W. Brown in "Some Questions arising in the Valuation of Special Class Policies" (*Trans. A.S.E.*, vol. iv, p. 387) suggests that a constant percentage on the sum assured might be substituted for $P'_x - P_x$ in the usual formula. The main object of the suggestion appears to have been to avoid the necessity

* The corresponding expression for the requisite extra reserve (in addition to $A_{x+n} - {}_tP_x a_{x+n-t}$) has been deduced in a rather more general form by Mr. A. Fraser in his Note on "The Special Reserve required for Future Profits on Limited Payment Policies" (*T.F.A.*, vol. vi, p. 93), to which reference should be made. The writer is indebted to Mr. Fraser for the information that in a fairly large limited-payment class of 'some years' standing the total reserve by the usual method was almost exactly double the true reserve, and that in that class the mortality was found to be 61 per-cent of the $O^{(M)}$ as compared with 74 per-cent in the whole-life class and 52 per-cent in the Endowment Assurance class—all three classes being with profits.

of valuing the loading for each policy individually, but this becomes unimportant when—as is presumably now customary—the whole-life loading is classified with the sum assured in the whole-life schedules as a negative net premium. Apart from this consideration the suggestion involves only a modification of the ordinary loading—which is itself a percentage on the sum assured—and the practical difficulty in adopting it would be to decide precisely what the modification should be. Mr. Brown points out that “the constant may be taken rather less than the average “whole-life bonus loading on account of the larger interest “profit on limited-payment policies”, but that excess profit cannot be expressed in terms of a constant reduction for all durations. No doubt it would be possible, by comparison of the policy-values for various terms and durations with the estimated true values, to arrive at some reduction which would be on the safe side, but this process would be laborious and not in the end very satisfactory. If the reduction could be so fixed that the policy would have no value for duration 0, there would be a good deal to be said for the method, but although in some cases (as in the example given below) this would produce fair results, in others—especially in the case of a single-premium policy—it would give insufficient values for the longer durations.

A simpler modification, which has not, I think, been previously suggested, is to substitute for ${}_tP'$ in the usual formula a hypothetical office premium calculated by commutation of the whole-life premium on the valuation-basis, so that the value after n years becomes $A_{x+n} - P'_x \cdot a_x / a_{\overline{t}|} \cdot a_{x+n} / a_{\overline{t-n}|} + (P'_x - P_x) a_{x+n}$. This method—which may be called the hypothetical office premium method—has the practical advantages (1) of entailing no alteration of the usual method of classification, and (2) of making the policy-value 0 at entry and distributing over the payment term the valuation-loss due to a premium of ${}_tP'$ instead of $P'_x a_x / a_{\overline{t}|}$ having been charged. On the other hand it becomes identical with the usual formula as soon as the payments are complete and consequently does nothing to meet the difficulty in the case of a single-premium policy. It is also open to the objection that it takes no account of the actual premium paid and would consequently assign the same values to policies subject to different office-premiums. This objection would be serious if obviously inadequate premiums were charged, but on the assumption that the premiums charged are reasonably

justified by the interest and mortality experience it is of no practical importance.

For purposes of illustration a 20-payment policy effected at age 30 may be taken. On the assumptions that the premium charged is a whole-life premium of £2. 10s. per-cent commuted by $O^{[M]}$ 4 per-cent (which has been found to be approximately the average basis of commutation), and that the valuation is made by $O^{M(5)}$ $2\frac{1}{2}$ per-cent, the values by the several methods will be as shown in the following table :

Limited-Payment Policy-Values.

$x=30$; $t=20$; $P'=2\cdot5$; ${}_tP' = P'a_{[x]}/a_{[x]t}$ by $O^{[M]}$ 4 per-cent.

$O^{M(5)}$ $2\frac{1}{2}$ per-cent.

n	BY COMMUTATION-BASIS METHOD		BY USUAL METHOD (with whole-life loading)	BY MODIFICATION OF USUAL METHOD (with loading such that ${}_0V=0$)	BY HYPOTHETICAL OFFICE PREMIUM METHOD
	Interest 4 % constant	Interest decreasing in 40 years from 4 to 3 %			
0	0.00	1.49	5.31	0.00	0.00
5	12.23	12.99	18.26	13.31	14.00
10	26.65	28.80	33.08	28.52	30.02
15	43.81	46.19	50.10	45.96	48.44
20	64.50	66.87	69.86	66.16	69.86
30	74.46	76.21	77.47	74.70	77.47
40	83.29	84.29	84.71	82.83	84.71

Even if $O^{[M]}$ 4 per-cent be considered a suitable basis for the commutation of the whole-life premiums, it will no doubt be agreed that an extra reserve—as compared with ${}_nV_x$ —of more than the $O^{[M]}$ 4 per-cent value of $P'a_{x+n} - {}_tP'a_{x+n|t-n}$ ought to be gradually accumulated (whether from interest profit or light mortality) during the payment period. The values in column (3) may therefore be regarded as more correctly representing the true values, at any rate for the longer durations. Judged by these values those in column (5) tend to become too small. The hypothetical office-premium values, while always on the safe side, appear to offer a reasonable compromise between the inconvenience of a true valuation and the unnecessary rigour of a valuation by the usual method.

II.—DECREASING DEBTS ON ENDOWMENT ASSURANCES.

Decreasing debts in lieu of extra premiums have been discussed in two papers read before the Institute (A. W. Sunderland, *J.I.A.*, vol. xxix, p. 419; G. F. Hardy, *J.I.A.*, vol. xxxii, p. 153), but in each case with special reference to whole-life assurances. Under present-day conditions the question more often arises in relation to endowment assurances, and I propose in this note to discuss it in that connection.

It may be assumed that in any given case the extra premium that would be charged has been determined according to the usual office practice—even if the life has been rated up in the first instance by an addition to the age—so that the problem is not to interpret a specified addition to the age in terms of a decreasing deduction from the sum assured, but to find the deduction that would be equivalent to a specified annual extra. For this purpose it is, of course, necessary to make some assumption as to the distribution of the extra risk. The representation of extra mortality by an addition to the age is not now considered to be, as a rule, in accordance with the probable facts, but Sir George Hardy's convenient and practical assumption of equal reserves (or equal expected death-strain) gives a distribution that may be regarded as fairly applicable to most of the cases arising in practice—at any rate if select mortality be employed (as it should be), for in that case the extra mortality rises steeply and naturally with the select curve instead of starting in the abrupt way indicated in Diagram I of Mr. Burn's Berlin Congress paper on "The treatment of under-average lives by assurance companies" (*Congress Trans.*, V, p. 205) and rightly, I think, described by Mr. Burn in that paper as extremely improbable. Sir George Hardy's method, however, involves the calculation of a more or less complete set of tables, inasmuch as it starts with the debt and arrives at the extra premium by way of the implicit extra mortality on the assumption of equal reserves, and in dealing with a particular case it is probably more usual in practice to determine approximately the constant extra mortality corresponding to the given extra premium by reference to the similar effects on the annuity-value of an increase in the rate of mortality and an increase in the rate of interest, to apply the principle of equal death-strain to find the corresponding debt at intervals of 5 to 10 years, and finally to substitute a roughly equivalent debt decreasing regularly for the somewhat

irregular series thus obtained. An alternative method sometimes adopted, when the constant extra mortality has been determined, is to calculate the value of an increasing temporary assurance (and thence that of a decreasing assurance) by the approximate relation $(IA)_{xn}^1 = 100(1+i)(A_{jn}^{1-i-005} - A_{xn}^{1-i+005})$, in which, of course, effect must be given to the altered mortality by expressing the A 's in terms of a 's and suitably altering the rates of interest at which the latter are taken.

Both these methods are open to the objections (1) that they over-represent the extra mortality in the first few years of assurance, (2) that they involve a certain amount of approximation and careful calculation with consequent liability to error, (3) that they are inelastic and consequently do not lend themselves to an adaptation of the debt to practical requirements. The last-mentioned objection is perhaps the most serious. The chief obstacle to the acceptance of a decreasing debt is its prohibitive appearance at the outset, and it is therefore better to quote a truncated debt—that is, to substitute for the objectionable and comparatively useless top of the right angled triangle (if one may consider a debt geometrically) an equivalent area later on—but any added complication of this nature is almost out of the question when approximate methods are employed. It is, I think, better for practical purposes to tabulate the value of $a'_{[x]t}$ and $A'_{[x]\overline{t}}$ for, say, quinquennial ages at entry on the basis of two or three equi-different rates of extra mortality. Any distribution of extra mortality that may be considered generally suitable can be assumed, and when the values of ${}_t a'$ have been calculated those of ${}_t A'$ can be obtained at once by entering the Conversion Table with ${}_{t-1} a'$ and deducting ${}_t E'$ (which will have been obtained in calculating ${}_t a'$) from the result. The value of a debt of any requisite shape can then be found by summing the appropriate section of the ${}_t A'$ column; for example, the debt corresponding to an extra premium of $P'_{[x]\overline{t}} - P_{[x]\overline{t}}$ on a t -year endowment assurance will be $(P' - P) {}_t a' / \sum_t^{n+1} A'$ remaining constant for n years and then diminishing by equal annual instalments during the subsequent $t - n$ years. The debt corresponding to $P'' - P$ —and also that corresponding to $P''' - P$ if tables have been constructed for three scales of extra mortality—will be found in the same way, and the debt equivalent to the given extra premium can then be found by interpolation. A deduction of $2\frac{1}{2}$ per-cent may of course be made from the result in cases in which com-

mission would be payable on the premium. The particular distribution of extra mortality assumed will be a matter of opinion, but I think that an addition to the select q increasing in arithmetic progression to a constant after 4 years is fairly suitable. It might be better to effect a smooth junction with the normal curve at about age 55-60 as suggested by Mr. Burn in the paper already referred to, but this would make little difference so far as endowment assurances and decreasing debts are concerned, since the net amount at risk becomes small at the older ages. The assumption of a constant extra after the first few years has, moreover, the practical advantages (1) that the extra premium, for a given endowment term, is practically independent of the age at entry, (2) that the extra premium is approximately proportional to the addition to the mortality—this follows at once from the fact that $1/a$ at successive rates of interest is nearly in arithmetic progression—and (3) that the decreasing debt on this assumption accords fairly well with the convenient practice of making normal reserves. These considerations would be of no importance if it were possible to determine with any certainty the incidence of the extra mortality; but as in the present state of actuarial knowledge any assumption that may be made is necessarily somewhat arbitrary, I think that they may carry some weight.

For practical purposes the values of ${}_t a'$ and $\Sigma {}_t A'$ are the most convenient to work from as they give with very little trouble the corresponding debts and extra premiums—with any desired constant period of debt to start with—for any endowment term. But in order to illustrate their use I have prepared the following tables :

Corresponding Debts and Extra Premiums on the assumption that

$$q'_{[x]+n} = q_{[x]+n} + \frac{n+1}{5}c \text{ or } q_{[x]+n} + c, \text{ according as } n \text{ is } < 4 \text{ or } = \text{ or } > 4.$$

$O^{[M]}$ normal mortality.

Interest $3\frac{1}{2}$ per-cent.

Endow- ment term (t)	Extra Premium $100(P' - P)_{[x]\bar{t}}$	Debt constant for 11 years, and decreasing by equal amounts yearly thereafter during remainder of Endowment term			
		Age at entry			
		25	30	35	40
15	<i>s. d.</i> 4 1	22.1	20.5	18.3	15.8
„	8 2	32.0	30.3	28.0	24.9
20	4 8	26.1	23.8	21.2	18.0
„	9 5	37.9	35.7	32.5	28.8
25	5 2	28.7	25.8	22.6	19.0
„	10 5	41.8	39.0	35.2	30.7
30	5 7	30.1	26.9	23.3	...
„	11 3	44.5	41.0	36.6	...
35	5 11	31.0	27.4
„	12 0	46.0	42.0
40	6 3	31.2
„	12 7	47.2

Endow- ment term (t)	Extra Premium $100(P' - P)_{[x]\bar{t}}$	Debt constant for 6 years and decreasing by equal amounts yearly thereafter during remainder of Endowment term			
		Age at entry			
		25	30	35	40
15	<i>s. d.</i> 4 1	27.7	25.8	23.1	20.0
„	8 2	39.8	37.9	35.0	31.4
20	4 8	31.2	28.6	25.6	21.9
„	9 5	45.1	42.6	38.9	34.6
25	5 2	33.3	30.1	26.4	22.3
„	10 5	48.3	45.2	40.9	35.8
30	5 7	34.2	30.7	26.7	...
„	11 3	50.3	46.5	41.6	...
35	5 11	34.6	30.7
„	12 0	51.2	46.8
40	6 3	34.5
„	12 7	51.5

Suppose, for example, that the extra premium on a 25-year endowment assurance at age 30 were 7*s.* per-cent. A simple interpolation between 0, 25·8 and 39·0 gives a corresponding debt of 31·9 constant for 11 years and decreasing thereafter by 2·13 yearly.

A point that may be worth mention is that the value of $n A_{x-1}^1$ varies very little during the usual endowment term, as can be seen by inspecting any column of C_x . Consequently the shape of a debt can be varied (within reasonable limits) without materially altering its value provided the area be unaltered. For instance, in the case of a 25-year endowment assurance a debt of 17½ decreasing after 11 years would be practically equivalent to one of 21, decreasing after 5 years, because both represent a total debt of 315. The principle has no doubt been applied to truncate triangular debts obtained by an approximate method.

It is not usual to make a theoretically accurate reserve for a policy subject to an extra premium, and it would not presumably be considered necessary in the case of a policy subject to a debt, but it is of some interest to compare the decreasing debt as obtained on the assumption of a constant extra mortality after the first four years with that provided for on the same assumption by the normal reserve. The following examples, in which a simple reversionary bonus of 30*s.* per-cent per annum has been assumed and the basis of valuation has been taken as O^M 3 per-cent, will suffice for purposes of illustration :

Age at Entry 30. Endowment Term 30 years.

Extra Mortality ·01 after 1st 4 years.

Years in force	Sum Assured and Bonus	Death Strain	Debt provided by normal reserve*	DEBT OBTAINED AS ABOVE	
				Decreasing after 11 years	Decreasing after 6 years
5	107·5	92·44	53·5	41·0	46·5
10	115·0	82·38	41·5	41·0	37·2
15	122·5	69·23	31·4	30·8	27·9
20	130·0	52·64	20·7	20·5	18·6
25	137·5	29·92	9·7	10·2	9·3

Age at Entry 40. Endowment Term 20 years.

Extra Mortality ·01 after 1st 4 years.

Years in force	Sum Assured and Bonus	Death Strain	Debt provided by normal reserve*	DEBT OBTAINED AS ABOVE	
				Decreasing after 11 years	Decreasing after 6 years
5	107·5	83·52	41·0	28·8	34·6
10	115·0	62·57	24·6	28·8	23·1
15	122·5	35·68	11·6	14·4	11·5

* $\cdot 01 \times \text{Death Strain} \div (q^{O.M.} + \cdot 01)$.

It will be seen from the comparatively close agreement of the debts that the normal reserve gives a practically true reserve, and that after the first few years the debt decreasing after 11 years gives better results for the longer term endowment and the debt decreasing after 6 years for the shorter.

Since an increase can be regarded as a negative decrease, it may be permissible, notwithstanding the title of this note, to refer to a particular kind of increasing debt which is sometimes offered as an alternative to an extra premium on a with-profit policy, namely, the debt represented by a deferment of the vesting of the bonus. An increasing debt would not appear, in general, to be an attractive option, but under the comparatively inoffensive guise of a deferment of bonus it may in some cases prove more acceptable than any other form of surcharge. Two minor objections to this method of imposing an extra are (1) that it involves practically discounting a bonus without compensation if the rate of bonus discounted should not be realized, and (2) that it does not accord very well with the practice of making normal reserves, since the latter will obviously be too large in the early years. These objections are not, however, of much importance. A more serious difficulty is that it is not possible to obtain in this way an equivalent for more than a comparatively small extra premium on an endowment assurance. Mr. Burn has shown, in the Congress paper already referred to, that a considerable amount of extra mortality can be provided for by deferring an average bonus, but this is equally true of a small extra premium. On the basis of the distribution of extra mortality assumed in this note the extra premium that could be provided for, on a life aged 30, by deferring a simple reversionary bonus of 30s. per-cent per annum for the whole endowment-term, that is, by making the bonus payable only in the event of the life assured attaining the endowment-age, would be approximately as follows:—

Endowment term	15	20	25	30	35
	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>	<i>s. d.</i>
Extra premium	2 6	3 9	5 3	7 0	8 9

ABSTRACT OF THE DISCUSSION.

Mr. R. C. SIMMONDS said he proposed to offer some remarks under three heads: first, limited-payment premiums; secondly, limited-payment reserves; and, thirdly, contingent debts under endowment assurances—following the general lines of the paper.

With regard to limited-payment premiums, he noticed that the author adopted the ordinary whole-life premium as the basis for commutation. Such a course was obviously convenient, but he thought it should be remembered that the ordinary whole-life premium itself depended upon certain assumptions as to interest, &c., and was not merely a fixed quantity which could be operated upon in any way desired. Referring to the ratios which the author deduced, he thought one possible reason for the continued use by some companies of the $H^{(M)}$ or the H^M 4 per-cent basis might be found in the fact that probably their ordinary whole-life tables were still founded upon the old data, and that this being the case they would not think it worth while to alter their practice with regard to commutation.

With reference to the three reasons commonly adduced for the more favourable treatment of limited-payment cases, he was quite in agreement with the author regarding the expenses; it seemed to him that no abatement could be made under that head. With respect to interest, he thought that many felt that Mr. Moir's arguments were based upon certain assumptions which were hardly likely to be fulfilled in practice. It appeared to him that the argument for using a better rate of interest rested upon at least two assumptions: first, that the rate of interest in future would fall, and secondly, that it would be possible to invest the comparatively large premiums received over a short term of years (and, of course, the dividends upon such investments) at a rate of interest which would be permanently greater, *i.e.*, a rate of interest which would keep up after the general rate had fallen. Personally he thought that the second assumption was hardly justifiable, particularly when one considered the prevalence of redeemable securities. Therefore, he had come to the conclusion that on the question of interest no allowance of any consequence could be made. In respect to mortality, as the author pointed out, there was considerable divergency of opinion. Mr. Chatham, in the third volume of the Transactions of the Faculty of Actuaries, expressed the opinion that limited-payment policies were subject to distinctly better mortality, and he supported that opinion by a table of premiums which showed that the limited-payment rates were only about 90 per-cent of the whole-life. He gathered that the author was somewhat doubtful as to the alleged superiority, and it was interesting to look at his reasons. The first was the non-elimination of duplicates in the limited-payment experience. Personally, he hardly thought that that would have any serious effect. The second reason was the possible prevalence of concurrent paid-up assurances and annuities taken out as investments. That was, to him, a rather curious reason, because, from one or two tests that he had made, he found that the rate of interest yielded would be generally not more than $2\frac{3}{4}$ per-cent, so that it did not seem that such a system of dual transactions could be very attractive. He did not think that such cases would be very common. The third reason was the modern popularization of limited-payment policies, and this needed

serious consideration. The author suggested that the demand for those policies had been stimulated by the publication of tables and so forth, and also, possibly, by the special conditions as to paid-up policies, and he thought that these things had a decided effect, but he was loth, in the absence of further evidence, to consider that the limited-payment mortality was not somewhat better than that shown by whole-life cases. One could only depend upon evidence, and he was rather interested in seeing the figures which were given by Mr. A. Fraser. The fourth reason, the inclusion of under-average lives at tabular rates under a limited-payment table, depended for its validity upon the prevalence of the practice. He believed that Messrs. White and Whittall had shown (*J.I.A.*, vol. xxiv, p. 385) that such a practice was quite unsound, and it certainly re-acted unfavourably upon the mortality shown by the limited-payment policyholders. The question whether the mortality varied with the duration during which premiums were payable was a very interesting one. Mr. Moir, in assuming that those in a position to pay single premiums were the best lives, was possibly adopting a law which was presumed to apply in other spheres of life. He did not think that such an assumption was necessarily correct, and he was afraid that in the present state of actuarial knowledge one could not attempt to solve the problem. If superior mortality were proved to exist and if Mr. Moir's assumption were accepted, an increase in the rate of interest was a very convenient method of making the necessary allowance.

Passing to the question of valuation, he thought that it was agreed that the Text Book method yielded extremely awkward results. Taking the author's example, that method demanded a reserve at duration 0 of 5·3, which was more than $1\frac{1}{2}$ times the office premium of some 3·5, and even after ten years the reserve was 33·1, as compared with total premiums received of 34·8. In the circumstances it was not surprising that other methods were brought forward. The first modification suggested was the average loading method. That was easy, but rough, and there was no particular object in adopting it, unless, as might happen, the limited-payment cases were valued separately from the general body of whole-life policies. In that case the average loading method would be very convenient, because it would be possible to get the value of loading for the whole class by simply taking the total of the sums assured, deducting their total value, dividing the difference by the force of discount, and applying the average loading to the result. The "hypothetical premium" method was quite convenient and doubtless the somewhat complicated premium needed at the outset could be computed from prepared tables. In connection with the method employed in the fifth column of the table of Limited-payment Policy Values, it had been suggested to him by a colleague that one might deal with the matter by spreading the value, at the outset, of the limited-premium loading over the whole duration of the policy. Such a process gave the equated loading for use at a valuation instead of the ordinary whole-life loading assumed by the Text-Book

method. The effect, in the particular case taken by the author, was roughly to reduce the loading from $\cdot 5$ to $\cdot 3$, which reduction sufficed to make the reserve at duration 0 nil instead of $\cdot 5 \cdot 3$. The figures which he himself had calculated appeared to reproduce those which were given in the fifth column of his table, so that there seemed to be no doubt as to the identity of method. The commutation basis method, originally, was doubtless good, but it suffered from one rather serious defect, in that it appeared to involve the use of valuation factors at two rates of interest. With the ordinary method of grouping that feature seemed to be a distinct bar. He inclined to the view that the modified loading method was, on the whole, the best of the various methods that had been brought forward. It seemed to him that the capitalisation and re-spreading of the loading was in accordance with the general principles of current valuation practice. He would like, however, to ask, without venturing to digress unduly from the subject, why there should be the anomaly at all? It arose solely from the use of an artificially low valuation rate of interest, and it was not by any means the only anomaly that so arose. Possibly the growth of limited-payment business in future might strengthen the forces which made for the consideration of some other method of valuation more in accordance with actual experience. He would go so far as to hope that such a method might be adopted.

With regard to the question of contingent debts, whereas Sir George Hardy took a deduction and found a rating-up, the author proposed to take the cash extra and by finding the constant mortality to get the debt by working out a series of simple equations. The method of equal death-strain enabled him to do that with the utmost facility. There was no doubt that the decreasing debt was liable to considerable criticism on account of its extreme heaviness at first, and he was interested in the suggestion that a piece should be cut off at the beginning and added later. It would seem that by such a method of, as it were, making debts to measure, the wishes of proposers could be met and the plan made more popular. The actual variation of the debts could be made by reference to a table of temporary assurance values. The method consisted in taking the present value of the extra premiums that ought to be received and spreading it out by a combination of single-premium term assurances, much as was done in office practice in calculating a single premium decreasing-term rate. The plan was new to him and very interesting. The extra mortality which the author assumed was doubtless a fair average distribution, but it did seem to be a reflection on actuarial knowledge that actuaries were obliged to assume a distribution which they knew to be quite unrepresentative of certain types of extra risk. It pointed to the urgent necessity of having as soon as possible an investigation into the question of the incidence of extra risk, and he hoped that after the war such an investigation might be put in hand without delay. The idea of altering the truncation by assuming the approximate constancy of ${}_n A_{x1}^1$ was quite feasible in practice for short-term endowments, but he thought that for any term of more than medium length it would lead to

considerable variations. He had not, however, been able to test the matter. The deferment of bonuses seemed to him to be unsuitable for cases where the extra mortality was concentrated in the early years, as there the office was not sure of receiving adequate consideration for the risk of loss. The debt system, on the other hand, could be made suitable to any particular incidence of extra risk, because the consideration was received at the outset and all that had to be done was to adjust the valuation reserves so as to make proper provision in the later years.

Mr. C. C. MONKHOUSE said the problem, as regards the calculation of limited premiums, was to estimate what rate of interest could be reasonably expected in the future. The only data available to estimate the future rate of interest were the rates that had been realized in the past, and the influences that were likely to operate in the near future. During the last twenty years the average rate of interest had certainly not been much more than, if as much as, $3\frac{3}{4}$ per-cent after deducting tax. Mr. Hutton, in his Presidential address to the Faculty of Actuaries, estimated that about 16 millions had been written off for depreciation in the last 16 years, representing 4 per-cent of the total invested funds. That would be equivalent to taking $\frac{1}{4}$ per-cent off the rate of interest over that time, which reduced the actual rate of interest realized—taking depreciation into account, as he thought one ought to do, in estimating the rate of interest—to about $3\frac{1}{2}$ per-cent net. Therefore, it seemed to him that $3\frac{1}{2}$ per-cent was the utmost that one could expect to realize, without taking into account any influences in the future. With regard to those influences, there was of course the war, with the depreciation in capital, and consequent increase in the rate of interest; there was also the high income tax which would to a very large extent nullify that increase; and also the practice of some offices, and a growing practice, of distributing their investments over a much wider field and so earning a higher rate of interest. That practice, however, involved the risk of loss of income in particular investments, and that must be very evident at the present time. Taking everything into consideration, he could not see that it would be safe to expect a much better rate of interest in the future or at least in the next ten or twelve years, than had been received in the past. If his argument was correct, $3\frac{1}{2}$ per-cent was the utmost rate that should be used in calculating premiums. Why, then, should limited-premium rates be treated differently? Mr. Moir's argument appeared to be that if limited-premium rates were calculated by a limited-premium table and loaded to a lesser extent than whole-term premiums owing to the smaller expense a smaller premium would be obtained, and this could be allowed for by commuting a premium based on one rate of interest at a higher rate of interest. To start with, he did not see that limited-premium policies were such an important class that they ought to be based on a special mortality table. As the author had said,

that was not done in the case of endowment assurances, which were a much larger class, and probably limited premiums were subject to the same influences as endowment assurances with regard to the mortality to be expected in the future. It seemed rather curious that limited premiums alone should always be considered from the point of view of the assured instead of from the point of view of the office. If the mortality of limited-premium policyholders was so good they were the one class which should be induced to pay premiums throughout life. Why, then, should they be given special terms and the office so forego the legitimate profit which would arise if they paid premiums throughout life? He thought with regard to the longer terms it was quite probable that if the office did not give such good terms for limited-premium policies the proposers would take ordinary whole-life policies. With regard to the shorter terms and the single premiums, the argument did not apply, because the policies were effected for quite different reasons; they were not assurance but investment, and on account of the income tax allowance they worked out as a rather good investment for the assured. The only point that arose was whether those policies in themselves, not as compared with ordinary whole-life policies, were profitable to the office. He had worked out a few cases at with-profit premiums—he realized that the fact that they were with profits made the result rather better than one would expect—and allowing only for commission, and no other expense, he found that whenever the insured died, provided the bonuses were maintained, he would always get his premium back with from $3\frac{3}{4}$ to 4 per-cent interest. From that point of view it did not appear that the policies could be very profitable, and therefore they should not be given exceptionally good terms.

There was another disparity that seemed to him unjustifiable. If a policyholder wished to commute his premium he was charged probably $O^{[M]} 3\frac{1}{2}$ per-cent terms, but if he wished to buy an annuity he was charged approximately $O^{[AM]}$ net 3 per-cent terms. The $O^{[AM]}$ and $O^{[M]}$ mortality were practically the same. On the annuity the office had the benefit of the income tax; while on the commutation there was no such benefit. The policyholder, from his point of view, was buying exactly the same thing, and he was being charged considerably less under one method than under the other. Also by commuting a man's premium the office was presumably foregoing mortality profit. Why should the office forego interest profit as well? There was another reason why he thought short-term limited-premium policies and short-term endowment assurances should be discouraged; they seemed to be the ultimate refuge of unpatriotic millionaires who took advantage of a Government concession, a concession which under no circumstances was ever meant to apply to them.

Mr. C. R. V. COUTTS suggested that the author might explain more clearly the connection between the impossibility of accumulating a pound for a thousand years at 4 per-cent and a probable decrease in the rate of interest during the next forty years. The question of the future rate of interest was one that hardly came within the scope of the paper, but he would suggest that it depended on the relation between the supply of and demand for capital. Even in the last twenty years there had been so many changes that he would hesitate to suggest that it was more likely to fall from 4 to 3 per-cent than to go up from 4 to 5 per-cent. That led him to another point arising out of the rate of interest, namely, the rate of interest which was to be used in calculating the whole-life premiums, which formed the basis for the commutation for the limited premiums. The author suggested that those premiums would be based on something like $3\frac{1}{2}$ per-cent and discussed the effect of commuting whole-life premiums by annuities, which the author assumed were at a higher rate of interest than the rate of interest used in the calculation of the whole-life premiums in the first instance. But it was pointed out in the paper that an average whole-life premium at age 30 of £2 10s., represented a $3\frac{3}{4}$ per-cent net premium for a whole-life assurance with a compound bonus of 30s. per-cent, that was to say that a man who paid that premium was getting a $3\frac{3}{4}$ per-cent. investment. In a good many cases, if the same test was applied to the ordinary whole-life premiums, taking into account the rate of bonus given and treating it as part of the benefit, it would be found that the interest basis was at least 4 per-cent, if not more. That only applied roughly, because the premiums were not calculated to provide a certain rate of bonus as a definite liability; they might have originally been based on a $2\frac{1}{2}$ per-cent or 3 per-cent net premium rate, with a loading of a constant and a percentage; but the actual result, however calculated originally, was to give the man something like a 4 per-cent investment. If that were so, and it was desired to get the corresponding limited-payment rate to a whole-life premium which gave a 4 per-cent investment, it seemed to him that one was bound to use a rate at least as high as 4 per-cent for the annuities. If the commutation were based on annuities at a lower rate of interest than 4 per-cent, the limited-payment policyholder was being penalized; he was being given a premium which represented an investment of, say, $3\frac{3}{4}$ per-cent. Leaving out of account the question whether limited-payment policyholders deserved any better treatment—and on that point he was rather inclined to agree with the author and Mr. Monkhouse, that the considerations of expense and mortality were not sufficiently important to be taken into account—they were at any rate entitled to simple justice. In that connection he would like to remind Mr. Monkhouse that there was another point of view besides that of the insurance office. One did not always look on a policyholder as a person out of whom the company had to make as much profit as

possible, but sometimes as a member of a mutual society, and the duty of the office was to try and hold the balance equally between the different classes of policyholders and do equal justice between them—not to see which could be fleeced to the greatest extent. If the whole-life premium did as a matter of fact give a 4 per-cent investment, the office ought not to use a lower rate of interest than 4 per-cent in calculating corresponding limited-payment premiums.

The next point he wanted to refer to was the author's reference to the anomaly that there might be a stage when a non-profit limited-payment policyholder could take up a proportionate paid-up policy, and, if a first-class life, insure for the balance of the sum assured at a lower premium than he paid under his original policy. No doubt all actuaries had come across small anomalies of that kind. There was another one comparable to it, and that was the case of the man who wanted a term insurance for one year, and was able to claim commission from the office; he took out a non-profit whole-life policy, deducted his 1 per-cent commission, and thus got his term assurance at appreciably less than the normal term rate. He had heard of a case where that had been done for some years by a man who thought he was the sole discoverer of the method; the man told the medical officer that he went round from office to office taking out a whole-life policy and taking the whole of the commission. He (the speaker) did not know whether the policyholder ever succeeded in getting commuted commission as well! He did not see that actuaries should allow such little anomalies seriously to affect the basis on which they calculated premiums.

Dealing with the question of policy values, the author had suggested several ingenious ways of meeting the difficulty arising under the old-fashioned method of valuing limited-payment policies, under which a reserve had to be set up against a policy before payment of the first premium, which was certainly undesirable. All those methods had reference to a net premium valuation. He entirely agreed with Mr. Simmonds that the sooner offices got back to more natural conditions and more into touch with things as they were, with the rate of interest actually being earned, with the rate of mortality experienced, and the rate of expenses they were working on, the better. If a gross premium valuation were made with a reserve for future profits, none of those difficulties would arise.

With regard to the question of debts in lieu of extra premiums, he could not see why an office should not assess a man, if he were an under-average life, by a debt in the first instance and measure the extra risk in this form just as accurately as it was measured as a money extra or an addition of so many years to the age. They were all equally unscientific, because there were practically no data on which to calculate extra risks. If an office wished to give effect to an extra risk by means of a debt on the policy he could see no advantage in measuring the risk in the first instance in the form of an extra premium and then converting this extra premium into a deduction from the sum assured.

Mr. R. G. SALMON agreed with the author that the saving of

expense due to the premiums being limited instead of being payable throughout life might be regarded as negligible. His own experience had been that, apart from management of assets and interest income, which was the same for limited-payment and for whole-life premiums, the renewal expenses were exceedingly small, and any difference between the two classes was certainly negligible.

On the question of mortality, the author referred to the British Offices Experience, 1863-93. The difference there shown between limited-payment and whole-life mortality had been attributed to several causes, but his own experience led him to believe that there was no appreciable difference between whole-life, limited-payment, and endowment assurance mortality, amongst policies issued in recent years and under similar conditions in any one office. The Institute experience was compiled from the experience of offices over many years; if the mortality of assured lives had improved during that period, which he thought was highly probable, and the limited-payment and endowment assurance experience was on the average of more recent date than the whole-life, it would necessarily have shown a better rate of mortality than the total whole-life, even if the rate had actually been the same as the whole-life during the latter part of the experience. The author had referred to Mr. Fraser's experience with regard to endowment assurance mortality, but it was quite impossible to judge which was the better mortality without knowing the ratio of the actual to expected deaths at the different ages; the whole-life ratios might be very high at the older ages and low at the younger, and thus although the total ratio for the limited-payments was lower than the whole-life, the ratios at the younger ages might be equal. One investigation he was conversant with showed this to be the actual fact. Again, the British Offices experience was the aggregate of the separate experiences of many different Companies. It was quite possible and likely that the limited-payment and endowment experience portions of those returns might have been contributed by the lower mortality offices. That defect, of course, was inherent in all combined office mortality, and should give cause for hesitation before assuming that the O^M, or any combined office table, truthfully represented the rate of mortality obtaining in any particular office as a whole or in any one class. In fact, its use by an individual office as a basis for premiums might be actually unsafe.

The future rate of interest presented a difficult problem, especially in view of the increased income tax; but in the valuation part of the paper the author was not so much concerned with the absolute rate as with the difference between the experience and the valuation rates. There he thought the difficulty arose from their unpractical methods of valuation. Mr. Coutts had referred to the matter, and he agreed that valuations ought to be conducted on a rational basis and true experienced rates of mortality, interest, and expense used. Unless that were done, at any rate in theory, anomalies would frequently occur. The Board of Trade and public valuations might be on a 3 per-cent net premium basis if they liked, but actuaries

in all their theoretical considerations ought to recognize that the 3 per-cent valuation was only put forward because it was easily understood by the public and, generally speaking, gave in total reserves greater than those which were really required for present liabilities and future bonuses. In theory the rates of premium, the reserves, the surrender values and the distribution methods ought all to be consistent, and that was only possible if they were all based on the true experience rates, whether of mortality, interest or expense. In regard to reserves for limited-payment policies he therefore submitted that the true limited-payment formula should be based on that given in the first paragraph of the section of the paper dealing with policy values, namely, the commutation basis, but so modified as to provide for future bonuses as a liability and for expenses of renewal only. A point on which he felt rather strongly, although perhaps it did not arise directly out of the paper, was that valuations should more and more tend to true results based on experience. So long as the total reserves on their published basis were at least as large as on the true it mattered little to the actuary what basis was adopted, although of course the nearer it approximated in detail to the truth the more it should please the actuary.

Mr. W. P. ELDERTON said that in the first table given by the author he showed that the commutation basis adopted by certain offices was more generous than the O^M 4 per-cent. Of course it was open to any office to maintain that it was worth while to give some particular benefit on specially good terms, in order to attract business. In general if that argument were used there was not very much to be said in reply, unless one cared to expend one's energies in criticising either the so-called morality or otherwise of such an action. But in the particular case under discussion such an argument would hardly hold good. It might explain one generous rate, but it could hardly explain the large number of cases in which commutation was made upon very generous terms. The explanation was supposed to be either expenses, or interest, or mortality. The expenses, all agreed, had little influence. Apparently there was a slight difference of opinion in regard to interest. Mr. Monkhouse estimated it at $3\frac{1}{2}$ per-cent, and one had to deduct a good deal from that $3\frac{1}{2}$ per-cent for war depreciation. Mr. Coutts, from a different point of view, seemed to think a defence could be made out for 4 per-cent, because certain premiums accumulated at 4 per-cent gave certain results that had obtained in the past. Looking back to 1906, an office which commuted premiums then on a 4 per-cent basis for, say, a five-year limited-payment must have made a very bad investment. With regard to mortality, he had never been able to satisfy himself that the separation of the Institute experience into classes proved anything at all. It seemed to him that it was very largely a question of the dates when policies were effected, and that, combined with the suggestion that the selection of the few might be of importance and the selection of the many of no importance, possibly explained any differences in mortality that had arisen.

As an offset against the supposed advantages, there were the unduly large paid-up policies and the surrender values; the latter were frequently based on the inflated paid-up policy values. In connection with single-premium policies, he believed it was sometimes arranged that the surrender values should be very nearly equal to the one premium paid, or that the policyholder might borrow 95 per-cent or some other portion of that premium. In either case he was getting a far better surrender value than he would get under a whole-life policy if he effected it. That meant that as compared with ordinary whole-life policies, with premiums payable throughout life, the limited-payments were charged lower premiums and obtained better benefits. It might be that the ordinary whole-life premiums were too high, but he thought that some such factors as low mortality and high interest would have to be brought into account to justify some of the non-profit premiums that were charged.

With regard to the valuation of single-premium policies, he would suggest that an $O^M 2\frac{1}{2}$ per-cent or $O^{M(s)} 2\frac{1}{2}$ per-cent valuation possibly included some allowance for loading. Taking an ordinary whole-life policy on the $O^{M(s)} 2\frac{1}{2}$ per-cent basis they did not have a real loading at all, and a corresponding anomaly was found in the single-premium policy valuation.

As regards the second note, he thought that it would be useful to take a premium and find out the various types of mortality that could give that premium, and to work out from those mortalities the debts that arose. If that were done, a very large amount of information about both debts and mortality generally might be obtained. That would, of course, be following out the lines of Sir Gerald Ryan's early paper and the paper by Messrs. White and Whittall. He must confess to a prejudice in favour of a level debt. He did not think it was less acceptable to a proposer than a decreasing debt, if only because it appealed to what was once called in the Institute the "gambling tendencies of the human race."

He would like to try and summarize the conclusions that the paper and discussion appeared to have brought out. First, that commutations had been too generous. The reason for that was that the wish was father to the thought, and perhaps not a good father. Secondly, that there were difficulties in valuation and that those difficulties arose owing to the inconsistency of assuming one rate of interest one minute and five minutes later assuming a very different one. Thirdly, that debts had to be worked out on a hypothetical mortality table, because up to the present no data had been collected showing what the extra mortality was like on a large enough scale to make it worth the use. Those were not very satisfactory conclusions, because they implied a lot of criticism, but certainly that criticism was well deserved.

The PRESIDENT was sure the members would pass a hearty vote of thanks to Mr. Todhunter for his paper. To the duties of Honorary Secretary he had added, like Mr. Besant, the labour of writing a paper, and the Institute owed him a double debt of gratitude.

The motion was carried unanimously.

Mr. TODHUNTER said that he had been asked by Mr. Coutts to state a little more fully his reasons for using a decreasing rate of interest. The difference of opinion between Mr. Coutts and himself appeared to arise from the fact that Mr. Coutts referred to the rate of interest, whereas he (the author) was considering the rate of accumulation. The important thing was the rate of accumulation, and he thought theory showed that whatever might happen to the rate of interest the rate of accumulation must in the end be a decreasing rate. Mr. Monkhouse's investigations as to the effect of depreciation had a direct bearing on this question. It seemed clear that in considering the accumulation of premiums, depreciation must be taken into account, and that their calculations should be based on a rate of accumulation and not on a rate of interest. He was much interested in Mr. Simmonds's independent method of obtaining the modified loading formula, but he was not personally in favour of the use of that formula, as it seemed to give inadequate values for the longer durations. He had to congratulate himself on having had the pleasure of hearing Mr. Monkhouse's admirable reasons for not commuting whole-life premiums on such favourable terms as had been customary. The idea that a limited-payment policyholder should be deterred from taking a limited-payment policy, and encouraged to take a whole-life policy, was a most subtle and ingenious one. The reason given by Mr. Salmon for the superiority of the 1863-1893 limited-payment mortality was very suggestive. It would certainly go much farther than such a small matter as the inclusion of duplicates to explain the difference between the mortality in the different classes.

LEGAL NOTES.

By WILLIAM CHARLES SHARMAN, F.I.A., *Barrister-at-Law.*

**Lien on Policy
Monies.
Premiums paid
voluntarily and
without request.**

THE question as to whether a lien can be acquired upon a policy of assurance in respect of the payment of the premiums voluntarily and without request was considered in the case of *In re Jones' Settlement*, *Stunt v. Jones*, reported 1915 L.R. 1 Ch. 373.

The facts are as follows: Certain policies for £2,000 were assigned to the trustees of an ante-nuptial settlement dated 25 January 1876. The husband covenanted with the trustees to pay the premium, and it was also provided that the trustees, with the consent of the wife during her life and at their absolute discretion after her decease, might apply any part of the income, or if that was insufficient any part of the capital of the trust premises in paying the premium, but it should not be obligatory

on the trustees to enforce the husband's covenant in relation to the policies, or to apply any part of the income or capital in payment of the premiums, and no omission or neglect in respect thereof should be chargeable as a breach of trust. The husband died on 28 September 1914, intestate, and the wife took out administration to his estate, which was worth £108. The policy monies, amounting with bonuses to £3,257, were received by the trustees.

On 28 October 1914, the wife wrote to the trustees informing them that, her husband having no sufficient means of his own, she had for quite twenty-five years paid the premiums and thereby saved the policies from lapsing, and claiming recoupment accordingly. It appeared that from 1890 to 1914 the wife had paid £890 premiums direct to the insurance office, and she had also paid various sums to her husband out of which he had paid a few premiums, but her claim was limited to recoupment of the £890 directly paid by herself. These payments were admittedly made without any request by the trustees, and the wife had never requested them to pay the premiums out of the income or capital under their power in that behalf, the existence of which she had forgotten. She paid the premiums to prevent the policies from lapsing and without any agreement or understanding with her husband in relation thereto. On 18 November 1914, the trustees issued this summons to determine (*inter alia*) whether the wife was entitled to recoupment of the £890 out of the policy monies.

Mr. Justice Astbury in his judgment said : " The first question " in this summons is whether the wife is entitled to repayment " out of the policy monies of the £890 premiums paid by her in " her husband's lifetime. She found that her husband was unable " to pay these premiums, and unfortunately without any request " by her husband or the trustees she voluntarily paid them for " twenty-five years, with the result that the policies and bonuses " have accrued to the trust. There was a provision in the settle- " ment that the trustees with the wife's consent might apply " income or capital in payment of premiums, but this was not " acted on and no request in that behalf was made by the wife. " In *In re Leslie* (23 Ch. D. 552, 565) it was held that when a " person not the sole beneficial owner pays the premiums to keep " up a policy of life assurance, he is entitled to a lien on the " policy or its proceeds in the following cases : (1) By contract " with the beneficial owner ; (2) By reason of the right of " trustees to an indemnity out of their trust property for money

“ expended by them in its preservation ; (3) By subrogation to
 “ their right of some person, who at the request of the trustees
 “ has advanced money for the preservation of the property ;
 “ (4) By reason of the right of a mortgagee to add to his charge
 “ any money paid by him to preserve the property. In no
 “ other cases can a lien on a policy for premiums paid be acquired
 “ by a stranger or by a part owner of the policy. In the present
 “ case I regret I am obliged to come to the conclusion that the
 “ wife’s payments were voluntary, and that she is not entitled
 “ of right to recover them.”

The case of *Hampton v. Toxteth Co-operative Provident Society (Limited)* reported 31 T.L.R. 314 ; C.A. (1915) ; W.N. 143, is concerned with the question as to whether a trading Society granting insurance based on the amount of a Member’s purchases comes within the provisions of the Assurance Companies Act, 1909. The case was an appeal from a decision of the County Palatine Court.

The defendants were an industrial and provident society established in 1891 and registered under the Industrial and Provident Societies Act of 1876. They carried on the trade of general dealers, both wholesale and retail. In 1911 the defendants amended their rules so as to provide that they should have the power to carry on the business of insurance as provided by Rule 14 (a) which was added to their rules and was as follows :
 “ The committee of management may from time to time invest
 “ in any manner permitted by the rules of the society, or may
 “ appropriate out of any investment so made, or from the
 “ profits of the business such amounts as are authorised by the
 “ general meetings of the society, to form a fund for insuring
 “ the buildings, fixtures, and stock against losses by fire or
 “ otherwise, also for providing a sum to be paid on the death of
 “ a member, or the wife or husband of a member, such sum to
 “ be proportioned to one year’s average purchases of the member
 “ from the society during the three years immediately preceding
 “ death. No payment to be made on account of any person whose
 “ membership has not extended for a period of twelve months.”
 After the adoption of this rule, the society advertised “ free life assurance ” and that it paid 4s. in the pound on the average twelve months’ purchases on the death of a member or a husband of a member, and 2s. in the pound on the death of a married

Assurance Com-
 panies Act, 1909.

What is business
 of life assurance ?

woman member or wife of a member, and sums were in fact paid on the deaths of members and of the husbands and wives of members.

The plaintiff, a member of the society, brought this action, alleging that the society was carrying on life assurance business and was an assurance company within the Assurance Companies Act, 1909, and that the business was *ultra vires*, and in any case could not be carried on without a deposit of £20,000 in accordance with the Act. She claimed a declaration that certain purchase-books issued by the society either alone or with the membership cards or the rules of the society were policies of assurance on human life and void, an injunction to restrain the society from issuing any further policies of assurance on human life and from carrying on the business of life assurance, and an account of monies expended in carrying on the business of life assurance. The Vice-Chancellor held that the purchase books or vouchers, advertisements, membership cards, and rules of the society together constituted policies of assurance, and that the society was unlawfully carrying on a life assurance business without having made a deposit of £20,000 in accordance with the Assurance Companies Act 1909, and he made a declaration accordingly. The society appealed and the appeal was heard by Lord Cozens Hardy, M.R., Phillimore, L.J., and Joyce, J. Their Lordships (Phillimore, L.J., dissenting) allowed the appeal.

The Master of the Rolls said: "It is necessary to consider "the provisions of the Assurance Companies Act. It applies " (section 1) to every corporation carrying on within the United " Kingdom (a) life assurance business, that is to say, the issue " of or the undertaking of liability under policies of assurance " upon human life. By section 30 (a) a policy on human life means " any instrument by which the payment of money is assured on " death (except death by accident only) or the happening of any " contingency dependent on human life, or any instrument " evidencing a contract which is subject to payment of premiums " for a term dependent upon human life. Now it is clear that a " policy within the meaning of the Act must be in writing, or " at least be evidenced by writing. The Life Assurance Act, " 1774, provides this, and the use of the word 'instrument' in " Section 30 conclusively shows this. It is necessary next to " consider the nature of the defendant society and what it is " they are doing which is said to amount to carrying on a life " assurance business. The defendant society is an industrial

“ and provident society duly registered under the Act of 1876.
“ Its objects as defined by the rules are to carry on the trades
“ of general dealers, both wholesale and retail, and it appears
“ to be a highly prosperous company. By rule 14, the net profits
“ are to be applied in paying or providing for (1) expenses
“ of management ; (2) and (3) interest on paid up capital and
“ loan capital ; (4) depreciations ; (5) such other sums as any
“ quarterly meeting may vote for the objects of the society ;
“ (6) a reserve fund for any purpose, whether charitable, philan-
“ thropic, of public utility or any other purpose whether within
“ the objects for which the society is formed or not ; (7) any
“ other sums the committee consider necessary for contingencies,
“ and (8) $1\frac{1}{4}$ per-cent to the educational fund and the remainder
“ to be divided among members and non-members in proportion
“ to their purchases from the society, but the non-members are
“ only to receive one-half the rate which is allocated to members.

“ The society for some years effected what is described as a
“ collective policy with the Co-operative Insurance Society
“ (Limited) to provide for each member who should die during
“ the currency of the policy 4s. in respect of each £1 of his average
“ purchases. No question arises on this policy. It occurred,
“ however, to the managers that it might be advantageous to
“ drop this policy and to arrange for equivalent benefits to be
“ given to members out of the profits of the company, and on
“ 31 October 1911, new rules were passed by adding to Rule 1
“ the words : ‘ and to carry on the business of insurance as
“ provided in rule 14 (a) ’ and by adding to rule 14 (b) after
“ reserve fund, the words : ‘ to an insurance fund, by such
“ ‘ appropriation as the general meeting may from time to time
“ ‘ determine.’ Rule 14 (a) was a new rule headed ‘ Insurance
“ Fund.’ On 8 June it was resolved that the benefits of the
“ collective assurance scheme be extended so as to provide for
“ the payment of 2s. in the £1 on purchases on the death of
“ married women members or the wives of members. All these
“ new rules have been registered under the Act.

“ The defendant society is not subject to the statutory
“ restrictions imposed by the Companies Acts ; the members
“ can apply both income and capital to any purpose which they
“ think fit. A person desiring to become a member applies to
“ the committee, pays £1 for a share, and receives a membership
“ card, in which his or her name is inserted, and a registered
“ number together with a member’s purchase book, in which the

“ amount of purchases is entered. There is no other document
 “ of any kind except so far as the rules may be considered a
 “ document.

“ These being the admitted facts, I think there is no carrying
 “ on life assurance business within the meaning of the Act. No
 “ policy has been issued, no premium has been paid, there is no
 “ obligation upon the society to appropriate any further sum to
 “ the insurance fund. It seems to me that it is an abuse of
 “ language to call the membership card a policy or an ‘instru-
 “ ment evidencing a contract.’ The arrangement seems to me
 “ to lack every element necessary to constitute carrying on the
 “ business of life assurance. It is nothing more than appropriation
 “ of income or possibly of capital, which may at any moment
 “ be terminated by the general meeting. In my opinion the
 “ decision of the Vice-Chancellor cannot be supported, and the
 “ appeal must be allowed. The Vice-Chancellor was apparently
 “ under the impression that the arrangement was intended to
 “ include non-members as well as members, but this was a
 “ mistake. It is not suggested by the plaintiff that anything
 “ has been done outside of, or beyond, rule 14 (a).”

The case of *Ziman v. Komata Reefs Gold Mining Company, Limited*, was briefly mentioned in *J.I.A.*,
 Courts (Emergency Powers) Act, 1914.
 Mortgagee in possession.
 vol. xlix, p. 169, but it is now possible to refer to
 it more fully. The case is reported 1915 L.R.,
 2 K.B. 163, and is concerned with the question as to whether
 the exception introduced into section 1 (1) (b) of the Courts
 (Emergency Powers) Act 1914 “ except by way of sale by a mort-
 gagee in possession ” refers only to a mortgagee in possession
 whose security is real property or also to a mortgagee in
 possession of personal property.

The facts are as follows: On 30 July 1913, the plaintiff
 signed a memorandum of deposit whereby in consideration of
 the defendant company advancing to him by way of loan £4,000,
 he deposited with the company debentures to the amount of
 £5,000 secured on personal estate, to be held by them as collateral
 security for the repayment of the loan; he authorised them to
 sell the security if the loan and interest were not paid at maturity,
 and to call in the loan on or after 30 January 1914, by giving sixty
 days’ notice.

The debentures were forthwith transferred to and registered in the names of two directors of the company. They received the interest on the debentures and applied it towards payment of interest on the loan; but the interest on the loan was not paid in full, and on 14 November 1914 they gave notice to call in the loan, and that they intended to sell the debentures. The plaintiff obtained an order *ex parte* in chambers under section 1, sub-section 1 (b) of the Courts (Emergency Powers) Act 1914, restraining the defendants from disposing of the debentures until after the hearing of a summons dealing with the question. On the hearing of the summons, Lord Coleridge, J., refused to continue the *ex parte* injunction. The plaintiff appealed and the Court of Appeal (Lord Cozens Hardy, M.R., Phillimore, L.J., and Joyce, J.) unanimously dismissed the appeal.

Lord Cozens Hardy, M.R., said: "This is an appeal from an order of Lord Coleridge, J., in chambers, by which he refused to continue an *ex parte* injunction which had been granted by him. It raises a point under the Act of 4 and 5 Geo. 5 c. 78, the short title of which is the Courts (Emergency Powers) Act, 1914. The question is whether a sale which, according to the terms of an instrument which I am about to read, is clearly authorised, can be effected by the defendants, or whether such a sale is liable to be stayed under the provisions of the statute. The question turns upon the meaning of a few words in the Act. The Act provides that: '1 (1) From and after the passing of this Act'—which was August 31 1914—'no person shall (a) proceed to execution on, or otherwise to the enforcement of, any judgment or order of any Court (whether entered or made before or after the passing of this Act) for the payment or recovery of a sum of money to which this subsection applies, except after such application to such Court and such notice as may be provided for by rules or directions under this Act.' That does not apply to the present case; there is no order of the Court involved in the present case. Next we have these words: 'No person shall' sub-section 1 (b) 'levy any distress, take, resume, or enter into possession of any property, exercise any right of re-entry, foreclose, realize any security'—and then come these words—' (except by way of sale by a mortgagee in possession.)' Now what is this security? Is it a mortgage? That is not disputed. Are the defendants mortgagees in possession? That again is not and cannot be disputed according to

“the ordinary meaning of terms as understood in this Court
“to be applicable to English law. It seems to me that it is
“impossible to escape from the conclusion that the defendants
“here fall within the description of mortgagees in possession,
“unless we are to twist the construction of these words by reason
“of something else in the Act. It is said that this only applies
“to a mortgagee in possession of real estate. Why? It is impossible
“to suggest any reason for such an exception. Mortgagees in
“possession of the personal property are perfectly common;
“enormous sums of money are advanced in the City by banks
“and other persons who are mortgagees in possession of
“debentures, or shares, or other *choses in action*. Why are they
“not within the words? I confess I cannot see any reason unless
“it is this: it is said that, although ‘mortgagee in possession’
“according to the English law, is a perfectly general term,
“yet you find sufficient in the section which applies the Act to
“Scotland to give it a narrower meaning. . . We are asked to
“say that because in Scotland the exception only applies to a
“creditor in a hereditary security, it ought to be limited in
“this country to a mortgagee of real estate. That seems to me
“to be quite an impossible construction. It does not shock me
“at all to find that there may be differences in the application
“of this Act to Scotland and to England. At all events, I think
“it is our duty, in administering the law in England and construing
“this Act, to give the term that which is its undisputed
“meaning in this Court. Then it is said as a second point, that
“the expression ‘mortgagee in possession’ ought to apply,
“not to a mortgagee who has taken possession as part of the
“original transaction long before the war began, but only to a
“case where possession has been obtained adversely. Why?
“I never heard of two classes of mortgagees in possession:
“one who takes possession adversely by giving a notice to the
“tenants or otherwise, in spite of anything the mortgagor can
“do, and the other a mortgagee who, as part of the original
“transaction, has entered into possession. I think that the
“judgment of Lord Coleridge, J., was quite right, and that this
“appeal fails and must be dismissed with costs.”

Phillimore, L.J., in a concurring judgment said. . . “It is
“said this only applies to a mortgagee who has taken possession
“adversely. The statute does not say a mortgagee who has
“taken possession; but says a mortgagee who is in possession.
“Every mortgagee takes possession to get into possession.

“ Every mortgagee takes it by reason of some gift or grant of the mortgagor. It is part of the transaction of the mortgage which entitles him to do it, whether he does it *ab initio* or where he does it later ; in either case it is equally given to him and it is equally taken by him. Whether it is given or whether it is taken, or whether it is both, what the statute speaks of is a mortgagee *de facto* or *de jure* in possession, however it comes to be. He will have to account for the proceeds of the sale, but he is allowed to sell ; therefore I agree that this appeal must be dismissed.”

ACTUARIAL NOTE.

Practical Formulæ for the Value of a Loan repayable by a Cumulative Sinking Fund operating at every pth interest date.

A loan of 1 bearing interest at the rate of g per period, payable p thly, is repayable by a cumulative sinking fund operating at the end of each period and repaying the entire loan in n periods. What is the present value of the loan to yield the rate of j per period, convertible p thly ?

In a previous Note [*J.I.A.*, vol. xlv, p. 401] Mr. Lidstone gave two exact formulæ (in a different notation) for the present value required.* He also pointed out

that except in extreme cases a good approximation is $\frac{a_{\overline{pn}|j}^{jp}}{a_{\overline{pn}|g}^{gp}}$,

which is the value of the loan when the *equivalent* sinking fund operates at each *interest date* over the same number of periods. It was, however, remarked that the error involved

in the approximate formula $\frac{a_{\overline{pn}|j}^{jp}}{a_{\overline{pn}|g}^{gp}}$ increases as n decreases ; and

it appeared from the table given that the error becomes too large to be neglected when n is small. This approximation may be called (α).

It has since been found that a much closer approximation is given by the formula

$$\left. \begin{aligned} & \frac{a_{\overline{pn}|j}^{jp}}{a_{\overline{pn}|g}^{gp}} \left(1 - \frac{p-1}{2p} j \right) + \frac{p-1}{2p} g, \\ & \frac{a_{\overline{2n}|j}^{j2}}{a_{\overline{2n}|g}^{g2}} \left(1 - \frac{j}{4} \right) + \frac{g}{4} \end{aligned} \right\} \dots \dots (\beta)$$

which becomes

* Two alternative formulæ by Mr. Herbert Petter and Mr. S. E. Macnaghten were given in the *Journal*, vol. xlvii, pp. 94-97, where the notation is the same as that used in this Note.

when p equals 2, which includes the common case when the sinking fund operates yearly and the interest is payable half-yearly. It will be found that this very simple formula gives the value correct to within a few pence per £100 of loan for practically all cases that are likely to occur in practice. In view of the great simplicity of this approximate formula, and the very small error involved, it is very convenient, and amply accurate enough for most practical work.

The new approximate formula was obtained as follows: Let A' be the value of the loan and K' the value of the capital, when the sinking fund is applicable p thly, with the interest. Then

$$A' = 1 + \frac{g-j}{j} (1 - K') = \frac{g}{j} - \frac{g-j}{j} K'$$

whence
$$\frac{g-j}{j} K' = \left(\frac{g}{j} - A' \right)$$

If A and K be the corresponding values when the sinking fund is applicable at the end of each *period*, *i.e.*, at every p th interest date, we have, similarly,

$$\begin{aligned} A &= \frac{g}{j} - \frac{g-j}{j} K = \frac{g}{j} - \frac{g-j}{j} K' (1 - \kappa) \text{ say} \\ &= A' + \kappa \left(\frac{g}{j} - A' \right) \end{aligned}$$

By means of approximate expansions it is found that

$$\frac{K}{K'} = 1 - \frac{p-1}{2p} j + \{\text{terms involving } n \text{ and its powers, } i, j, \text{ and their powers and products}\}$$

Neglecting terms after the second, we have, approximately,

$$\kappa = \frac{p-1}{2p} j,$$

$$\begin{aligned} \text{and } A &= A' + \frac{p-1}{2p} j \left(\frac{g}{j} - A' \right) = A' \left(1 - \frac{p-1}{2p} j \right) + \frac{p-1}{2p} g \\ &= \frac{a_{\overline{pn}}^{jp}}{a_{\overline{pn}}^{gp}} \left(1 - \frac{p-1}{2p} j \right) + \frac{p-1}{2p} g \end{aligned}$$

The consistent accuracy of the approximation, whatever the value of n , appears to be due to the fact that when n is small the neglected terms in the approximation to κ are

insignificant; and as n increases and these neglected terms become more important, the value of $\left(\frac{g}{j} - A'\right)$, into which they are multiplied, rapidly diminishes; and it thus happens that, whatever the term n , the product (neglected terms) $\left(\frac{g}{j} - A'\right)$ is very small, *i.e.*, the error is very small.

The formula may also be obtained by consideration of the difference between the incidence of the repayments in the two cases. The p repayments made in any period, when the sinking fund is applicable p thly, operate on the average $\frac{p-1}{2p}$ ths of a period earlier than the single repayment made at the end of the period by the sinking-fund applicable at every p th interest date. Consequently a closer approximation than A' might be expected to result from postponing all the p thly repayments $\frac{p-1}{2p}$ ths of a period. If A'' be the value of the loan when the repayments are postponed $\frac{1}{p}$ th of a period, then by equating values at the end of the first $\frac{1}{p}$ th of a period, $A''\left(1 + \frac{j}{p}\right) = \frac{g}{p} + A'$, whence $A'' = A'\left(1 - \frac{j}{p}\right) + \frac{g}{p}$ approximately. Hence, by simple proportion, the value of the loan when the repayments are postponed $\frac{p-1}{2p}$ ths of a period will be $A'\left(1 - \frac{p-1}{2p} j\right) + \frac{p-1}{2p} g$ approximately.

Another very simple approximation is given by the formula

$$\frac{\alpha_n^j}{\alpha_n^g} \quad . \quad . \quad . \quad . \quad (\gamma)$$

This shows no error for $n=0$ and $n=\infty$, and between these extremes the error increases with n up to a maximum and then decreases for higher values of n . For the great majority of cases occurring in practice the error is less than that occurring in (α) and greater than that occurring in (β) . A further and very accurate approximation may be obtained from (γ) as follows. When $g=j$ the error in (γ) is 0, and when $g=0$, the error is $\frac{\alpha_n^i}{n} - \frac{\alpha_n^j}{n}$ (where i is the effective rate equivalent to j convertible p thly), since in the latter case

each sinking-fund payment is $\frac{1}{n}$ and the true value is $\frac{a^i}{n}$.

Hence, by simple interpolation, which will be sufficient because we are dealing with a small correction, the error in the general

case is $\frac{j-g}{nj} (a_n^i - a_n^j)$, or $\frac{j-g}{n} \cdot \frac{i-j}{j} \cdot \frac{d}{dj} a_n^i$ approximately.

The exact value of the differential coeff. is $-(a_n - nv^{n+1})/j$, but it will be sufficiently accurate to use $-(a^{j-\theta} - a^{j+\theta}) \div 2\theta$.

Putting $\theta = \frac{j-g}{2}$, the divisor 2θ will cancel the factor $(j-g)$,

and we shall have for the further approximation

$$\frac{a_n^j}{a_n^g} + \frac{i-j}{nj} (a_n^{(3j-g)/2} - a_n^{(j+g)/2})$$

Since $\frac{i-j}{j} = \frac{p-1}{2p} j$ nearly, we may write this in the form

$$\left. \frac{a_n^j}{a_n^g} + \frac{p-1}{2pn} j (a_n^{(3j-g)/2} - a_n^{(j+g)/2}) \right\} \dots \dots \dots (\delta)$$

$$\text{or when } p=2, \left. \frac{a_n^j}{a_n^g} + \frac{j}{4n} (a_n^{(3j-g)/2} - a_n^{(j+g)/2}) \right\}$$

This will be found to give values differing from the true values only slightly in the third place of decimals per 100 of loan.

A somewhat rougher value of $(j-g) \frac{d}{dj} a_n^i$ is $a^j - a^g$, and using this we have

$$\left. \begin{aligned} &\frac{a_n^j}{a_n^g} + \frac{p-1}{2pn} j (a_n^i - a_n^g) \\ \text{or, when } p=2, &\frac{a_n^j}{a_n^g} + \frac{j}{4n} (a_n^i - a_n^g) \end{aligned} \right\} \dots \dots \dots (\epsilon)$$

This simple formula involves the annuity-values for rates j and g only and gives a very good approximation.

The following table of exact and approximate values includes the cases given *J.I.A.*, vol. xlvi, p. 405, and some additional cases.

In view of the various discussions of the problem and the different notations which have been used it has been thought convenient to collect the formulæ, reduced to a consistent notation, in the appended Synopsis.

G. J. L.
R. T.

Table of Exact and Approximate Values.

		APPROXIMATE VALUES								
<i>n</i>	<i>p</i>	True Value	Approx. β		Approx. γ		Approx. δ		Approx. ϵ	
			Error	Error	Error	Error	Error	Error		
			<i>g</i> = .050				<i>j</i> = .055			
48	2	92.848	92.836	-.012	92.881	+.033	92.846	-.002	92.844	-.004
48	4	92.831	92.814	.017	92.881	.050	92.828	.003	92.825	.006
36	2	93.854	93.842	.012	93.891	.037	93.855	+.001	93.852	.002
24	2	95.273	95.265	.008	95.312	.039	95.276	.003	95.275	+.002
24	4	95.254	95.242	.012	95.312	.058	95.258	.004	95.256	.002
12	2	97.210	97.205	.005	97.239	.029	97.211	.001	97.211	.001
6	2	98.402	98.399	.003	98.421	.019	98.403	.001	98.402	.000
6	4	98.393	98.388	.005	98.421	.028	98.393	.000	98.393	.000
			<i>g</i> = .045				<i>j</i> = .055			
48	2	85.881	85.860	-.021	85.947	+.066	85.877	-.004	85.868	-.013
48	4	85.847	85.816	.031	85.947	.100	85.842	.005	85.829	.018
36	2	87.867	87.848	.019	87.943	.076	87.870	+.003	87.862	.005
24	2	90.653	90.639	.014	90.730	.077	90.658	.005	90.653	.000
24	4	90.614	90.594	.020	90.730	.116	90.622	.008	90.614	.000
12	2	94.458	94.450	.008	94.516	.058	94.461	.003	94.459	+.001
6	2	96.816	96.809	.007	96.853	.037	96.816	.000	96.815	-.001
6	4	96.796	96.788	.008	96.853	.057	96.798	.002	96.797	+.001
			<i>g</i> = .050				<i>j</i> = .045			
48	2	108.105	108.120	+.015	108.068	-.037	108.104	-.001	108.102	-.003
48	4	108.124	108.146	.022	108.068	.056	108.123	.001	108.120	.004
36	2	106.804	106.816	.012	106.764	.040	106.801	.003	106.799	.005
24	2	105.087	105.096	.009	105.050	.037	105.084	.003	105.083	.004
24	4	105.106	105.119	.013	105.050	.056	105.101	.005	105.099	.007
12	2	102.907	102.912	.005	102.881	.026	102.905	.002	102.905	.002
6	2	101.635	101.639	.004	101.619	.016	101.635	.000	101.635	.000
6	4	101.644	101.648	.004	101.619	.025	101.643	.001	101.641	.003
			<i>g</i> = .055				<i>j</i> = .045			
48	2	116.425	116.458	+.033	116.351	-.074	116.424	-.001	116.416	-.009
48	4	116.463	116.512	.049	116.351	.112	116.461	.002	116.448	.015
36	2	113.790	113.819	.029	113.710	.080	113.784	.006	113.777	.013
24	2	110.293	110.314	.021	110.218	.075	110.285	.008	110.281	.012
24	4	110.331	110.361	.030	110.218	.113	110.321	.010	110.313	.018
12	2	105.855	105.865	.010	105.802	.053	105.851	.004	105.849	.006
6	2	103.282	103.289	.007	103.250	.032	103.281	.001	103.280	.002
6	4	103.299	103.307	.008	103.250	.049	103.297	.002	103.296	.003

for the value of a loan of 1, bearing interest at rate g per period payable at j thly intervals and repayable in n periods by a Cumulative Sinking Fund of z ($=1/s_n$, at rate g) operating at the end of each period, to yield the nominal rate j convertible j thly, equivalent to the effective rate $i = \left(1 + \frac{1}{j}\right)^j - 1$ per period.

Reference to J.I.A.	Formula	General case, $p = p$	Particular case, $p = 2$
xlvi, 403, (4)	Exact	$\frac{a_{\overline{pm} i}^{j/p}}{pa_n^u} + \frac{(1+j/p)^p - (1+j)}{(1+j/p)^p - (1+g)} \left(\frac{g}{j} - \frac{a_{\overline{pm} i}^{j/p}}{pa_n^u} \right)$	$a_{2n }^{j/2} + \frac{(1+j/2)^2 - (1+j)}{(1+j/2)^2 - (1+g)} \left(\frac{g}{j} - \frac{a_{2n }^{j/2}}{2a_n^u} \right)$
xlvi, 404, (8)	Exact	$1 + (g-j) \frac{s_{\overline{p} i}^{j/p} - \frac{a_{\overline{pm} i}^{j/p}}{p}}{(1+j/p)^p - (1+g)}$	$1 + (g-j) \frac{s_{\overline{2} i}^{j/2} - \frac{a_{2n }^{j/2}}{2}}{(1+j/2)^2 - (1+g)}$
xlvi, 96/7	Exact	$\frac{g}{j} + \left(1 - \frac{g}{j}\right) \frac{z - v^{pm}(z+g)}{(1+j/p)^p - (1+g)}^*$	$\frac{g}{j} + \left(1 - \frac{g}{j}\right) \frac{z - v^{2n}(z+g)}{(1+j/2)^2 - (1+g)}^*$
xlvi, 96/7	Exact	$\frac{a_{\overline{pm} i}^j}{a_n^u} + g \frac{s_{\overline{p} i}^{(p)} - 1}{i - g} \left(1 - \frac{a_{\overline{pm} i}^j}{a_n^u}\right)$	$\frac{1}{2(1+j/4)} \cdot \frac{a_{2n }^{j/2}}{a_n^u} + \frac{1}{4} \frac{gj}{(1+j/2)^2 - (1+g)} \left[1 - \frac{1}{2(1+j/4)} \cdot \frac{a_{2n }^{j/2}}{a_n^u}\right]$
xlix, 290	Approx. (3)	$\left(\frac{a_{\overline{pm} i}^{j/p} + a_{\overline{pm} i}^{u/p}}{pm} \right) \left(1 - \frac{p-1}{2p} j\right) + \frac{p-1}{2p} g$	$(a_{2n }^{j/2} + a_{2n }^{u/2}) (1 - j/4) + g/4$
xlix, 292	Approx. (7)	$a_{\overline{pm} i}^j + a_{\overline{pm} i}^u$	$a_n^j + a_n^u$
xlix, 293	Approx. (8)	$\frac{a_{\overline{pm} i}^j}{a_n^u} + \frac{p-1}{2pm} j \left(a_{\overline{pm} i}^{(3j-g)/2} - a_{\overline{pm} i}^{(j+g)/2} \right)$	$\frac{a_n^j}{a_n^u} + \frac{j}{4n} (a_n^{(3j-g)/2} - a_n^{(j+g)/2})$
xlix, 293	Approx. (e)	$\frac{a_{\overline{pm} i}^j}{a_n^u} + \frac{p-1}{2pm} j \left(a_{\overline{pm} i}^j - a_n^u \right)$	$\frac{a_n^j}{a_n^u} + \frac{j}{4n} (a_n^j - a_n^u)$

* v^{pm} at rate j/p , v^{2n} at rate $j/2$.

Obituary.

EDGAR CORBLE, Associate of the Institute, Private, 1st Battalion London Rifle Brigade.

Killed in Action 6 January 1915.

FREDERICK CHRISTMAS MANN, Associate of the Institute, 2nd Lieutenant, 1st Battalion North Staffordshire Regiment.

Killed in Action 12 March 1915.

CHRISTOPHER JONATHAN ELLIOTT, Probationer of the Institute, Private, 1st Battalion London Rifle Brigade.

Died of Wounds received in Action about 26 April 1915

GEORGE HENRY POLLOCK, Probationer of the Institute, Lieutenant, 4th Battalion South Staffordshire Regiment (attached to 1st Battalion Royal Warwickshire Regiment).

Killed while on Patrol Duty 18 June 1915.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

*The New National Life Tables.** By GEORGE KING, F.I.A., F.F.A.,
Consulting Actuary.

[Submitted to the Institute, 26 April 1915.]

THE Council invited me to open at a Sessional Meeting a discussion on the new National Life Tables, and I was glad of the opportunity thus given to set forth in a little more detail than heretofore the objects which had to be kept in view in preparing the tables, and the bearing of these objects on selecting a method of construction. At first I had intended to open the discussion by an unwritten speech, but on looking at the subject from this point of view it was found that a mere speech would hardly meet the necessities of the case, and that it would be better to write an informal paper. This I have now the honour to submit.

Conversations with friends showed that misconceptions on some important points prevailed, mainly through the magnitude of the problem to be solved not having been realized, some of these misconceptions being reflected in the two reviews which have appeared in the *Journal*. An opportunity has therefore arisen to deal with these misconceptions, and if in doing so I seem to enter into controversy with the two reviewers, I hope to be forgiven. It is not that I have any cause of complaint

* *References :*

Census of England and Wales, 1911, vol. vii (Cd. 6610), p. xxxix. Report on the Graduation of Ages. By GEORGE KING.

Supplement to the 75th Annual Report of the Registrar-General of Births, Deaths, and Marriages in England and Wales. Part I, Life Tables (Cd. 7512).

First Review, *J.I.A.*, vol. xlviii, p. 207.

Second Review, *J.I.A.*, vol. xlix, p. 96.

against them, but merely that on matters of principle I differ in some respects from them, and that it will be useful to set out the reasons for that difference.

I had the honour of being retained by the Registrar General on two occasions in connection with the Census of 1911. He first invited me to graduate for England and Wales as a whole the populations as enumerated at each year of age, and, also, similarly to graduate the populations for various separate sections; and when that work was finished he then invited me to construct the new life tables which it has been customary to issue from Somerset House after each decennial Census. Thus, I sent in two reports, and these reports have been reviewed separately in the *Journal*, apparently by different writers, who may be called, respectively, the First Reviewer and the Second Reviewer.

It is convenient to take up first the graduation of the enumerated populations, called in the official report the "Graduation of Ages." Two main tables had to be prepared for England and Wales as a whole, one for males, and the other for females, and each of these had to be supplemented by tables for unmarried, married, and widowed, respectively, there being thus eight separate tables for England and Wales as a whole. These are the main tables dealt with.

The enumerated populations presented a rough appearance, that roughness being due to various causes. First of all there must be inherent irregularities in enumerated populations, due to past changes in the birth rate, the death rate, and the migration rate. The object of a graduation of such data is not to produce a table conforming to any particular ideal. The table is not intended to forecast the future, but merely to give accurately the present populations. Thus it is analogous to a table of Exposed to Risk in a life office experience, and differs from an ordinary mortality table, which is intended to measure probabilities of future events, the probabilities of living and of dying. What is required is a table showing the exact population in each year of age, existing on a given day, which would have been enumerated on that day had every individual in the population been included, and had he stated his true age. These inherent irregularities should not be removed by graduation. For instance, if in any particular year, say twenty years ago, there had been an unusual number of births, then at age 20 at the Census there should be an excess in the number enumerated as

compared with the adjacent ages below and above. The excrescence so produced should remain. Again, suppose that, say twenty years ago, there had been an unusual amount of emigration of young folk, aged, say, from 18 to 23. That would produce a depression at present ages 38 to 43, and that depression should not be removed. It is impossible to find a method of graduation which will not remove the irregularities caused by variations for one year only in the birth rate, and unfortunately such irregularities are unavoidably removed. But a good method of graduation should leave the depressions or elevations due to past changes in the migration rate and in the death rate, even if these changes had been for one calendar year only, because such changes do not affect only one year of age, but at least several years of age, and are spread over a certain length of the population curve, and a good method of graduation will retain waves in the curves of populations arising from these causes.

Then there are irregularities not of the kind which are called here "inherent", and these are very fully discussed in my first report, paragraphs 28 to the end. It is rather curious that the reviewers do not appear to have mentioned these paragraphs. They seem to me very important. Attention is called to accidental errors which any good method of graduation will smooth away. Then attention is called to deliberate errors due to mis-statements of age. These may be divided into two kinds, the minor and the major, and in my report I show that those which are minor can be cleared away, and that a good graduation does clear them away. The major deliberate errors, however, we cannot with our present knowledge correct. At any rate, I am not aware of any method of graduation which will reach them, and I do not think that the reviewers will maintain that the National Health method they both advocate has any useful effect upon them. Moreover, not knowing at what points they occur, or to what extent, we have no guide to assist in adjusting the data preparatory to graduation. These major deliberate errors, however, I personally do not consider to possess great weight except at the old ages, and in each of my reports I have said so. The Second Reviewer alleges that the supposed tendency among women to under-state their age at certain periods of life is not even mentioned by me, but he has failed to notice that in paragraph 30 I say, "for instance, a person aged 40 may purposely give the age of 30." I had women in my mind in writing these words, and evidently it would have pleased the reviewer better had I

said "woman" instead of "person," and had I used for the illustration ages ten years younger. Lower down I shall put forward reasons for the opinion that these deliberate errors, except at the old ages, do not possess great weight, and here I may explain that I have no preconceived theory to maintain on this point. When I entered on the investigation I expected to find very strong marks of these major deliberate errors, and it was only a careful examination of the figures that led me to the conclusion that they are unimportant from the practical point of view.

A method of graduation had to be adopted which would smooth away accidental errors and minor deliberate errors, the chief of which is a remarkable selection of particular digits of age, but which would not remove more than could be helped the inherent irregularities spoken of above. Under my instructions it was also necessary that the graduation should reproduce the total populations exactly, and it was evidently also desirable that the method should be such that when applied separately to each of the sections of the population which make up the whole, the sum of the populations of the several sections at each year of age should be identical with the corresponding total population.

Any good summation formula of graduation would produce this last result, but it was found that summation formulas would not sufficiently clear away the minor deliberate errors, and that there would be humps in the curves at the places where digits of age 0 and 8 occur, these digits coming close together. Moreover, it was to me self-evident that any method of graduation which involved a definite law was unsuitable, because it would remove all trace of the inherent irregularities spoken of above. I had some years ago given a great deal of time and work to devising a method of constructing life tables from Census Returns and Records of Deaths with a humble hope that the results would be useful for the Census of 1911, but not even in my dreams had I imagined that the honour would fall on me to carry through the work. The method evolved had not been intended specially for such a graduation as that of enumerated populations, but naturally when other methods failed it was tried, and I venture to submit that for the purposes in view as set forth above it is suitable. The errors to be removed were more or less of a cyclical character, decennial, and, to a lesser extent, quinquennial; and a method had to be adopted of a quinquennial character to

counteract these cyclical errors. This I submit is one of the features of the method I deduced, and which the reviewers have called the Census Method. It is not, however, final, and if anyone can devise a better, none will be more pleased than myself. In both my reports—paragraph 23 of the first, and paragraph 6 of the second—it is explained that the fullest details are given so that other investigators may pursue their enquiries, and they are directly invited to do so.

The errors in the recorded ages due to selection of digits of age, and especially the great favour shown for digit 8 brought out now for the first time, are very interesting, but perhaps not of any great practical importance. The First Reviewer has tried to estimate the effect of this selection, but I venture to think that his method is incomplete, and that it confuses two causes of apparent excessive enumerations at particular digits of age. He takes for males and for females the recorded populations from age 10 to age 99 inclusive, and for each digit of age he brings together the populations recorded at that digit. Starting with digit 0 he adds the populations at ages 10, 20, &c., up to 90; for digit 1, the populations at ages 11, 21, &c., up to 91, and so on, until for digit of age 9 we have the populations for ages 19, 29, &c., up to 99. He does not, however, mention that each of the digits of age 0 is nine years higher up the table than the corresponding digit 9, and that, therefore, on account of deaths and emigration there must be a great excess of population at the digit of age 0 over that at the digit of age 9, even were there no special selection of any particular digit at the enumeration. The result is, that while his table and his diagram include the excess at digit of age 0 due to special selection, they also include the excess due to lesser age, and in his diagram the lines at digit 0 are tilted very much too high as compared with digit 9. If his method were complete, it would not matter at what digit the count commenced. I have taken the count starting at age 9, and running to age 98, each one of my numbers in the series from the first to the last being for one year younger than the corresponding number of the reviewer. This would make a small difference, hardly appreciable, if the method were complete, but it has a very marked effect on the table and on the diagram. In Table I annexed there are given for males, in the first two columns the reviewer's figures, and in the next two columns the figures due to moving all the ages one year up, and in Diagram 1 these figures are translated into lines.

The dotted line represents the table as calculated by the reviewer, and the continuous line the similar table starting from age 9; while the broken line represents the figures on the reviewer's method starting from age 10 taken from the Text Book table. The Text Book table is as smooth a table as can be made, and there is no selection whatever of digits of age, and yet it will be seen that by the reviewer's method there is a very considerable selection of digit 0 over 9, and a gradually decreasing selection of all the other digits. As to the enumerated populations, the reviewer's method, starting from age 9, gives lines following very much the same form as when the start is made from age 10, except that at digit 8 the positions are reversed, and that at other points the preference for certain digits is understated. It is admitted that digit 8 is a favourite, and yet from the second edition of the reviewer's method it would seem as if 9 were preferred. That is because in this second edition in every case digit 9 is an age nine years younger than digit 8, and the effects of deaths, &c., are much greater than those of the selection of the digit.

Table II for males of my first report, and Table V for females supply a much better method of estimating the effect of the selection of age digits. In these tables the deviation for a population of 100,000 at each age is given between the enumerated and the graduated populations. It may be assumed for present purposes that the graduation gives correct populations, and that the deviations shown are real. This assumption seems legitimate, because any reasonably good graduation would give deviations very similar, although not quite the same, at the different points of age. Table II annexed, based upon the deviations for 100,000 at each age, is a summary of Tables II and V of the report, and shows for 100,000 graduated population at each age what would be the total deviation at each digit of age, and what the corresponding enumerated population; and in an adjacent column, following the example of the First Reviewer, the ratio of these enumerated populations at each digit of age is taken to digit 9. In Diagram 2 the ratios are set out for both males and females, the upper set of two lines being a reproduction of the reviewer's diagram, and the lower set being the approximately correct lines showing the actual effect of digit selection. The two sets of lines are of the same form, thus showing that the reviewer's method does include correctly the effects of digit selection, but the reviewer's lines are tilted up on the left, digit

of age 9 being the pivot, and the amount of that tilt shows the disturbance introduced through failing to take account of deaths and migrations.

It is perhaps worth while to point out that in the first report tables corresponding to II and V are given also for the sample populations both in 1901 and 1911, and that in paragraphs 31 to 41 of the report the question of mis-statements of age is discussed at length. The reviewer has not mentioned this, but I think it is of importance, and I think that a further study of the sample population figures might result in useful conclusions. For the Census of 1901 the populations in certain selected districts were published for each year of age, and the same districts were again segregated, and the populations similarly given for the Census of 1911. This was intended by the authorities to supply means of investigation. It will be seen from my report that if the methods there adopted are legitimate, there is reason to think that an improvement took place at the Census of 1911 over that of 1901 in the statements of age, and that females seem to be more accurate in this respect than males.

A more important matter is that of the major deliberate errors, which is dealt with at considerable length by both reviewers. They share the opinion, which is very general, that these deliberate errors, especially in the case of females, are of serious importance, and vitiate to a large extent deductions which may be derived from the Census figures. I agree with them in so far as advanced ages, from about 85 onwards, are concerned, but my investigations have led me to think that at younger ages they have practically very little weight. The reviewers speak more particularly of young folk, and they suggest that the apparent excessive populations recorded at somewhere about ages 20 to 25 are due to under-statements of age, and that a considerable number of the enumerated at these young ages should be transferred to older ages, say about age 30 onwards. For present purposes the exact ages involved are not of any consequence. Another cause that might have led to the comparatively excessive populations at the young adult ages is immigration, but that seems to be rejected by both reviewers. When the populations are used to estimate the rate of mortality, then, if there be to any great extent the under-statements of age at the Census in favour of these young adult ages, the rate of mortality brought out at these ages should be low, because there would be too many living in the denominator of the mortality

fraction, and as a consequence at older ages in the table the rate of mortality should be comparatively high, because of the undue reduction in the enumerated numbers living. This would be so unless in the case of the deaths there were equivalent under-statements of age. When constructing the Life Tables I had this point in mind, and by means of ungraduated central death rates investigated it sufficiently to convince me that it was unimportant. For the purpose of this paper I have gone into the matter more fully. For males and females, respectively, in England and Wales we have the populations recorded at each age at the Census, and we have also the deaths registered in the three years 1910-12. From these data we can obtain the central death rate m_x for each age, and that for ages 15 to 74 is given in Table III annexed under the heading " m_x Rough." The sequence of values runs remarkably smoothly considering the irregularity of the material, but it is desirable to effect graduation, and it is necessary to select a formula that can have no bias, but will give smooth results without any cyclical tendency, or any kind of selection. For this purpose formula C in my paper on Summation formulas of Graduation, *J.I.A.*, xli, 531, is suitable, because it is a formula of very considerable graduating power, better than Woolhouse's, and yet very easily applied. The results are included in Table III under the heading " m_x Graduated," and both the rough and the graduated values are set out in Diagram 3. The upper pair of lines in the diagram relate to males, and the lower to females, the rate of mortality among males being somewhat higher than among females. It will be seen that the graduated curve runs very accurately through the ungraduated lines, and is very smooth, and what is more important, it shows no wave-like bends. The ungraduated lines bring out distinctly the preference at the Census for certain digits of age. For instance, at ages 30, 40, and 50, the ungraduated value of m_x is considerably lower than at the adjacent ages on each side. That is because of the excess in the enumerated populations at these ages which is not compensated for by a corresponding excess in the recorded deaths, from which it would appear that the ages at death are given much more accurately than the ages at the Census enumeration. It has been pointed out above that if the theory of the reviewers were correct, and if there were to any considerable extent under-statements of age of persons really aged 30 to 35, the central death rate at ages about 20 to 25 should show a depression, while there should be a corresponding elevation at

ages 30 to 35. No such features appear in the graduated curve, and this seems to me to be conclusive evidence that the alleged under-statement of age at this period of life does not exist to any great extent, and that the apparent excess in the populations enumerated at the younger ages must be due to some other cause, and the only other cause that occurs to me is migration.

We have no means of estimating the amount of immigration into England and Wales from Scotland, Ireland, and abroad, and we can only surmise from the data before us that such immigration must take place. We can imagine, however, how it may be produced. In the case of females, for instance, we may have a considerable number of young women coming from the Continent as governesses, and returning again to the Continent when the work for the families they have visited has ceased, and there may be a similar cause, such as becoming waiters, &c., among the males. Or again, it has been the custom for many years for parents to send their children, boys and girls, to school on the Continent to be educated, and these children during school age fail to be enumerated at the Censuses. Their school days over, they return home, and at early adult ages they are enumerated. Hence at early adult ages there may be more persons enumerated at one Census than might have been expected to survive from those enumerated at school ages at the last preceding Census.

We can, however, estimate the migrations from different sections of the country to other sections by means of the figures reproduced in Appendix I of my second report, for England and Wales as a whole, and for the sections, County of London, County Boroughs, Urban Districts, and Rural Districts. In Table IV annexed I give for ages 16 to 36, and for males and females separately, the graduated population for each of these sections, along with the total population for England and Wales. All the graduations have been effected by the Census method, and here we have an illustration of the great advantage of employing a method which when applied to sections will reproduce the total population. It will be seen that the proportionate numbers living at each age differ widely in the different sections. A diagram was prepared to illustrate this, but it was very difficult to get it in a form that would bring out the peculiarities clearly, and it was therefore thought better, where L_x represents the population living in the year of age x to $x+1$, to tabulate the ratio $L_{x+1} \div L_x$, this ratio showing the rate

of increase or decrease with increasing age in the population. This ratio is given in Table V annexed, and is also set forth in curve form by the continuous line in Diagram 4 for males, and in Diagram 5 for females, and the same ratio derived from the mortality tables, from which migrations are excluded, for the respective sections given in my second report is shown in the dotted line. These diagrams disclose how remarkably migrations affect the populations enumerated in the different sections. As regards the males, the total for England and Wales shows that from age 18 to age 25, the rate of fall age by age in the population is small and diminishing, but the populations never actually increase with age. In the County of London, however, there is the same feature accentuated, and the populations actually increase between ages 18 to 23, the ratio $l_{x+1} \div l_x$ being greater than unity. The curve for the County Boroughs is somewhat like that for the County of London, but the peculiarities are not so marked, while for the Urban Districts and Rural Districts the curves have rather an opposite character. Similar remarks apply to Diagram 5 for females, but there the peculiarities are more marked than in the case of males, and in the County of London at the young ages from 16 to 24 there is immense immigration of females.

These extraordinary bends in the curves must be due almost entirely to migrations, and cannot be due to any extent to mis-statements of age, and the resultant of the whole is given in the curves for the total of England and Wales. That the contortions of the curves are due to migrations and not to mis-statements of age is proved by the remarkable regularity of the dotted lines representing the ratios derived from the graduated mortality tables. If the contortions had been due to mis-statements of age there would have been corresponding contortions in the curves derived from the mortality tables, but the mortality tables do not seem to be in the slightest degree affected by the migrations, which means that the populations are real, and that the deaths recorded correspond to them. If this be so for sectional tables, it seems to me conclusive that it must also be true for the total of England and Wales. Notwithstanding the opinion which I thus express, an opinion formed not rashly and requiring defence, but from a careful study of all the figures, I invite further investigation. The question is far too important to be ignored, and I feel sure that young men with sufficient actuarial knowledge, and with a certain amount of scientific imagination,

and who do not fear hard work, might prepare useful papers to be read at our Sessional Meetings. The whole of the Census data, and, if I may say so, the whole of my two reports, bristle with subjects to be investigated, and which would give opportunities to our rising young actuaries to distinguish themselves.

The First Reviewer has applied the National Health method to graduate the enumerated populations for England and Wales Males, and in the Diagrams 2 and 3 of his review he gives the results, where the first differences of his table taken in quinquennial groups are set forth in curve form. The curve in his Diagram 2 certainly is smooth, and looks like a well developed cranium, but I would point out that it is not quite correct at age 29, the point at that age being placed too high, and if it were properly placed there would be an indentation in the curve which would rather spoil its outline. On the same diagram he sets forth the similar function for the ungraduated values, and for the values by the Census graduation. May I, however, mildly protest against the method he has adopted for displaying the Census graduation? The points on his own curve he has joined by curved lines, whereas the points on the Census curve he has joined by angular lines. The Census method points should have been joined also by curved lines if the comparison is to be fair. The function is derived from the population living not at a point of age, but in a year of age, and, therefore, we have not to deal with points to be joined by straight lines, but with points to be joined by curved lines. I think that the reviewer will admit that this little grumble is not unjustified.

A far more important question is, however, the method of testing the smoothness of a curve. To tabulate the first difference in quinquennial groups does not test the smoothness, but the nature, of the curve. A curve may be absolutely continuous, that is, it may pass from mathematical point to mathematical point (let me add, not from integral age to integral age) without the slightest break, and yet the first differences may run very strangely. I should like to see the reviewer set out in diagram form, as he has done for the Census figures, the figures that may be represented by a curve of the form of the letter S. That curve would be perfectly smooth, and yet the first differences would have a most extraordinary character. The regularity of the reviewer's first difference curve is due to the fact that he has removed all trace of the inherent irregularities which must

exist in a population curve, and which ought to be retained in a good graduation. He has produced a fancy article which does not represent the population as it would be seen really to exist were only accidental and minor deliberate errors removed. In fact, I should say that the National Health method of graduation is entirely out of place when applied to a population curve, however good it may be under suitable conditions for a mortality table, because it distorts the data, and removes inherent peculiarities. In Table VI I give the graduated populations from ages 15 to 90 for England and Wales males, and the first and second differences. The second differences show the changes in curvature of the curve. It is, I think, generally admitted that the Census method adheres very closely to the original data. It must also be admitted that it gives a smooth curve, that is, a curve that shows no break in passing from mathematical point to mathematical point, yet the second differences show very considerable variations in the curvature, and these are due to the inherent irregularities existing in the actual populations, and which the Census method does not obliterate. These inherent irregularities must not be removed as is done by the National Health method, but must remain. I do not say that the Census method is the best possible, but I do say that it is a good attempt to solve the problem, and I repeat that I should be delighted if it could be improved upon.

Before leaving this point I would ask the reviewer how he would apply the National Health method of graduation separately to the populations of the four sections, namely, the County of London, the County Boroughs, the Urban Districts, and the Rural Districts, where the contortions in the curves due to migration are much greater than in the curve for England and Wales as a whole. It has already been said that the method employed must be such that it can be applied independently to each of the several sections, but that is impossible with the National Health method.

Passing now to the more important branch of the subject, the construction of the Life Tables, it may be well for me to quote here the principles kept in view in selecting the method to be employed. These principles are set out in paragraph 4 of my second report as follows :

“ In constructing the tables it was desirable that a method “ should be employed, simple in theory, easy in application,

“and which would produce curves of smooth graduation, and “curves which would adhere closely to the original data.” It seems to me under all circumstances to be desirable to employ a method simple in theory, and easy in application; if that method be really effective, and not inferior in its results to others more complicated. The higher mathematics are invaluable to the Actuary, mainly as an instrument of research, but should be avoided in actual practice wherever possible without loss of efficiency. There were, however, other reasons why a method simple in theory and easy of application should be employed for the National Life Tables. Medical Officers of Health and others are possessed of a strong desire to construct life tables for their own particular districts, and it was the wish of the authorities to supply them with a method which they could use with facility. The Medical Officers are gentlemen of high attainments, but it cannot be expected of them that they should master complicated methods of life table construction which only skilled mathematicians can understand. My own method was devised very much with such considerations in mind, and it is gratifying to me that the authorities at Somerset House should have been pleased to sanction its adoption, and to arrange for a full demonstration of it to be given in the report. The Second Reviewer remarks that no labour is in vain which is directed to the ascertainment of a method of graduation that will command general assent as the best obtainable in all circumstances. He need not be afraid that any labour was spared in this respect, and in testing the Census method before it was finally applied. He can scarcely imagine the amount of work that was expended, and the abortive calculations that were made, before the exact plan to be followed was settled. The reviewer favours the National Health method, and his main objection to the Census method appears to be that it does nothing to correct what he calls “systematic errors”, and which I have called “major deliberate “errors”, and he also seems to hold that the population statistics should receive “proper treatment” in order to disperse these errors, before they are used for the purpose of constructing a table. On this point I would ask him, seeing that he favours the National Health method, does he maintain that it does effectively disperse the large systematic errors, and does he assert that in preparing the National Health Table the statistics received “proper treatment” before being employed? So far as I am aware the data received no “proper

treatment" for the National Health Table, and surely the National Health method makes no attempt to deal with large systematic errors. It only conceals them, but we do not want our National Life Tables to be whited sepulchres. It is granted that the Census method also does not disperse large systematic errors, but where they exist it does not conceal them, and in Paragraph 6 of my second report I refer specifically to this point, and add that "in constructing the present life tables no correction has been attempted to counteract the possible mis-statements of age. An unbiassed position has been assumed, and the effort has been to construct the tables absolutely in accordance with the facts as recorded, and to allow those who are interested in the subject to pursue further investigations, and to make such corrections as in their opinion may be thought necessary." These last words quoted are a reply in advance to the remark of the reviewer that the subject should still be regarded as open to discussion. He is directly invited to prepare tables with such corrections as he may think necessary, but certainly if in the present state of our knowledge I had made corrections I should have met with very severe criticism from many quarters, and from persons of every shade of opinion; and perhaps the Second Reviewer himself would not have spared me. At the end of paragraph 6 above quoted I added that it was not intended to cast doubt upon the accuracy of the tables as presented, but that I felt sure that as far as about age 85 they do really represent the mortality prevailing, while beyond that rather advanced age they may be accepted without much hesitation. The further investigations I have made for the purpose of this paper have more than confirmed me in this opinion, and I feel sure that the large systematic errors do not appreciably affect the tables until extreme old age. Already in what has gone before I have given good reasons for this opinion, and have shown that the waviness of the population curves is due to migrations much more than to mis-statements of age. I need now only call attention again to Table VI in connection with Table VII annexed. In Table VI are given the graduated populations for England and Wales, with the first and second differences, and in Table VII the corresponding differences of q_x in the final mortality table. It will be seen that while in Table VI the second difference displays very frequent changes of sign, thus showing frequent changes in the direction of curvature, yet Table VII shows only changes of sign in the

second difference at the youthful and early adult ages, which is also the case with many other tables, but after age 23 these changes in curvature no longer take place, and the table runs very regularly.

I quite admit that under suitable circumstances the National Health method may be an excellent one, and may be the proper method to adopt, but I submit that our knowledge at present is not sufficient to enable us to apply it with confidence to Census statistics, and especially is this so when a large number of tables have to be prepared which are closely co-related, and when it would not be possible to use the same constants for them all.

It cannot fairly be said that I am averse to mathematical formulas of graduation and construction, and perhaps here it will not be held to be egotistical if I indulge in a little personal historical retrospect. In my early days Makeham's formula was looked upon as hardly more than a beautiful scientific curiosity. I was then much occupied in coaching students for the Institute examinations, and Makeham's formula interested me greatly, and I set to work to try whether some method could not be found to apply it practically. This was in about the year 1877, and I had made a great many investigations and a great many calculations, and had on the whole progressed satisfactorily, when in 1878 the late Sir George Hardy came to me as a student to read for the final examination. Naturally I gave him the run of my work, and I was soon struck by his genius and intuition, and presently I invited him to join me in completing the paper for the Institute which was already on the stocks. The result is the paper read by us jointly on 26 January 1880 before Hardy had passed his examination. Early in 1882 I was invited by the Council to write Part II of the Text Book, and before long I submitted to the Text Book Committee the question of whether it would not be well to include for purposes of illustration a table graduated by Makeham's formula, but it was not easy at that time to prevail upon the Committee to give their official sanction to such an innovation. The table, however, is in the Text Book, and I think I may claim that since then the formula has ceased to be a scientific curiosity, and has become a practical reality. Perhaps even this fact was not without effect in stimulating research, and leading to the discovery of other mathematical methods.

Even now, when it is proposed to use Makeham's formula, the mortality table must first be very carefully examined to see

whether the formula is suitable, and various trials must be made to see in what form it can best be applied. We have not sufficient knowledge yet of the qualities of the tables derived from National statistics to enable us to apply safely mathematical methods, and it seems to me much better to construct a table which confessedly keeps close to the original facts as recorded, without assuming any definite law. At next Census, when, no doubt, populations will be given again age by age, a start may be made from the base so prepared, and that may make it possible to ascertain better what are the real facts as regards age mis-statements, and migrations of populations. This seems to me to be the most promising course to pursue, and the most scientific.

It is not necessary to say much regarding the smoothness of the curves produced by the Census method of table construction, as that is proved not only by the figures given in my report, but also by the comments on them by the Second Reviewer. He takes exception to the statement that "it would be difficult to imagine" a curve of smoother graduation unless it were a curve really "of the third order with a constant third difference", and he compares the third differences of q_x in English Life Table No. 8 Males with the third differences by the National Health method, which, having been constructed by a mathematical formula, is essentially smooth. He shows that from ages 20 to 59, ignoring signs, the sum of the English Life Table third differences is 57 with 17 changes of sign, and of the National Health Table, 36 with 14 changes of sign. There is really no distinction here to speak of. The differences are derived from the values of q_x to five places of decimals, but a loss of smoothness results from cutting down the figures. In Tables VII and VIII annexed the first, second, and third differences for Life Table No. 8 are given constructed from q_x to seven decimal places, the differences then being cut down to correspond to q_x to five places of decimals. These revised third differences show, for English Life Table No. 8 Males, a sum of 44 for the third difference in the section 20 to 59, with only seven changes of sign. All this seems to me, however, to be hair-splitting. The reviewer does not mention that for English Life Table No. 8 males from age 75 onwards I apply the very severe test of taking out the fifth order of differences of q_x , and he does not notice the further criterion of a good graduation which I put forward, namely, that the differences, if not small, should follow a definite law. This is an important

point, because, although it is true that very small third or fifth differences mean a good graduation, yet the converse is not true, that if the third differences are large the graduation is defective. In English Life Table No. 8 males a constant fourth difference of $\log p_x$ was taken starting from age 88, but that does not become fully effective until age 92, but from age 92 onwards the column of $\log p_x$ has differences all based upon a constant fourth difference, and every function derived from $\log p_x$ must also run with absolute smoothness. In Table IX annexed I give, up to the eighth order, the differences of q_x taken to seven places of decimals, and it will be seen that not only are these differences large, but that they swing backwards and forwards between positive and negative in a remarkable way. Nevertheless that table is absolutely smooth. This seems to me to be a good illustration of what I have said that we must not look merely at the magnitude of the differences in judging of the smoothness of a table, but also at the law which they follow.

The Second Reviewer finds it to be "disturbing" that in comparing the actual with the expected deaths of English Life Table No. 8 for males and females respectively I use the same age groups as were employed in the construction of the tables. There was no sinister intention in so doing, and, in Tables X and XI, I now give a complete statement of the expected and the actual deaths for both males and females, age by age, and a summary for the grouping usually selected for the purpose, 0-4, 5-9, &c. I trust that the reviewer with this information will have peaceful nights in future. The grouping 0-4, 5-9, &c., shows somewhat larger transferences from one group to another than does the grouping 4-8, 9-13, &c. Both these groupings, however, have the digits of age 8 and 0 in separate groups, and necessarily show fewer transfers from group to group than would any of the other three possible methods of grouping. Throughout the greater part of the tables there is a great excess of expected deaths at digits of age 0 and 8 because of the excessive numbers enumerated at these age digits. Hence, when both these age digits are included in one group there must, if the graduation is successful, be a considerable transfer of deaths from the groups including them to the adjacent groups. Therefore, it is rather a sign of good graduation than otherwise that in a comparison such transference is shown to have taken place. Every comparison, however, that can be instituted will prove that the tables throughout adhere closely to the

original facts, and that is the main object that was in view in constructing them.

The reviewer thinks that before comparing the actual with the expected deaths, the deaths should have been graduated, but I do not see how that would do any good. It would be more to the point to graduate the populations, and so to get rid of the excrescences at digits 0 and 8. Here, however, it must be remembered that the populations used in constructing English Life Table No. 8 are not the populations actually enumerated at the Census, these populations having been brought down to 1 July 1911 in order to be central to the deaths. The changes in the populations thus produced are, however, very small, and of only the slightest consequence.

This portion of my paper, which, perhaps not inappropriately, may be called a "Review of Reviews", has occupied more space than was intended, and there is not room now to call attention to other points which are worthy of discussion. Everyone, however, will formulate his own points for discussion, and it is perhaps well, therefore, that I should not give a lead. But one or two matters occur to me which are of real importance. The first is the extraordinary discrepancy between the enumerated children under 2 years of age and the number of births by which these children must have been produced. I refer at some length to this subject in paragraphs 14 and 15 of the first report, and paragraphs 18 to 21, and 34 to 43 of the second. I am quite unable myself to account for the discrepancy, and it would be interesting to enquire whether it exists in the returns for Scotland and Ireland. It seems to me very desirable that the question should be thoroughly threshed out before the next Census, so as to arrange if possible for more accurate returns being then secured.

In paragraphs 17 to 20 and 26 of the first report the difficulties experienced in connection with the marital condition of the population are mentioned, and here again there is a wide field for investigation. Also, I would much like to see an attempt made by some of our younger actuaries to graduate the population columns, single, married, and widowed, for both sexes, and to make them correspond exactly with similarly graduated columns for the total population. I confess that I have more or less failed in my graduation according to marital state, and I should be very pleased to see something better produced.

In paragraph 23 of my second report I explain the method

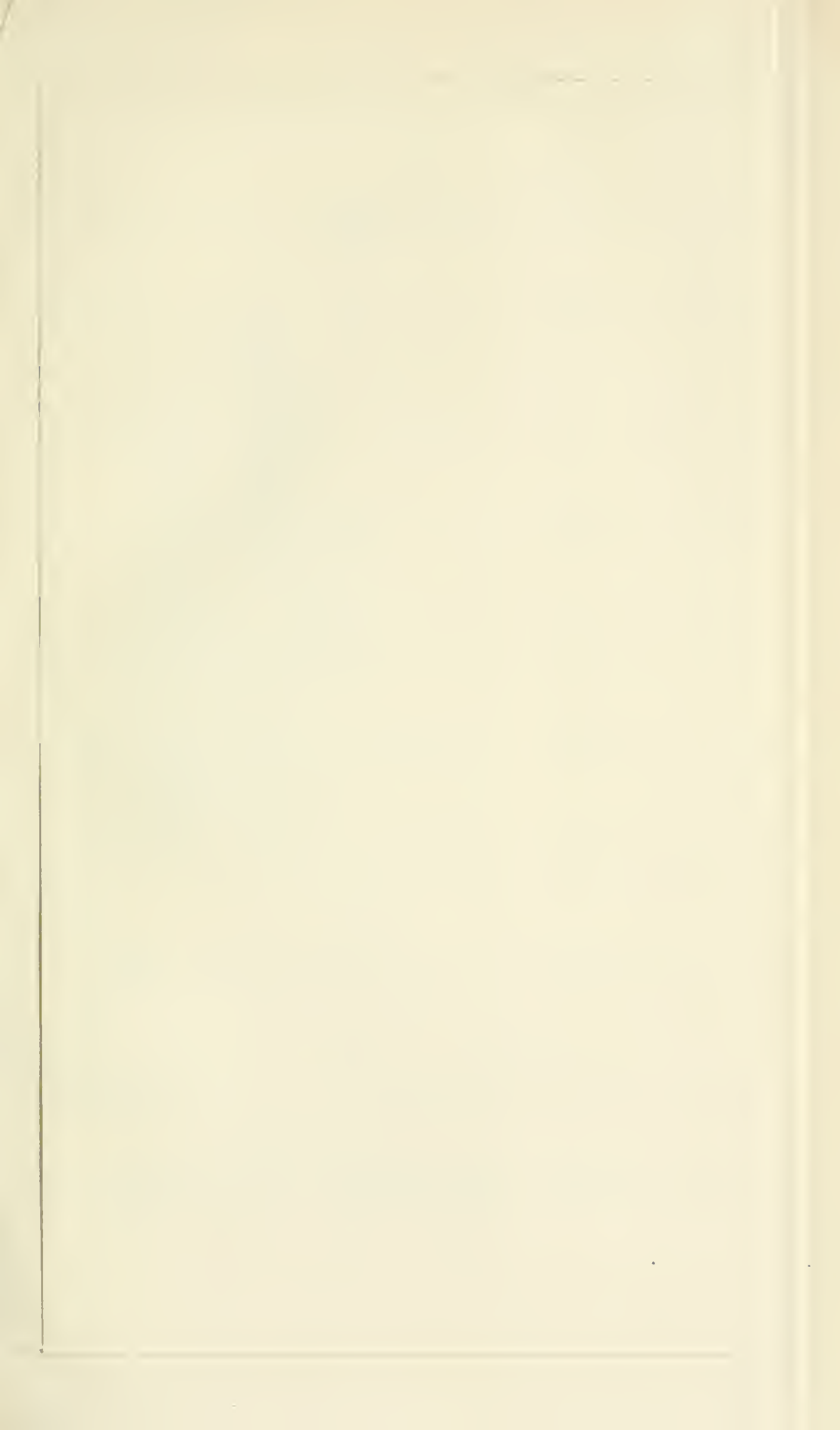


Diagram No 1

The Diagram represents the ratios in Table No I
England & Wales, Males

Starting from age 10 Starting from age 9

Text Book Table. Starting from age 10

The digits of age are on the abscissa axis and the
ordinates represent the ratios

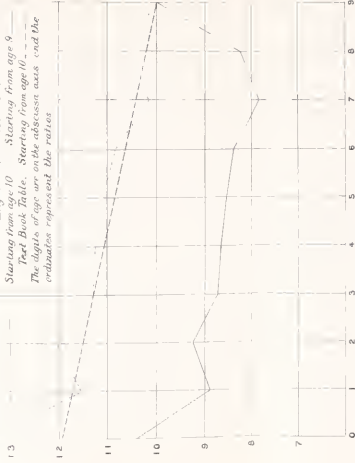


Diagram No 2

The Diagram represents the ratios in Table No II.

England & Wales Males. England & Wales Females.

The upper pair of lines are according to the method

of the First Reviewer and the lower according

to the revised method

The digits of age are on the abscissa axis.

and the ordinates represent the ratios

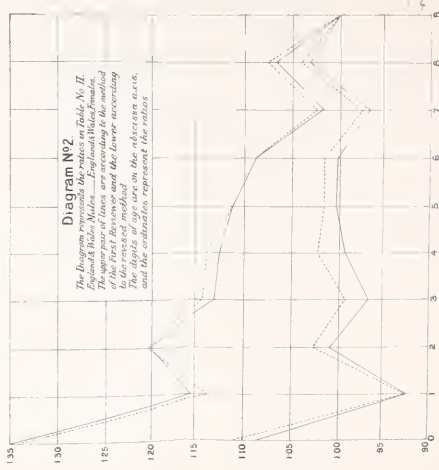


Diagram No 3

The Diagram represents Table No III and shows as a curve the values of the Central Death Rate, m_x , derived from the original data, namely, the populations enumerated at the Census of 1911, and the deaths of the three years 1910 to 1912

The higher pair of curves relates to males and the lower pair to females

The broken line gives the ungraduated values and the continuous line the graduated

The ages are on the abscissa axis, and the ordinates represent the values of m_x

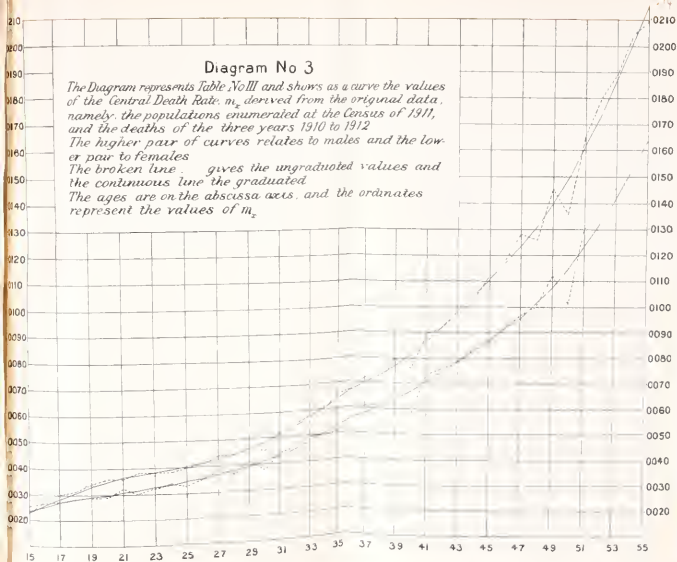


Diagram N^o4.

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The Diagram represents the ratios $L_{x+1} \div L_x$ in Table N^o V. Males, and has a separate division for each column of the Table. It also represents the same ratios according to the Life Table prepared for the corresponding section of the population.

Graduated Enumerated Population —————

Life Table Population - - - - -

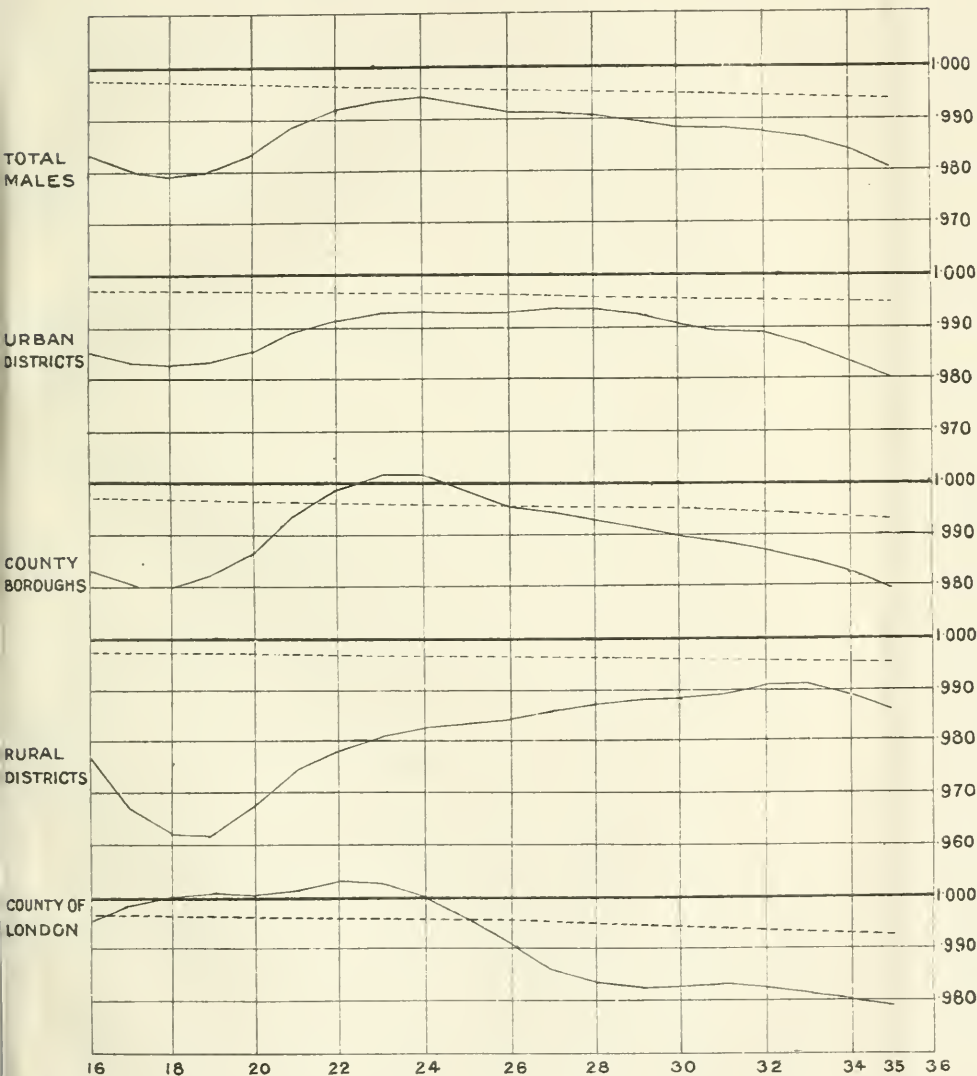
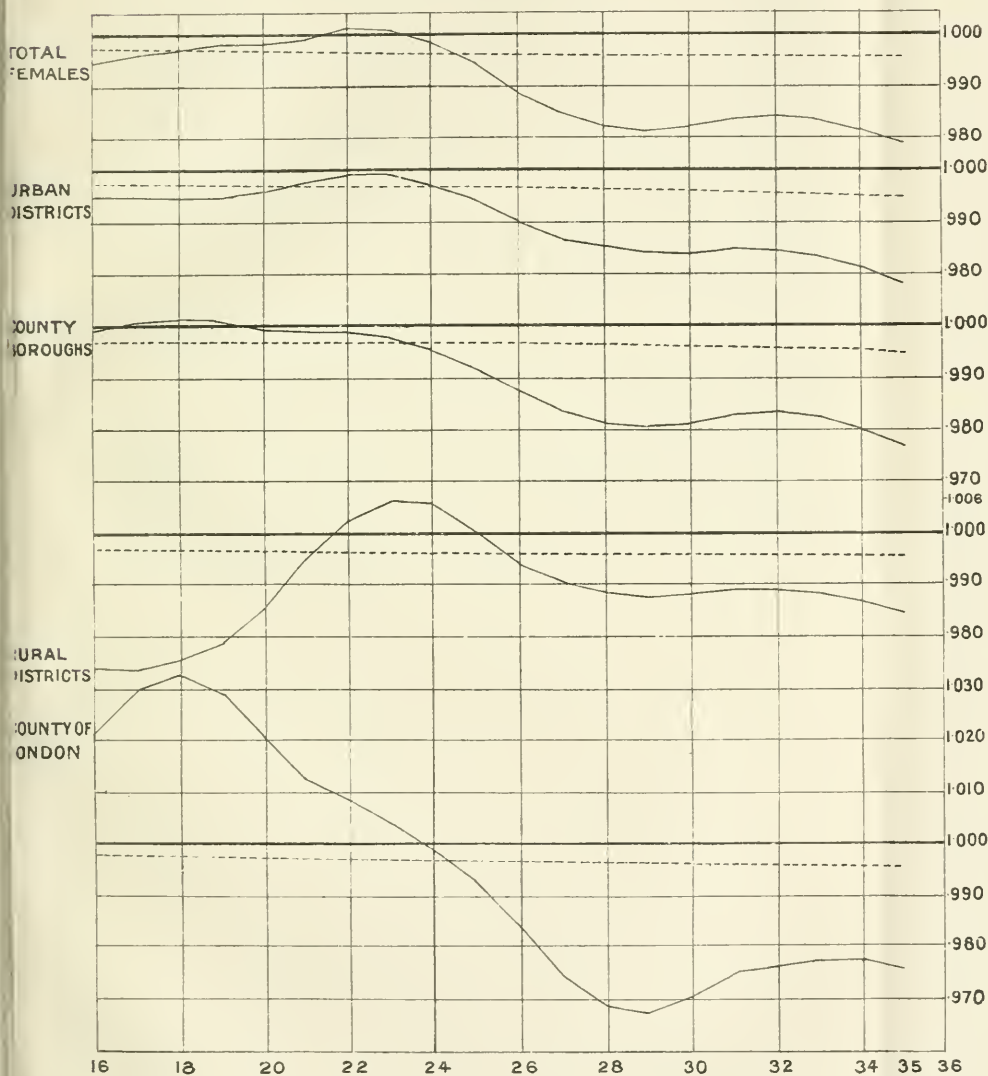


Diagram N° 5.

The Diagram represents the ratios $\frac{L_{x+1}}{L_x}$ in Table N° V, Females, and has a separate division for each column of the Table. It also represents the same ratios according to the Life Table prepared for the corresponding section of the population

Graduated Enumerated Population. _____

Life Table Population.....



which has been followed in the case of all the Life Tables to complete them at the old ages. In my papers in the *Journal* a different method was followed, but the late Mr. A. C. Waters took strong exception to it, and in deference to his opinions, and out of respect for his memory, I tried to find a method that would meet his objections. His view was that there was no absolute limit to human life, and that to assume unity for q_x at any definite point of age was wrong. The new method assumes no such limit, but, by taking a fourth difference suitably, the tables can be brought practically to an end at any point desired. Perhaps the tables would have been more artistic had the fourth differences of $\log p_x$ been started at an earlier age. For instance, by a suitable equation a fourth difference might be found at age 80 which would give q_x equal to .99999 at, say, age 105 for males and 108 for females, and for practical purposes very good tables would result. I throw out this idea merely as an illustration, but it would not have been suitable on the present occasion, where the object was to keep as close as possible to the facts as recorded to the oldest practicable age.

In paragraphs 58 to 62 of the second report, the National Health Table is dealt with, and it is desirable that that table should be reconstructed on the new estimates of population. Perhaps some industrious young actuary will perform this piece of work. It would be interesting to see a comparison of the new q_x he would produce with the approximations given in my report.

TABLE I.

Preference for Digits of Age.

England and Wales. Males. Populations at each digit of age for 90 years of life, first starting from age 10, and secondly starting from age 9; and the ratio of the population at each digit of age to the population at digit of age 9. Also similar figures according to the Text Book Table starting from age 10.

Digit of Age	ENGLAND—WALES. MALES				TEXT-BOOK TABLE	
	Starting from Age 10		Starting from Age 9		Starting from Age 10	
	Population	Ratio to Digit 9	Population	Ratio to Digit 9	Population	Ratio to Digit 9
0	1,638,755	1·349	1,638,755	1·040	547,785	1·196
1	1,402,153	1·154	1,402,153	·889	537,596	1·174
2	1,461,802	1·203	1,461,802	·927	527,470	1·152
3	1,373,018	1·130	1,373,018	·871	517,400	1·130
4	1,367,895	1·126	1,367,895	·868	507,387	1·108
5	1,350,062	1·111	1,350,062	·856	497,412	1·087
6	1,320,912	1·087	1,320,912	·838	487,475	1·065
7	1,234,881	1·016	1,234,881	·783	477,569	1·043
8	1,297,629	1·066	1,297,629	·823	467,686	1·022
9	1,215,057	1·000	1,576,503	1·000	457,826	1·000

TABLE II.

Preference for Digits of Age.

Measured by Deviations from 100,000 Graduated Population at each age as given in Table II for Males and Table V for Females in the Report on Graduation of Ages.

England and Wales Males and Females, ages 10 to 89. Total Deviation at each digit of age for 100,000 of Graduated Population at each age, and the corresponding Enumerated Populations. Also the ratio of the population at each digit of age to the population at digit of age 9.

N.B.--Positive quantities, heavy Ionic type.

Negative quantities, light Italic type.

Digit of Age	MALES			FEMALES		
	Total Deviation	Corresponding Enumerated Population	Ratio to Digit 9	Total Deviation	Corresponding Enumerated Population	Ratio to Digit 9
0	70,989	870,989	1.090	76,072	876,072	1.111
1	61,618	738,382	.924	72,490	727,510	.922
2	7,513	807,513	1.010	11,778	811,778	1.029
3	25,060	774,940	.970	15,369	784,631	.995
4	4,743	795,257	.995	6,646	806,646	1.023
5	2,496	802,496	1.004	401	800,401	1.015
6	1,246	801,246	1.003	231	800,231	1.015
7	27,094	772,906	.967	31,788	768,212	.974
8	25,584	825,584	1.033	20,689	820,689	1.040
9	736	799,264	1.000	11,253	788,747	1.000

TABLE III.

Values of m_x , Ungraduated and Graduated, derived from the population enumerated at the census of 1911 and the deaths of the three years 1910-1912.

Age	MALES		FEMALES		Age	MALES		FEMALES	
	m_x Rough	m_x Graduated	m_x Rough	m_x Graduated		m_x Rough	m_x Graduated	m_x Rough	m_x Graduated
15	·0023	·00231	·0025	·00239	45	·0112	·01107	·0086	·00864
6	·0026	·00258	·0026	·00254	6	·0118	·01170	·0091	·00908
7	·0030	·00285	·0027	·00268	7	·0128	·01237	·0097	·00957
8	·0031	·00311	·0028	·00280	8	·0126	·01313	·0100	·01012
9	·0034	·00333	·0029	·00289	9	·0145	·01400	·0112	·01076
20	·0036	·00350	·0029	·00297	50	·0136	·01500	·0101	·01150
1	·0036	·00363	·0032	·00305	1	·0165	·01615	·0130	·01234
2	·0038	·00374	·0030	·00313	2	·0180	·01740	·0136	·01326
3	·0038	·00382	·0032	·00321	3	·0189	·01872	·0146	·01425
4	·0038	·00391	·0034	·00331	4	·0204	·02009	·0151	·01529
25	·0040	·00401	·0033	·00341	55	·0209	·02153	·0155	·01639
6	·0041	·00412	·0035	·00352	6	·0227	·02304	·0179	·01754
7	·0044	·00425	·0038	·00365	7	·0260	·02471	·0200	·01881
8	·0044	·00441	·0037	·00379	8	·0263	·02661	·0201	·02024
9	·0046	·00460	·0041	·00395	9	·0288	·02877	·0217	·02186
30	·0045	·00484	·0038	·00412	60	·0281	·03122	·0207	·02369
1	·0052	·00511	·0044	·00432	1	·0366	·03391	·0279	·02570
2	·0055	·00542	·0046	·00454	2	·0359	·03672	·0280	·02718
3	·0058	·00574	·0049	·00477	3	·0406	·03954	·0307	·02990
4	·0060	·00606	·0050	·00501	4	·0425	·04233	·0325	·03206
35	·0065	·00638	·0051	·00527	65	·0470	·04514	·0347	·03436
6	·0067	·00668	·0054	·00553	6	·0471	·04815	·0358	·03697
7	·0072	·00699	·0061	·00580	7	·0523	·05171	·0409	·04018
8	·0070	·00733	·0061	·00608	8	·0551	·05605	·0439	·04413
9	·0080	·00771	·0067	·00638	9	·0578	·06129	·0453	·04886
40	·0076	·00815	·0059	·00670	70	·0655	·06738	·0528	·05425
1	·0089	·00866	·0073	·00705	1	·0776	·07410	·0645	·06009
2	·0092	·00923	·0076	·00742	2	·0831	·08121	·0671	·06616
3	·0100	·00983	·0078	·00782	3	·0890	·08853	·0732	·07238
4	·0103	·01045	·0083	·00822	4	·0974	·09606	·0786	·07882

TABLE IV.

Graduated Enumerated Populations.

Age	Total England and Wales	County of London	County Boroughs	Urban Districts	Rural Districts	Age
<i>Males.</i>						
16	335,427	38,387	99,220	118,210	79,610	16
7	329,866	38,233	97,493	116,413	77,727	7
8	323,299	38,185	95,545	114,419	75,150	8
9	316,477	38,201	93,605	112,395	72,276	9
20	310,150	38,240	91,902	110,505	69,503	20
1	305,070	38,261	90,665	108,917	67,227	1
2	301,573	38,315	90,061	107,697	65,500	2
3	299,159	38,427	89,936	106,740	64,056	3
4	297,322	38,525	90,042	105,935	62,820	4
25	295,557	38,530	90,127	105,188	61,712	25
6	293,360	38,368	89,940	104,395	60,657	6
7	290,783	37,998	89,492	103,614	59,679	7
8	288,165	37,470	88,951	102,914	58,830	8
9	285,423	36,846	88,300	102,205	58,072	9
30	282,477	36,185	87,524	101,401	57,367	30
1	279,248	35,546	86,608	100,418	56,676	1
2	275,895	34,932	85,591	99,312	56,060	2
3	272,472	34,304	84,483	98,140	55,545	3
4	268,739	33,657	83,226	96,817	55,039	4
35	264,454	32,990	81,762	95,249	54,453	35
6	259,378	32,297	80,032	93,353	53,696	6
<i>Females.</i>						
16	338,515	41,160	105,409	122,270	69,676	16
7	336,801	42,030	105,309	121,611	67,851	7
8	335,599	43,296	105,336	120,925	66,042	8
9	334,736	44,703	105,405	120,246	64,382	9
20	334,037	45,992	105,426	119,617	63,002	20
1	333,330	46,906	105,313	119,075	62,036	1
2	333,033	47,474	105,141	118,747	61,671	2
3	333,263	47,863	104,968	118,606	61,821	3
4	333,390	48,044	104,681	118,465	62,200	4
25	332,784	47,960	104,165	118,132	62,527	25
6	330,817	47,573	103,308	117,419	62,517	6
7	327,119	46,749	102,003	116,246	62,121	7
8	322,111	45,518	100,326	114,738	61,529	8
9	316,345	44,079	98,436	113,016	60,814	9
30	310,373	42,632	96,492	111,199	60,050	30
1	304,748	41,379	94,652	109,403	59,314	1
2	299,642	40,342	92,935	107,687	58,628	2
3	294,687	39,387	91,387	105,971	57,942	3
4	289,624	38,480	89,752	104,171	57,221	4
35	284,196	37,587	87,977	102,201	56,431	35
6	278,146	36,676	85,957	99,978	55,535	6

TABLE V.

Ratio $L_{x+1} \div L_x$, Graduated Enumerated Populations.

Age	Total England and Wales	County of London	County Boroughs	Urban Districts	Rural Districts	Age
<i>Males.</i>						
16	·9833	·9961	·9826	·9848	·9763	16
7	·9802	·9988	·9800	·9829	·9669	7
8	·9788	1·0004	·9797	·9824	·9618	8
9	·9799	1·0010	·9818	·9832	·9616	9
20	·9836	1·0006	·9865	·9856	·9673	20
1	·9886	1·0014	·9933	·9888	·9743	1
2	·9920	1·0029	·9986	·9911	·9780	2
3	·9938	1·0025	1·0012	·9925	·9807	3
4	·9940	1·0001	1·0009	·9929	·9824	4
25	·9927	·9959	·9979	·9925	·9829	25
6	·9913	·9901	·9950	·9924	·9838	6
7	·9911	·9861	·9940	·9933	·9858	7
8	·9905	·9833	·9927	·9932	·9871	8
9	·9897	·9820	·9912	·9921	·9879	9
30	·9886	·9824	·9895	·9903	·9880	30
1	·9881	·9827	·9883	·9890	·9891	1
2	·9876	·9820	·9870	·9882	·9908	2
3	·9863	·9811	·9851	·9865	·9909	3
4	·9840	·9802	·9824	·9838	·9894	4
35	·9808	·9790	·9788	·9801	·9861	35
<i>Females.</i>						
16	·9949	1·0211	·9991	·9946	·9738	16
7	·9964	1·0301	1·0003	·9944	·9733	7
8	·9975	1·0325	1·0007	·9944	·9749	8
9	·9979	1·0288	1·0002	·9947	·9786	9
20	·9979	1·0199	·9988	·9956	·9847	20
1	·9991	1·0121	·9984	·9972	·9941	1
2	1·0007	1·0083	·9984	·9988	1·0024	2
3	1·0004	1·0037	·9972	·9988	1·0061	3
4	·9982	·9983	·9951	·9971	1·0053	4
25	·9941	·9919	·9917	·9940	·9998	25
6	·9888	·9827	·9873	·9900	·9937	6
7	·9847	·9737	·9836	·9870	·9905	7
8	·9821	·9684	·9811	·9850	·9884	8
9	·9811	·9672	·9803	·9839	·9874	9
30	·9819	·9706	·9809	·9838	·9878	30
1	·9832	·9749	·9824	·9844	·9884	1
2	·9835	·9763	·9828	·9840	·9883	2
3	·9828	·9770	·9821	·9830	·9876	3
4	·9813	·9768	·9802	·9811	·9862	4
35	·9787	·9758	·9770	·9783	·9841	35

TABLE VI.

*England and Wales. Total Males.**Graduated Populations.*

N.B.—Positive quantities, heavy Ionic type.

Negative quantities, light Italic type.

Age	Population	δ	δ^2	Age	Population	δ	δ^2
15	340.125	<i>4,698</i>	<i>863</i>	55	133.528	<i>6,249</i>	510
6	335.427	<i>5,561</i>	<i>1,006</i>	6	127.279	<i>5,739</i>	334
7	329.866	<i>6,567</i>	<i>255</i>	7	121.540	<i>5,405</i>	221
8	323.299	<i>6,822</i>	495	8	116.135	<i>5,184</i>	111
9	316.477	<i>6,327</i>	1,247	9	110.951	<i>5,073</i>	<i>1</i>
20	310.150	<i>5,080</i>	1,583	60	105.878	<i>5,074</i>	<i>5</i>
1	305.070	<i>3,497</i>	1,083	1	100.804	<i>5,079</i>	70
2	301.573	<i>2,414</i>	577	2	95.725	<i>5,009</i>	75
3	299.159	<i>1,837</i>	72	3	90.716	<i>4,934</i>	79
4	297.322	<i>1,765</i>	<i>432</i>	4	85.782	<i>4,855</i>	85
25	295.557	<i>2,197</i>	<i>380</i>	65	80.927	<i>4,770</i>	143
6	293.360	<i>2,577</i>	<i>41</i>	6	76.157	<i>4,627</i>	139
7	290.783	<i>2,618</i>	<i>124</i>	7	71.530	<i>4,488</i>	53
8	288.165	<i>2,742</i>	<i>204</i>	8	67.042	<i>4,435</i>	<i>56</i>
9	285.423	<i>2,946</i>	<i>283</i>	9	62.607	<i>4,471</i>	<i>121</i>
30	282.477	<i>3,229</i>	<i>124</i>	70	58.136	<i>4,592</i>	<i>273</i>
1	279.248	<i>3,353</i>	<i>70</i>	1	53.544	<i>4,865</i>	<i>214</i>
2	275.895	<i>3,423</i>	<i>310</i>	2	48.679	<i>5,079</i>	11
3	272.472	<i>3,733</i>	<i>552</i>	3	43.600	<i>5,068</i>	235
4	268.739	<i>4,285</i>	<i>791</i>	4	38.532	<i>4,833</i>	461
35	264.454	<i>5,076</i>	<i>1,054</i>	75	33.699	<i>4,372</i>	453
6	259.378	<i>6,130</i>	<i>892</i>	6	29.327	<i>3,919</i>	303
7	253.248	<i>7,022</i>	<i>502</i>	7	25.408	<i>3,616</i>	314
8	246.226	<i>7,524</i>	<i>108</i>	8	21.792	<i>3,302</i>	326
9	238.702	<i>7,632</i>	283	9	18.496	<i>2,976</i>	336
40	231.070	<i>7,349</i>	184	80	15.514	<i>2,640</i>	396
1	223.721	<i>7,165</i>	<i>77</i>	1	12.874	<i>2,244</i>	388
2	216.556	<i>7,242</i>	71	2	10.630	<i>1,856</i>	299
3	209.314	<i>7,171</i>	221	3	8.774	<i>1,557</i>	212
4	202.143	<i>6,950</i>	367	4	7.217	<i>1,345</i>	122
45	195.193	<i>6,583</i>	550	85	5.872	<i>1,223</i>	149
6	183.610	<i>6,033</i>	450	6	4.649	<i>1,074</i>	209
7	182.577	<i>5,583</i>	178	7	3.575	<i>865</i>	167
8	176.994	<i>5,405</i>	<i>91</i>	8	2.710	<i>698</i>	128
9	171.589	<i>5,496</i>	<i>365</i>	9	2.012	<i>570</i>	...
50	166.093	<i>5,861</i>	<i>540</i>	90	1.442
1	160.232	<i>6,401</i>	<i>361</i>
2	153.831	<i>6,762</i>	<i>94</i>
3	147.069	<i>6,856</i>	171
4	140.213	<i>6,685</i>	436

TABLE VII.

Differences of q_x taken to seven decimal places and then cut down to five places.

English Life Table No. 8.—Males.

N.B.—Positive quantities, heavy Ionic type.

Negative quantities, light Italic type.

Age	δ	δ^2	δ^3	Age	δ	δ^2	δ^3
10	12	13	2	50	103	12	1
1	1	10	3	1	115	10	1
2	11	8	3	2	126	11	1
3	19	4	4	3	136	12	1
4	23	1	4	4	148	12	...
15	24	4	6	55	160	12	...
6	21	3	2	6	172	12	2
7	23	...	2	7	185	14	2
8	24	2	2	8	199	16	2
9	22	4	2	9	214	18	3
20	18	6	1	60	232	15	...
1	12	4	3	1	247	15	6
2	8	2	3	2	262	21	7
3	6	1	3	3	283	28	7
4	7	4	...	4	310	35	1
25	11	3	2	65	345	36	2
6	14	2	...	6	382	35	3
7	16	2	...	7	416	38	4
8	18	2	...	8	455	42	4
9	20	3	...	9	497	46	10
30	23	3	...	70	543	57	4
1	27	3	1	1	600	61	6
2	30	2	1	2	660	55	7
3	32	2	1	3	715	48	8
4	34	1	1	4	763	40	10
35	35	75	803	50	12
6	35	1	1	6	852	62	8
7	36	2	1	7	914	54	9
8	38	4	1	8	968	45	11
9	42	5	...	9	1.012	33	4
40	47	5	1	80	1.045	29	10
1	51	4	...	1	1.075	39	8
2	55	4	...	2	1.114	47	9
3	60	5	...	3	1.161	57	11
4	64	5	2	4	1.218	67	77
45	70	4	...	85	1.285	144	9
6	73	3	3	6	1.429	153	104
7	77	6	3	7	1.583	49	126
8	83	9	3	8	1.632	77	146
9	92	12	...	9	1.555	223	247

TABLE VIII.

Differences of q_x taken to seven decimal places and then cut down to five places.

English Life Table No. 8.—Females.

N.B.—Positive quantities, heavy Ionic type.

Negative quantities, light Italic type.

Age	δ	δ^2	δ^3	Age	δ	δ^2	δ^3
10	10	12	3	50	77	10	1
1	2	9	3	1	87	8	...
2	12	6	4	2	95	9	...
3	18	2	4	3	104	9	...
4	20	2	5	4	112	9	1
15	18	7	6	55	121	8	...
6	11	1	1	6	129	8	3
7	10	1	1	7	137	11	3
8	9	...	1	8	147	14	3
9	9	9	161	17	6
20	9	60	178	11	1
1	9	...	1	1	189	10	10
2	8	...	1	2	198	20	11
3	9	1	1	3	218	31	12
4	10	2	...	4	249	42	8
25	11	2	...	65	291	51	4
6	13	1	...	6	342	47	5
7	14	1	...	7	389	41	6
8	16	2	...	8	430	35	7
9	17	2	...	9	465	28	1
30	19	2	...	70	494	27	8
1	21	2	...	1	521	35	7
2	23	2	...	2	556	43	8
3	24	1	...	3	599	51	9
4	25	1	1	4	650	60	26
35	26	75	710	87	5
6	26	...	1	6	797	92	26
7	27	1	1	7	889	66	31
8	28	2	1	8	954	35	37
9	30	3	1	9	989	...	22
40	34	3	1	80	986	24	21
1	36	2	1	1	962	3	25
2	38	3	1	2	959	21	28
3	41	4	1	3	980	50	34
4	45	5	1	4	1,030	84	25
45	50	3	...	85	1,114	109	12
6	53	3	2	6	1,222	97	23
7	56	5	2	7	1,319	74	27
8	60	7	2	8	1,393	47	33
9	68	9	...	9	1,440	14	72

TABLE IX.

Values of q_x and its differences. English Life Table No. 8.—Males.

N.B.—Positive quantities, heavy Ionic type.

Negative quantities, light Italic type.

Age	q_x	δ	δ^2	δ^3	δ^4	δ^5	δ^6	δ^7	δ^8
92	2.977.077	76.136	<i>20,942</i>	13.762	6.396	<i>408</i>	<i>1,053</i>	<i>679</i>	61
3	3.053.213	55.194	<i>7,180</i>	20.158	5.988	<i>1,461</i>	<i>1,732</i>	<i>618</i>	775
4	3.108.407	48.014	12.978	26.146	4.527	<i>3,193</i>	<i>2,350</i>	157	1.238
95	3.156.421	60.992	39.124	30.673	1.334	<i>5,543</i>	<i>2,193</i>	1.395	1.610
6	3.217.413	100.116	69.797	32.007	<i>4,209</i>	<i>7,736</i>	<i>798</i>	3.005	1.026
7	3.317.529	169.913	101.804	27.798	<i>11,945</i>	<i>8,534</i>	2.207	4.031	<i>922</i>
8	3.487.442	271.717	129.602	15.853	<i>20,479</i>	<i>6,327</i>	6.238	3.109	<i>3,631</i>
9	3.759.159	401.319	145.455	<i>4,626</i>	<i>26,806</i>	<i>89</i>	9.347	<i>522</i>	<i>5,431</i>
100	4.160.478	546.774	140.829	<i>31,432</i>	<i>26,895</i>	9.258	8.825	<i>5,953</i>	<i>3,106</i>
1	4.707.252	687.603	109.397	<i>58,327</i>	<i>17,637</i>	18.083	2.872	<i>9,449</i>	1.992
2	5.394.855	797.000	51.070	<i>75,964</i>	446	20.955	<i>6,577</i>	<i>7,457</i>	7.771
3	6.191.855	848.070	<i>24,894</i>	<i>75,518</i>	21.401	14.378	<i>14,034</i>	314	8.093
4	7.039.925	823.176	<i>100,412</i>	<i>54,117</i>	35.779	344	<i>13,720</i>	8.407	1.757
105	7.863.101	722.764	<i>154,529</i>	<i>18,338</i>	36.123	<i>13,376</i>	<i>5,313</i>	10.164	<i>5,436</i>
6	8.585.865	568.235	<i>172,867</i>	17.785	22.747	<i>18,689</i>	4.851	4.728	<i>7,127</i>
7	9.154.100	395.368	<i>155,082</i>	40.532	4.058	<i>13,838</i>	9.579	<i>2,399</i>	<i>2,740</i>
8	9.549.468	240.236	<i>114,550</i>	44.590	<i>9,780</i>	<i>4,259</i>	7.180	<i>5,139</i>	
9	9.789.754	125.736	<i>69,960</i>	34.810	<i>14,059</i>	2.921	2.041		
110	9.915.490	55.776	<i>35,150</i>	20.771	<i>11,118</i>	4.962			
1	9.971.266	20.626	<i>14,379</i>	9.653	<i>6,156</i>				
2	9.991.892	6.247	<i>4,726</i>	3.497					
3	9.998.139	1.521	<i>1,229</i>						
4	9.999.660	292							
115	9.999.952								

TABLE X.
English Life Table No. 8.
Comparison of Actual with Expected Deaths.

Age last Birth- day	MALES		FEMALES		Age last Birth- day
	Actual Deaths	Expected Deaths	Actual Deaths	Expected Deaths	
0	164,033	164,033	127,925	127,925	0
1	41,577	41,577	38,288	38,288	1
2	15,916	15,916	15,598	15,598	2
3	9,639	9,639	9,387	9,387	3
4	6,895	6,895	6,732	6,732	4
5	5,604	5,604	5,476	5,476	5
6	4,116	4,449	4,112	4,377	6
7	3,141	3,604	3,160	3,556	7
8	2,589	2,889	2,639	2,875	8
9	2,270	2,391	2,263	2,392	9
10	2,180	2,086	2,126	2,119	10
1	2,013	1,912	1,956	1,971	1
2	1,851	1,906	1,943	1,986	2
3	1,897	1,999	2,178	2,082	3
4	2,103	2,179	2,353	2,261	4
15	2,318	2,357	2,495	2,409	15
6	2,623	2,609	2,634	2,607	6
7	2,960	2,767	2,729	2,711	7
8	3,069	3,025	2,848	2,839	8
9	3,325	3,167	2,859	2,854	9
20	3,319	3,227	2,914	2,947	20
1	3,277	3,348	3,169	3,030	1
2	3,385	3,371	3,017	3,135	2
3	3,400	3,450	3,270	3,265	3
4	3,406	3,504	3,399	3,349	4
25	3,481	3,532	3,345	3,416	25
6	3,645	3,659	3,464	3,505	6
7	3,702	3,626	3,616	3,462	7
8	3,962	3,943	3,664	3,775	8
9	3,931	3,952	3,817	3,693	9
30	4,193	4,480	3,934	4,275	30
1	4,020	3,940	3,757	3,657	1
2	4,604	4,474	4,187	4,151	2
3	4,465	4,358	4,110	4,015	3
4	4,807	4,762	4,332	4,341	4
35	5,167	5,031	4,386	4,487	35
6	5,230	5,228	4,504	4,639	6
7	5,070	4,949	4,653	4,436	7
8	5,588	5,815	5,144	5,102	8
9	5,635	5,459	5,089	4,803	9
40	5,993	6,446	4,998	5,635	40
1	5,299	5,148	4,536	4,370	1
2	6,259	6,244	5,520	5,351	2
3	5,885	5,730	5,080	5,027	3
4	5,918	5,923	5,180	5,098	4
45	6,783	6,680	5,468	5,510	45
6	6,545	6,493	5,459	5,472	6
7	6,792	6,601	5,609	5,559	7
8	7,131	7,513	6,233	6,358	8
9	7,524	7,295	6,308	6,089	9

TABLE X—continued.

Age last Birth-day	MALES		FEMALES		Age last Birth-day
	Actual Deaths	Expected Deaths	Actual Deaths	Expected Deaths	
50	7,970	8,787	6,360	7,246	50
1	6,993	6,788	5,822	5,497	1
2	8,239	7,909	6,807	6,592	2
3	7,877	7,699	6,720	6,514	3
4	8,605	8,402	7,047	7,104	4
55	8,266	8,484	6,705	7,063	55
6	9,065	9,218	7,700	7,549	6
7	8,740	8,370	7,474	7,072	7
8	9,441	9,623	8,072	8,168	8
9	9,603	9,600	8,113	8,170	9
60	10,416	11,507	8,720	9,874	60
1	9,556	8,724	8,035	7,271	1
2	10,385	10,396	9,039	8,795	2
3	10,505	10,014	9,153	8,707	3
4	10,674	10,466	9,683	9,399	4
65	11,511	11,021	10,201	10,064	65
6	10,698	11,062	9,806	10,199	6
7	10,839	10,911	10,240	10,209	7
8	11,743	12,163	11,268	11,523	8
9	11,868	12,732	11,202	12,218	9
70	12,125	12,432	12,542	12,897	70
1	11,029	10,375	11,811	10,912	1
2	11,973	11,453	13,248	12,871	2
3	11,196	10,909	12,686	12,322	3
4	11,066	10,742	12,464	12,311	4
75	10,359	10,490	12,015	12,015	75
6	9,765	9,726	11,576	11,597	6
7	8,919	9,053	10,751	10,677	7
8	8,770	8,831	10,620	10,855	8
9	7,851	7,371	9,687	9,312	9
80	7,159	7,679	9,530	10,149	80
1	6,131	6,083	8,004	7,739	1
2	5,980	5,949	8,135	8,047	2
3	5,089	5,165	7,117	7,163	3
4	4,761	4,730	6,553	6,837	4
85	3,892	3,915	5,679	5,603	85
6	3,330	3,268	4,913	4,932	6
7	2,683	2,724	4,216	4,084	7
8	2,095	2,121	3,347	3,300	8
9	1,665	1,719	2,726	2,720	9
90	1,273	1,390	2,187	2,371	90
1	958	940	1,735	1,695	1
2	738	715	1,394	1,388	2
3	524	431	981	878	3
4	307	327	668	652	4
95	219	242	490	519	95
6	188	152	364	385	6
7	93	99	246	248	7
8	49	64	160	184	8
9	37	41	103	99	9
100	57	58	138	151	100
& over					& over
All ages	771,760	772,255	726,236	726,544	All ages

TABLE XI.

*English Life Table No. 8.**Summary Comparison of Actual with Expected Deaths.*

Ages last birthday	MALES				FEMALES				Ages last birthday
	Actual Deaths	Expected Deaths	Deviation Expected <i>less</i> Actual		Actual Deaths	Expected Deaths	Deviation Expected <i>less</i> Actual		
			Positive	Negative			Positive	Negative	
0 to 4	238,060	238,060	197,930	197,930	0 to 4
5 „ 9	17,720	18,937	1,217	...	17,650	18,676	1,026	...	5 „ 9
10 „ 14	10,044	10,082	38	...	10,556	10,419	...	137	10 „ 14
15 „ 19	14,295	13,925	...	370	13,565	13,420	...	145	15 „ 19
20 „ 24	16,787	16,900	113	...	15,769	15,726	...	43	20 „ 24
25 „ 29	18,721	18,712	...	9	17,906	17,851	...	55	25 „ 29
30 „ 34	22,089	22,014	...	75	20,320	20,439	119	...	30 „ 34
35 „ 39	26,640	26,482	...	158	23,776	23,467	...	309	35 „ 39
40 „ 44	29,354	29,491	137	...	25,314	25,481	167	...	40 „ 44
45 „ 49	34,775	34,582	...	193	29,077	28,988	...	89	45 „ 49
50 „ 54	39,684	39,585	...	99	32,756	32,953	197	...	50 „ 54
55 „ 59	45,115	45,295	180	...	38,064	38,022	...	42	55 „ 59
60 „ 64	51,536	51,107	...	429	44,650	44,046	...	604	60 „ 64
65 „ 69	56,659	57,889	1,230	...	52,717	54,213	1,496	...	65 „ 69
70 „ 74	57,389	55,911	...	1,478	62,751	61,313	...	1,438	70 „ 74
75 „ 79	45,664	45,471	...	193	54,649	54,456	...	193	75 „ 79
80 „ 84	29,120	29,606	486	...	39,439	39,935	496	...	80 „ 84
85 „ 89	13,665	13,747	82	...	20,881	20,639	...	242	85 „ 89
90 „ 94	3,800	3,803	3	...	6,965	6,984	19	...	90 „ 94
95 „ 99	586	598	12	...	1,363	1,435	72	...	95 „ 99
100 & over	57	58	1	..	138	151	13	...	100 & over
All ages	771,760	772,255	3,499	3,004	726,236	726,544	3,605	3,297	All ages

ABSTRACT OF THE DISCUSSION.

Mr. S. J. ROWLAND said that the tabulation of the population statistics age by age would be of great value in future years if it were continued, although—owing to the fact that the 1901 figures were in grouped ages—it had not been possible to derive the full advantage from it on the present occasion. It was very interesting to note the variations in the numbers returned at different integers in age. Mr. King had selected 19, 24, &c., as the initial ages for his groups, instead of the more usual 20, 25, &c.; and that, no doubt, rectified to a large extent the accidental errors in the age returns. At the same time there appeared to be great variations in the 1911 returns, when they were compared with the expected survivors of the 1901 enumeration. In the returns for males there was a great deficiency in the groups 15–34, no doubt due to emigration. In the returns for females there was a great excess for ages 20–30, and a deficiency over age 30. Mr. King attributed these variations also to migration, on the ground that if they were due to misstatements of age, as had been suggested, there would be a dip in the curve of q_x for ages 20–30, and a hump for ages 30–40. This argument seemed to him (the speaker), however, to support the misstatement of age theory. In most existing tables of female mortality, where both married and single women were included, and where the ages could be relied upon and the lives were not all subject to recent selection, for example, the H^F and O^F , there was a maximum value of q_x about age 25, with a corresponding minimum in the 30's. Neither of those features was to be found in Mr. King's tables, although one would have expected them to be more pronounced than in a table of assured lives. If there had been a transfer from the 30–40 group to the 20–30 group, this would have tended to smooth out the maximum in the 20's and the minimum in the 30's, and it seemed not unlikely, therefore, that such a transfer—due presumably to misstatements of age—had occurred. The excess in the recorded populations about age 65, for both males and females, was interesting in view of the Old Age Pension Act, although it would be difficult to say whether that excess was due to the fact that a prospective pecuniary advantage had led to true ages or incorrect ages being returned in 1911. There, again, it was suggestive to note that the excess was greater in the case of the females.

With regard to the death returns, in the examination of death certificates in connection with claims he had rarely come across any material errors in age, but it would be interesting to know if that was the experience of the large industrial offices, since it would show to what extent the returns were reliable. The errors in the population returns would probably be greatly reduced if Censuses were taken at more frequent intervals, and in view of the important effect of a decreasing death rate on the cost of sickness benefits it would seem that the extra expense of more frequent enumerations would be well repaid. Another point suggested by most Census Reports was that many of the difficulties not otherwise readily

explainable were put down to migration. It would seem that there should be no insuperable difficulty in records of migration being kept; such records would not only be valuable in themselves, but would help materially in solving the question as to the true age distribution of the population. To obtain the mean populations Mr. King had followed the method used by the late Mr. A. C. Waters for the 1901 Census. There was some doubt, however, whether that method—which assumed a geometrical rate of increase—was quite satisfactory, as the rate of increase, taken over a long period, had not been geometrical, but had varied between an arithmetical and a geometrical progression. That, again, pointed to the desirability of having more frequent Censuses.

In connection with the graduation of the data, Mr. King stated that the variations at successive ages should be to a large extent preserved in the final figures, because they were an inherent feature, being the result of fluctuations in the birth rate, &c. It was doubtful, however, if these variations were anything more than accidental fluctuations, for, as Mr. King had shown in connection with infantile ages, any close relationship between the enumerated populations and the actual births and deaths seemed to be lost in the first two years of life. The method of graduation employed by Mr. King assumed constant third differences for the grouped populations. If the National Health curve, according to which the differences never became constant, correctly represented the facts, then it would appear that Mr. King's method introduced an error, although in practice that error was probably small. On the other hand, it was doubtful if any convenient mathematical formula was sufficient to cover the data, say, from age 20 to the end of life, and to give effect to the variations in curvature which should be retained. The essential difference of opinion between Mr. King and his reviewers seemed to be to what extent those variations should be retained. A point which he thought the reviewers did not perhaps appreciate was that Mr. King had really two distinct problems before him, the one being to find the actual age distribution of the population at the time of enumeration, and the other to construct a mortality table, and it was not necessary that the same method would be applicable to both. It seemed to him that the reviewers, in effect, agreed that the National Health method would not apply to the first problem, since they had to re-distribute the deaths—which were, probably, approximately correct—before they graduated them.

Mr. King obtained a central value of L_x from three quinquennial group values, and then filled in the remaining values by osculatory interpolation. By a slight modification of Mr. King's method it was possible to obtain the five central values of L_x direct from the group values, when it would be found, as was obvious from the theory on which the method was based, that the sum of those five values was the same in the graduated and ungraduated values. That explained why Mr. King's figures followed the data so closely, and why there were the changes in curvature pointed out by the

first reviewer. Mr. King's method depended largely on interpolation. In the ordinary way interpolation was used either because the intermediate values could not be ascertained, or because of the amount of labour involved in obtaining them. In the case under discussion neither of those reasons applied, and it would seem that the sole reason for interpolation was that a smoother curve was gained by its use. Mr. King could have obtained the five central values direct from the data, and those values would be as reliable as the one central value used as the basis for interpolation. If those five central values were not reliable then it would seem that the probability was that the one central pivotal value and the values obtained from it were not reliable. If the five central values were correct then there was no need for interpolation with its attendant errors. He thought that, wherever possible, the final values should depend directly on the data and should not be obtained by interpolation from selected ages. In that connection it was to be noted that for ages 5 to 12, where the value of q_x depended almost entirely on interpolation, the expected deaths exceeded the actual by as much as 7 per-cent.

It seemed to him that Mr. King's and the National Health methods might, perhaps, be combined. Mr. King used a series of third difference curves, and the National Health formula consisted of a third difference function plus a geometrical progression. Would it not be possible to use the geometrical portion of the National Health formula and add that to the value of $\log T_x$ and then graduate the resulting figures by a finite difference method or a modification of Mr. King's method? The effect would be the same as that of using the National Health method, with a separate graduation of the differences between the graduated and ungraduated figures. The suggested method was analogous to that used by the late Sir G. F. Hardy for the Indian Census Tables, with the exception that the differences were in that case graduated graphically.

Dr. A. NEWSHOLME said that the subject of the paper was one in which medical men—and especially medical men engaged in the public health branch of the profession—were very deeply interested. Mr. King's Report was one of the most important documents that had been issued by the General Register Office, and it would form part of his official duty to draw attention to it in the future and to do his best to see that all medical officers of health had an opportunity of studying the important deductions which could be drawn from the life table. Although not technically competent to speak about the data and the method of construction of life tables, he would like to make one or two general remarks on those points. First, with regard to the data. He took the opportunity of congratulating the General Register Office on the very important reform introduced in 1911, without which the Local Life Tables constructed by Mr. King in conjunction with the National Life Table would have been to some extent under suspicion as regards the accuracy of the data. In that year the General Register

Office, with the co-operation of the Local Government Board, was able to inaugurate a more accurate system of relegating to their proper districts the deaths of persons who died outside the area in which they had previously lived. There had been some previous relegation of deaths in institutions, to an incomplete extent, but now it was possible to study the death rates of different areas with the assurance that so far as possible all the deaths relating to those areas were included in them. That fact rendered it possible for Mr. King to have data which were accurate so far as the different local areas with which he had had to deal were concerned.

With regard to the method of construction of the life table, he had to confess his personal disappointment that Mr. King had abandoned the only method of construction which he (the speaker) had any knowledge of at all, namely, the graphic method—which, as Mr. King had shown in his paper on the construction of the Carlisle Table, was not only an elegant method, but one capable of a high degree of accuracy. At the same time he had no doubt that Mr. King had good reasons for not constructing the recent life tables by the graphic method. Whatever method was employed, it was a matter of the first importance that the results of one life table should be comparable with those of another, and in that connection he congratulated Mr. King and the General Register Office on the fact that they had published the full details of the construction of the life tables, so that anyone who wished to check the results could do so. He believed that in the National Life Table for 1891–1900, some of the details were given, but not as fully as on the present occasion. The importance of the publication of full details might be seen by reference to the case of the 1881–1890 table. That table was constructed by methods which were considerably more accurate than those used by Dr. Farr in the Life Table No. 3, but in introducing the improved methods an element of comparability had been lost, and the late Dr. Hayward, in a little-known paper published in the *Journal of the Royal Statistical Society*, took the trouble to calculate by the same method the National Life Tables for 1871–1880 and for 1881–1890. The official figures for 1881–1890 had shown that the expectations of life for males were lower for all ages over 45 than those given by the 1871–1880 table, and on the strength of that there had appeared in all the medical and other journals lucubrations as to the degeneration and approaching decadence of the British race, similar to those which appeared at intervals in connection with the differential birth rate and infant mortality, and so on. But the falsity of the supposition appeared when Dr. Hayward proved, by reconstructing the two tables on the same basis, that so far from the expectation of life having decreased at ages over 45, it had increased at every age. At age 45, for instance, the expectation in 1881–1890, as compared with 1871–1880, was more than .21 of a year greater. That illustrated the extreme importance of publishing the methods by which the life tables were constructed, and as far as practicable adopting the same method for successive tables. In

that connection he would ask Mr. King whether his Life Table No. 6, which dealt with the experience of 1891-1900, was constructed by the same method as the Life Table for 1901-1910, because that might affect to some extent the comparability of the two tables.

Turning from the question of methods to the question of results, from the public health point of view the most important functions were L_x , e_x , and the function now commonly known as life capital, which was introduced first, he believed, in Dr. Tatham's Manchester Life Table, and gave for the whole community what the expectation of life gave in regard to the individual. Mr. King had referred to the probabilities of living five and ten years in connection with the different life tables, and he would ask in what way those probabilities were superior, for the purpose of comparing one decade with another, to the quinquennial or decennial death rates at the corresponding age periods, which every self-respecting medical officer of health in a large community now gave in his annual reports for the two sexes separately. Medical officers of health for their respective districts, and the Registrar General for the whole country, stated the death rate for every five-yearly period up to 25, and for every ten-yearly period thereafter. Did not that give as much information as a life table could give by means of functions of ${}_5p$ or ${}_{10}p$?

In conclusion, he wished to make a suggestion which he hoped the General Register Office might see its way to take up. He would like to see all the National Life Tables—including the 1838-1854 Life Table of Dr. Farr—re-calculated on one and the same basis, and the lessons of the comparative life tables brought out, in detail, by some master of the subject like Mr. King. He thought, furthermore, that the value of such an investigation would be greatly increased if, in addition, the experience of selected lives, those of assurance societies, were compared with that of the National Life Tables. He was aware that the Institute had collected most valuable information on that point, and if the two sets of information were combined on a comparable basis, there would be a document of immense national importance.

Dr. T. H. C. STEVENSON said that he had to represent not only himself but his chief, Mr. Mallet, who had intended to be present to express his appreciation of Mr. King's services in connection with the graduation of the Census data and the construction of the life tables, but had been called to France on business connected with the war. When the idea of inviting a member of the actuarial profession to undertake the work was first mooted the name of Mr. King at once suggested itself, on account of his published work upon the subject, and in that connection it might be of interest to mention that when he (the speaker) was in Washington some five years ago, investigating the tabulation methods in use in the Census office there, the question of constructing an American life table for the registration area of the United States was under consideration, and the conclusion arrived at was that Mr. King's published method had on the whole most to recommend it from the American point of view.

He wished to draw attention to one or two points that had struck him on reading Mr. King's paper, because to some extent Mr. King had been independently covering ground that had been already covered in the General Register Office. First there was the question of migration *versus* misstatement of age on the part of young women. One approached that subject with a strong prejudice against the women. It had been officially put forward in reports issued from the General Register Office, that the apparent anomalies in the enumerated numbers at certain ages were due to under-statements of age by women who wished to remain in the twenties longer than they ought to do, and one had got into the habit of assuming that to be the true reason for the peculiarities of the figures. But the suggestion was made that alongside the females of England and Wales should be tabulated the females of the United Kingdom as a whole. That was accordingly done, and in the report on the ages of the Census, vol. vii, p. xxxiii, there was a diagram representing two curves of age distribution, one of females in England and Wales, and the other of females in the United Kingdom, and that diagram showed that the anomaly was very much less in the case of the United Kingdom than it was in the case of England and Wales alone. It had always seemed to him that that was fairly conclusive evidence in favour of its origin, in large part at all events, in migration from other parts of the United Kingdom, and he could only assume that no considerable number of workers came from Ireland and Scotland such as were known to come from the rural districts to the urban districts. That was shown in the same volume by the curves representing the age distribution of rural districts and the age distribution of urban districts, where it could be seen that at the critical age from the migration point of view, say 15 to 25, the distribution in one case was almost exactly the converse of the distribution in the other, *i.e.*, the numbers at 20 to 25 were low in the case of rural districts and high in the case of urban districts.

Another case in which the Register Office had been travelling along the same lines as Mr. King was in regard to the curves showing mortality from crude figures. In the Annual Report of the Registrar-General for 1912 a curve was published for each sex, showing the mortality year by year upon the crude Census figures and crude death returns, and the main feature of interest was that the curve was very much smoother than either the crude Census curve or the crude deaths curve. It showed there was a very considerable amount of parallelism between the two. The death returns were somewhat better than the Census returns, because there was a dip in the mortality at each even year of age from 30 onwards, but the point came out very clearly that up to 30 at all events the crude mortality curve was really wonderfully smooth. It seemed to him that that must be regarded as an argument telling in favour of the returns of female ages at that period of life, because one could only assume either that the statements of both deaths and population were correct or that each was incorrect to an equal

extent, and such complete parallelism as the latter assumption would require seemed very unlikely.

He wished to say also that at the General Register Office they entirely agreed with Mr. King that what might be assumed to be wilful and major misstatements of age were not to be corrected in a table of the kind which Mr. King was called upon to prepare. The reason for that was that in a National Table of the kind one could only go so far as it was possible to carry every user of the table with one. It was common ground that the preference for round-figure ages ought to be corrected, but he thought the difference of opinion between Mr. King and his reviewers was a very good illustration of the difficulties which would occur if an endeavour was made to transfer a certain number of females from the twenties to the thirties. Mr. King took up the position that none of them ought to be transferred, while the reviewers took up the position that a good many of them should be. It was impossible to please everybody, and it seemed to him the only safe plan in work of the kind was to leave such errors as that to be corrected by subsequent workers, necessarily on divergent lines, in accordance with the views which they took as to their extent and even their existence.

Mr. A. HENRY said that in addition to the very unequal distribution of births, there were other disturbing factors which affected Census returns. There might, for example, be an exceptionally heavy rate of infantile mortality in a particular year, and that was almost as important as an exceptionally heavy birth rate. Mr. King attributed the fluctuations in his graduated curve mainly to migration and to the disturbing factors to which he (the speaker) had just referred. He would suggest, however, that the fluctuations in that curve were not necessarily due to migration. The formula used to construct the table was, in a sense, a summation formula. It had been said that there was an inherent tendency in the summation method to reproduce accidental features and fluctuations of the ungraduated data, and he thought Mr. King's formula did very much to exaggerate that. The central five terms had $13\frac{1}{2}$ times the weight of the other ten. In Mr. Spencer's well-known formula the weight given to the middle five terms was only 1.8 times that of the others, and in Woolhouse's it was 3.8. Mr. King's formula had, in fact, very little graduating power. It was an excellent formula for the purpose of distributing what Mr. King called the minor misstatements, but its graduating power, as such, seemed to him to be small. In fact, by merely taking one-fifth of the quinquennial group values as the values of the central ordinates, one could obtain very nearly the same results, except at old ages, as by using Mr. King's formula, which brought into account the second difference.

There was one other point to which he should like to direct attention. In the figures given in Table 6 for the values of the second differences of the graduated populations, he had found that there were twenty points at which either a maximum or a minimum occurred, and those twenty points were confined to the

first and last two ages of the quinquennial groups. The middle ages were not represented at all, and he ventured to suggest that that showed almost conclusively that the variations in the second difference were not due to migration, but to the formula.

With reference to the question as to what constituted smoothness, it seemed to him that if a curve merely passed by undulations from one point to another one was not justified in saying necessarily that it was smooth. He did not gather what criterion Mr. King proposed to apply in such cases. He would suggest that the summation of some of the differences, certainly the second and probably the third, would give a criterion as to the rate at which the slope of the curve, and the radius of curvature, changed. He thought if the radius of curvature was changing very rapidly it indicated that the curve was not a smooth one, considered purely from the point of view of the graduation of actuarial statistics.

Mr. T. G. ACKLAND said that when he received Mr. King's Report he looked in the first instance at the figures for the earlier years of life, because he had been engaged for some time in investigations into the mortality of infancy and childhood, a period which presented many points of difficulty, and in regard to which it was extremely desirable to have the most recent and accurate data available. It was very important, for instance, from the point of view of industrial insurance companies, to know what the real mortality of the country was in the first fifteen years of life, in rural, urban, and city districts, and in the different geographical areas of England and Wales. He had hoped that Mr. King would have been able to give some valuable information as to the mortality over this age-period, but it appeared from the Report that at the early ages the results could not be regarded, and were not regarded by Mr. King himself, as altogether reliable. Mr. King had had to make assumptions with regard to the first five years of life, because he found remarkable and at present unexplained anomalies between the results deduced from the births and deaths and the results deduced from the Censuses and the death returns, and he had, no doubt wisely, used the former alone in deducing the mortality rates for the first five years of life. That left some doubt as to how far the figures for those five years could be regarded as representing truly the mortality of infancy.

For the following ten years of age, namely, from 5 to 15, Mr. King's powerful and useful method of osculatory interpolation could not be used, as his graduated values were only available at ages 11 and 16. Mr. King, no doubt after very careful consideration, had adopted Lagrange's well-known method of interpolation, using the values for ages 4, 5, 11, 16 and 17 to determine those for ages 6-10 and 12-15. He had, therefore, in effect passed a curve of the fourth order through the facts in that section of the table. The rate of mortality between the ages 5 and 15 could not, however, be represented even approximately by a curve of the fourth order, and

it would be found that the method had produced, for both sexes, a considerable deviation in the age-group 4 to 8, and a much smaller one in the group 9 to 13. The difference between the actual and expected deaths was, for male lives 1,096 on 22,345 actual deaths, or approximately 5 per-cent, in the former age-group, and 83 on 10,211 actual deaths in the latter, and for female lives the figures were generally similar. Mr. King referred in his report to these deviations, but considered that they were "of no practical importance." No doubt they were of no practical importance if the table were considered as a whole, but when dealing, as he had been trying to do, and as many others had to do, with the ages of infancy and childhood, the matter appeared to him to be of very considerable importance. He was, therefore, led to try and devise some method by which these deviations could be reduced. After some unsuccessful experiments by other methods he took the value Mr. King had deduced, at age 1, from the births and deaths, the ungraduated value at age 6, and the values as adjusted by Mr. King at ages 11 and 16, and then deduced the intermediate values at ages 7 to 10 by the osculatory method of interpolation. He then dealt similarly with the values at ages 6, 11, 16 and 21, and deduced adjusted values between ages 11 and 16. The resulting graduation applied to $\log q_x$ for the Life Table No. 8 Males and Females, from 6 to 16 inclusive, was shown in the following Tables, also a comparison of the expected and actual deaths per annum :

Male Lives.

Age	Estimated Population 1 July 1911	Graduated Values of q_x	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation
6	373,128	·0036703	1,372	1,372	0	0
7	373,544	·0028412	1,063	1,047	+ 16	+ 16
8	366,769	·0023553	865	863	+ 2	+ 18
9	362,014	·0020677	749	757	- 8	+ 10
10	359,459	·0019011	684	727	- 43	- 33
11	351,676	·0018105	637	671	- 34	- 67
12	349,592	·0018165	636	617	+ 19	- 48
13	345,526	·0019344	669	632	+ 37	- 11
14	343,313	·0021312	732	701	+ 31	+ 20
15	334,480	·0023680	793	773	+ 20	+ 40
16	335,758	·0025868	870	874	- 4	+ 36
Total	9,070	9,034	+ 125	+ 140
					- 89	- 159
					+ 36	- 19

Female Lives.

Age	Estimated Population 1 July 1911	Graduated Values of q_x	Expected Deaths	Actual Deaths	Deviation	Accumulated Deviation
6	373,495	·0036640	1,371	1,371	0	0
7	374,016	·0028632	1,072	1,053	+ 19	+ 19
8	368,208	·0023932	882	880	+ 2	+ 21
9	361,853	·0021147	766	754	+ 12	+ 33
10	360,052	·0019525	704	709	- 5	+ 28
11	352,633	·0018617	657	652	+ 5	+ 33
12	350,717	·0018673	655	648	+ 7	+ 40
13	345,994	·0019856	688	726	- 38	+ 2
14	344,726	·0021773	751	784	- 33	- 31
15	335,945	·0023949	806	832	- 26	- 57
16	337,667	·0025700	869	878	- 9	- 66
Total	9,221	9,287	+ 45	+ 176
					- 111	- 154
					- 66	+ 22

The expected deaths were deduced by multiplying the estimated population at each age by $m_x = \frac{2q_x}{2 - q_x}$.

The total deviation, and total accumulated deviation, over ages 6 to 16 inclusive, shown by Mr. King's graduation, were, for male lives +425, and +3,876, and, for Female lives +257, and +3,128, respectively, the accumulated deviation being positive for both sexes at all ages in this group.

It would be seen that the values obtained by the method he had used accorded fairly closely with the original facts, and they were, he thought, entirely superior to Mr. King's. At the same time he did not put the method forward as by any means the best, nor as a final alternative to Mr. King's method, but merely as an effort to try and deduce, with some greater degree of accuracy, the very important progression of the rate of mortality over the ages of infancy and childhood.

Dr. E. C. SNOW said that he had had occasion to enquire closely into the figures for young females between the ages of 20 and 25, to attempt to ascertain whether the wave in the age-curve for England and Wales was entirely due to misstatement of age or could be explained by immigration of females. Comparing the number of females living aged 20-25 in 1911 with the number aged 10-15 in 1901, he found for the whole of the self-governing portion of the British Empire—between which aggregate and the rest of the world there was probably very little migration of young females—that the former exceeded the latter by about 2 per-cent. In the case of England and Wales, the corresponding figure was about 3 per-cent. For the German Empire there was no increase at all. The inference was that either misstatement of age by young women

was greater in England and Wales than in the rest of the Empire and elsewhere, or that there was some net immigration of young females into England and Wales, some explanation for which Mr. King had given. He was disposed to attribute about two-thirds of the wave to misstatement, and one-third to migration.

With regard to the construction of life table functions for the first year of life, he would suggest that the medical officer of health had, in his crude data, all the material to enable him to estimate the chance of living a single year from birth. In the case of the decade 1901-1911, for example, he had the births and the deaths under one year of age for the ten years, and the ratio of these would give him fairly accurately the chance of living one year after birth. The only obstacle in the way of a perfect measure was that some of the births occurred before the Census of 1901, and, therefore, it might be better to measure the births from six months before one Census to six months before the next. However, it made little difference—about .0002, he believed—in the probability of living a year. The difference between the value of p as obtained by the method he suggested, and that derived by Mr. King's method amounted to .004, or twenty times as much. This difference would have an appreciable effect on the expectation of life. He thought Mr. King's tables, in fact, understated the expectation of life at birth. The point was an extremely important one, and he was sure Somerset House would be very grateful if actuaries would assist in elucidating the matter.

Mr. S. J. GUNNINGHAM said that Mr. King had gone very fully into the question of the cause of the apparently excessive populations recorded at the younger ages. In his Diagram 3, he gave both the graduated and the ungraduated values of m_x , and in the latter it would be seen that where the male line dropped or rose so did the female line, except at the ages under discussion, namely, 19 to 25 or 26, and there it would be seen the two curves ran in opposite directions. Why was that? In the early part of Vol. vii of the Report of the General Register Office preceeding Mr. King's report on the Graduation of Ages, it was shown that up to age 26 or 27 there was really no serious preference on the part of the male population for even digits of age. After that age, however, there was a preference. Where there was an undoubted preference for digits of age both male and female curves showed similar irregularities, but below that there was a difference. He suggested that the difference was due to the tendency of females to understate their ages. A difficulty in connection with the migration explanation of the facts was that whereas urban districts were supposed to take their excess of females from the rural, the diagrams given in the early part of Vol. vii of the Census Report appeared to show that the exodus from the rural districts started at least five years earlier than the influx into the urban districts. Until the actual figures for migration, with ages, were obtained, he did not think it was possible to say definitely that one cause existed to the exclusion of the other.

In paragraphs 31 to 41 of his Report Mr. King attempted to estimate what improvement there had been in the accuracy of the statements of age in 1911 as compared with 1901. He stated that a person aged 40 returning his age as 30 caused two deviations of the graduated from the ungraduated results, namely, a positive deviation at 30 and a negative deviation at 40, and on that assumption he proceeded to compare the misstatements for the sample populations of 1901 and 1911 by taking one-half the total deviations in each case. But were the sample populations for 1901 and 1911 graduated by the same methods? Everything depended on the graduation. If a severer form of graduation were employed, there would be a larger number of deviations. Assuming the correctness of the method, he would apply it to Mr. King's graduated results both for males and females for the total and for the three divisions, according to marital condition from age 14 upwards. All these four tables were graduated by the same method. The deviations for the three divisions, when summed, should come to the deviations in the total (the total graduated populations being made to agree), but they did not. For the males, the deviations in the total curve came to about 8,000, whereas the deviations of the three divisions came to 14,000. For the females the figures were similar, 8,000 and 10,000. That seemed to throw considerable doubt on Mr. King's method of estimating the improvement in the accuracy of the age returns.

Mr. King referred to the fact that in Diagram 2, *J.I.A.*, vol. xlviii, p. 210, the first difference for age 29 was placed rather too high, and that if that were put down to the value as given in the table there would be a bend in the curve. That was so. But the figure for the next quinary age was also slightly too high. If both were put down to their true positions Mr. King would find there was no bend in the curve. With regard to the question of smoothness, Mr. King said that a curve in the form of a letter S, which was quite smooth, would show extraordinary variations in the first differences. But the curve $y = a \sin(bx + c)$, which represented a continuous succession of letters S, had a first difference which could not be called extraordinary, being merely another sine curve, but with a different maximum and epoch. With regard to the curve in the first review showing the preference of the population for certain digits of age, he thought that Mr. King had, perhaps, hardly appreciated what was the reviewer's intention. Supposing one had a perfectly smooth Population Table, age by age, with no bumps or kinks in it at all, and the numbers were added together for ages 10, 20, &c., 11, 21, &c., and so on, these, when plotted, would give a curve gradually dropping with outline convex to the horizontal, and any departure from that at any digit would show that there was a preference for that digit. In his second edition of the reviewer's method, Mr. King had taken the reviewer's figures for the units ending in 0 up to 8, but instead of starting at 19, for digit 9, he started at 9 and made that his common denominator in getting out the ratios. The only effect

of that was to make the slopes slightly different, and to throw out the sequence of ages. The fact that Mr. King found a preference for digit 9 showed that he had not followed the reviewer's method. Mr. King then followed up his criticism by giving a more correct method of estimating the effect of the preference; but it must be remembered that when the first review was written the ungraduated figures were the only ones available; there were no graduated figures, and the former had to tell their own tale. One had the rough figures and wished to see if there was any outstanding irregularity in them, and that could only be done to a certain extent quantitatively; it was not suggested in the first review that the figures should be used quantitatively, the diagram being regarded rather as showing the nature of the irregularity than its amount.

Sir ALFRED WATSON, in closing the discussion, said that there were two main subjects with which Mr. King had dealt, the first being the population statistics and the second the National Life Tables. He thought Mr. King had been more anxious than the circumstances really required to correlate the two and that it was not at all incumbent upon him, having graduated the population column for one particular purpose, to employ the same process in graduating the life table. With regard to the population curve, he agreed with Mr. King that the method he had adopted did, as nearly as the circumstances of the case allowed, provide a proper grouping of the population with reference to age. He also agreed with Mr. Henry when he said that Mr. King might almost have adopted the central ordinate of each group as the graduated value, because the value which he had obtained differed but very slightly from that central value, as indeed was bound to be the case in view of the character of the statistics.

When Mr. King, having got his quinary values, interpolated a smooth flowing curve through them he produced something which certainly represented a better graduation than the enumerated population would show if the real facts were brought out. Looking at the statistics issued by the Registrar General for many years past, one would see an extraordinary variation in the number of births from year to year, and that variation would be found to be reproduced if, by some simple process of interpolation, one passed from the numbers recorded in calendar years to the numbers estimated to have been born in years beginning on the 1st April, with which the numbers surviving at a Census taken at or about the beginning of April could be compared. He could not comprehend how those variations in the numbers born from year to year could fail to find some reflection in the numbers recorded at successive ages in the Census, if correct information were always given. He thought, on reading the official reports, that Mr. King was of opinion that the recorded numbers ought to form a perfectly smooth curve, whereas he (the speaker) believed that, if it were possible to reproduce the actual figures as they should be, a very irregular line would be obtained and not a smoothly-flowing

curve; but he now found from Mr. King's statement in his present paper that that was not his view, and that his opinion was that there were differences which ought properly to remain but which graduation did in fact remove. Such being the case, he found himself in a little difficulty in agreeing with Mr. King's statement in his first Report, that the graduation had removed misstatements of age representing 1·7 per-cent of the numbers enumerated, this being in fact the percentage borne by the mean number of deviations to the total numbers under graduation. It appeared to him that if graduation had itself introduced other errors which were not in the rough facts, the difference between the numbers as graduated and the numbers as first enumerated represented nothing of a concrete character such as the extent to which graduation had reduced the supposed errors in the enumerated ages. In view of the fact that Mr. King had apparently drawn that inference, he trusted he would forgive him for only realizing at so late an hour that he did agree that the figures, as they would leave the Census Office in a perfect world, would rather represent the edge of a saw than the line of a perfectly-graduated curve.

He desired next to add his opinion on the much-debated question of misstatements of age *versus* migration. The feature to which attention was called in the reviews had presented itself at every Census from 1851 onwards. Recently, at the Royal Statistical Society, Dr. Snow read a very interesting paper, in which he presented a diagram showing that among women during this long period the population at ages 20 to 25 had invariably been as much as or rather more than the population at ages 10 to 15, ten years earlier. If, therefore, the idea that women misstated their ages was a superstition it had at any rate acquired the respectability of age, and he did not think it ought to be dismissed without some further evidence of its unreliability than was at present available. In the second review (*J.I.A.*, vol. xlix, p. 100) there was a table giving the number of persons expected to have survived at the present Census from the number living at the ages ten years earlier at the previous Census, and this showed that the actual number of women surviving was, on a balance of excesses and defects, about 140,000 fewer than the number expected to have survived. It would, therefore, seem that somehow the population of women had lost within the last ten years 140,000, and it was inconceivable to him that at the ages under 30 the population should have really received a net increment of 90,000 but at the ages over 30 should have suffered a net efflux of about 230,000. It appeared to him that what one would expect to happen in regard to emigration probably had happened, that the women who had left this country left it in the main at the young adult ages and not in the middle ages and decline of life; and there was, in fact, an emigration of women which was expressed by a loss of population of about 140,000 in the last ten years; and, further, that at the young ages at which that loss was at its maximum it was entirely cloaked by the misstatements

of age of a section of the women who were enumerated. He did not suppose that the practice of misstatement of age was restricted to young women. He thought rather that it went right on through life until the pension-age was approaching, when different considerations came into play. It was quite clear that among girls aged 10 to 15 at the Census of 1901 there could be but very little misstatement of age. The difference between the numbers who survived and the numbers expected to survive at ages 20 to 25 was therefore very significant, and the fact, which everybody would admit, that migration from rural to urban districts accounted for many peculiarities in the sectional tables, did not appear to him to explain or indeed to have any bearing on this difference.

In regard to men, the position was quite different, and he was willing to accept Mr. King's suggestions as to the classes of immigrants which the exercise of a little imagination would discover. But as a matter of fact, among the men, so far from there being any indication at the young ages of systematic understatement, there was a heavy decline from the numbers expected to have survived—the deficiency being at its maximum at ages between 20 and 30. The shortage (representing about 250,000 in all) would have been considerably greater if it had not been for the return to this country of the men who were fighting in South Africa at the time of the Census of 1901. Dr. Snow, apparently, was so impressed by that feature of the 1901 Census as to have elaborately adjusted all his figures relating to survival at 1911, to give effect to it. As had been suggested, it was a point possibly affecting the exposed to risk, but he did not know whether Mr. King would agree that it was of such importance as sensibly to affect the rate of mortality at the ages round about 25 to 35. He did not himself suggest that the point was of much importance, but he thought it was clear that the calculated number of the exposed to risk at these ages in the ten years had been appreciably under-estimated by leaving out from the population, in 1901, those men who were in South Africa at that time, and who came back to this country in 1902 or 1903.

Mr. King had expressed very strong views with regard to what was called the National Health Insurance method of graduation. He sincerely hoped that the scheme of National Insurance was not going to be identified with a method of graduation, and one, moreover, for which the permanent professional advisers of the National Insurance Department were not entitled to take credit. The method was evolved by the late Sir George Hardy, and the question of its suitability in any particular case must be largely determined by the circumstances, and by the manner in which it was proposed to apply it to meet particular conditions. In the formation of the National Health Insurance Table the method was applied in two different ways. In the case of the men a single curve was fitted, while in the case of women the deaths were dealt with first by the ordinary formula, and a supplemental curve was laid on afterwards. There was no one definite, inelastic method by which the formula must be applied, and it could not, therefore, be criticised fairly on

the ground that it did not deal with this or that feature of the ungraduated statistics. The formula certainly involved no definite theory as to the law of mortality. It was a method of running a smooth curve through certain statistics, and it could be so applied as to give the most suitable approximation to the realities which appeared to underlie the rough statistics. Special stress should not be laid upon the particular method by which the plan had been applied by the Actuarial Advisory Committee who advised on the finance of National Health Insurance. What was now being considered was the graduation of a great National Life Table, and that graduation was the end in itself. In the case of the National Health Table the graduation, and, indeed, the Life Table, were merely stages on the road to a much bigger end. Because those responsible in that case took a certain course in regard to the preparation of the Life Tables it was not, therefore, to be assumed that they had laid down a working procedure for which they claimed acceptance in all, or, indeed, any other, cases. He believed it to be perfectly justifiable, in the present state of our knowledge of National vital statistics, to represent life tables by curves of a general character, based upon suitable mathematical formulas, believing that in so doing, if the work was performed with skill and care, they obtained as good results as the data would support. While he was of that opinion, this in no way abated his admiration for the learning and the industry which Mr. King's reports exhibited, and for the practical outcome of his labours as represented in the new National Life Tables.

The PRESIDENT having proposed a vote of thanks, which was carried unanimously,

Mr. KING, in reply, said that he had been much interested in the remarks at the close of Mr. Rowland's speech on possible improvements in the application of the method for deducing mortality tables from the Census Returns. He hoped Mr. Rowland would enlarge upon the subject, because it was desirable to have the very best method that could be produced, and he did not care whether it superseded his own or not. Mr. Rowland spoke about a constant third difference. He would remind him that it was a third difference that was changed at every quinquennial point in such a way as to make a smooth junction; there was no idea of having a constant third difference throughout the table.

Dr. Newsholme had asked what advantage there was in taking the probability of living five years or ten years instead of the average central death rate. He understood Dr. Newsholme to mean by the average central death rate the ratio obtained by dividing the deaths in any particular age period by the recorded populations in the same age period. By taking from the life table the probability of living five years or ten years, there was no confusion or error, but by taking the average death rate, in the way that he understood was intended, there was an error, and that error would not be the same in different sets of statistics; it depended on the numbers at the several ages in the

particular age period. By taking one population it was possible to obtain a larger number at the earlier parts of the age period and a smaller number at the later, and if that was reversed in another population no fair comparison could be obtained. He granted that taking small periods of age it would not make very much difference, but still there was an error which could be avoided by taking the probability of living five or ten years from a life table.

With regard to English Life Table No. 6, that was re-calculated as to males by himself, practically on identical methods with those adopted for Tables Nos. 7 and 8, except in finishing off the table at the old ages at 95 onwards. A similar re-calculation was made by Mr. Vyvyan Marr for females. Therefore Table No. 6, which was used for comparison, was constructed in the same way as Tables Nos. 7 and 8. Dr. Newsholme would find that he (Mr. King) had not quite given up the graphic method; it was used for the ages up to 14 in graduating the populations. The main reason why he did not like it was that he could not use it effectively as he could not draw well enough! All the same, he would like to see someone make a graphic graduation of the spinsters, wives and widows, separately, so as to get good curves for them, and obtain at each age, by summing them, the exact population for the total females, graduated in the same way. That would be a very difficult graphic graduation.

Mr. Ackland had explained an improvement which he (Mr. King) cordially welcomed at what might be called the juvenile ages in the construction of the Life Tables. It would be noticed that in his Report he said his own method was not quite satisfactory at those ages. He gathered from the figures Mr. Ackland had given that by his method the small divergence at that portion of the table was corrected, so that there was very little difference between the expected and actual deaths, taking the table as a whole. That result would be very satisfactory.

He might be excused for reminding Sir Alfred Watson that it was not he (Mr. King) who had coined the expression "National Health method." He had taken that expression from the reviewers. He did not in any way question the National Health Table, which was a very good one; he only said that when one tried to apply the same method to a great number of tables that had to be correlated he did not think it would be found to work.

Dr. W. F. SHEPPARD, who attended the meeting but was unable to take part in the discussion, sends the following comments:

In § 6 of the Report on the Graduation of Ages, Mr. King refers to his "graduated quinquennial values" as a means by which "to interpolate the numbers for intervening ages so as to produce a smooth table of populations age by age, from which, as far as possible, the errors in the statements of age given by the persons

enumerated had been eliminated." This seems to suggest two aims, which are not only different but inconsistent.

When we have data such as a population age-table, there are two possibilities: either the figures are correct, or they are not. The mere fact that the values do not proceed with the regularity of a mathematical table does not show that the figures are not correct. Even supposing (to avoid complications) that the numbers of births in successive years have been equal, or at any rate have proceeded according to some regular law, there will have been variations in the mortality which will cause irregularities, or "errors" (as distinct from mistakes), in the numbers of the survivors. (The accidental misstatements of age, to which Mr. King refers in § 28*, may be included in these.) The correct figures will therefore show irregularities. Conversely, if any process produces a smooth table, we may be sure that it does not represent facts.

We may, however, apply a method of graduation to the data in order to obtain a table which would show what the figures might have been expected to be if the accidental irregularities of mortality, &c., had not existed.

When the figures given are not correct, two courses are open to us. One is to try to obtain the most probable values of the correct figures. The other is to try to obtain a table of the kind just described above. It is not clear which of these two things Mr. King is aiming at, and consequently it is hardly possible to criticise his method as a whole.

It may, however, be mentioned that the so-called "graduated quinquennial values" are not obtained by graduation at all, in the ordinary sense, but by interpolation. Denoting by T_x the total male population enumerated as of age x and upwards, Mr. King, for very good reasons, rejects all the T 's except T_4 , T_9 , T_{14} . . . The T 's retained form a table such as the following:

x	T_x	1st Difference	2nd Difference	3rd Difference
⋮	⋮	—	+	⋮
⋮	⋮	⋮	⋮	⋮
39	4 898 452	⋮	⋮	⋮
		1 118 644	⋮	⋮
44	3 779 808	⋮	174 372	⋮
		944 272	⋮	— 30 500
49	2 835 536	⋮	143 872	⋮
		800 400	⋮	+ 18 942
54	2 035 136	⋮	162 814	⋮
		637 586	⋮	— 29 644
59	1 397 550	⋮	133 170	⋮
		504 416	⋮	⋮
64	893 134	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮

* Mr. King says that, when large numbers are involved, these errors tend to neutralise each other. This, of course, is only true relatively; absolutely, the net error increases as the total numbers increase.

From this table, assuming differences of the (true) T 's above the 3rd to be negligible, we might by central-difference interpolation (using, *e.g.*, Everett's formula) obtain a table at intervals of 1 year. This table would be :

x	T_x	1st Difference
⋮	⋮	—
⋮	⋮	⋮
49	2 835 536	⋮
		172 196
50	2 663 340	
		166 289
51	2 497 051	
		160 232
52	2 336 819	
		154 022
53	2 182 797	
		147 661
54	2 035 136	
		139 594
55	1 895 542	
		133 318
56	1 762 224	
		127 280
57	1 634 944	
		121 479
58	1 513 465	
		115 915
59	1 397 550	⋮
⋮	⋮	⋮
⋮	⋮	⋮

The 1st differences shown in heavy type in the above are the values of u_x (\doteq number of age x) which Mr. King adopts, the others being rejected.

From these selected values of u_x , obtained on the assumption that differences of the T 's above the 3rd—*i.e.*, differences of the u 's above the 2nd—are negligible, Mr. King obtains his remaining values by osculatory interpolation. The formula which he uses is correct to 2nd differences, so that its use is consistent with the assumption mentioned above. But the investigation can hardly be regarded as complete until the tables have been tested in order to find whether the assumption is sound.

The table found in this way runs from age 16 to the corresponding age at the other end—apparently 96. Mr. King then extends the table backwards to age 14 by the ordinary 3rd-difference formula, using u_{18} , u_{17} , u_{16} , and u_{11} . But it is not explained why 3rd differences, which previously have been neglected, are now taken into account, nor why this process of interpolation between u_{16} and u_{11} is not used to find u_{13} and u_{12} as well as to find u_{15} and u_{14} . As regards this latter point, it would appear that, at any rate in the case of the males, the agreement with the original data is better

if, with Mr. King, we stop at age 14. As the osculatory interpolation is performed on the u 's, not on the T 's, the sum of the u 's is altered, so that there is a discrepancy between the total enumerated and the total calculated. The former total, for ages 16 to 96, is 11 580 131, and the latter total (assuming the calculated u_{96} to be that shown in the table*) is 11 573 286, so that the method leaves 6 845 unaccounted for. If now we join up the values of u_{11} ($= 353\ 159$), u_{16} , u_{17} , and u_{18} by a curve of this 3rd degree, we get the following results:

x	Enumerated u_x	Calculated u_x	Alteration
15	334 241	340 125	+ 5 884
14	342 933	344 103	+ 1 170
13	345 144	347 507	+ 2 363
12	349 205	350 477	+ 1 272
11	351 287	353 159	+ 1 872

It will be seen that, if we enter the thus calculated values for u_{15} and u_{14} , we get an addition of 7 054, which just counterbalances the loss of 6 845, leaving a net addition of 209. This trifling discrepancy is apparently met by knocking 123 off ages under 14, and 86 off ages 97 and upwards.

SUPPLEMENTARY REMARKS BY MR. KING.

The discussion of the paper continued until a late hour, and at its close there was no time for me to do more than make a few brief remarks in reply, and I am therefore grateful to the Editors of the *Journal* for allowing me to supplement what I then said.

The most salient feature of the discussion, if an omission may be called a salient feature, was the absolute silence of all the speakers except Dr. Stevenson on the question of the principles which, it was laid down, should be kept in view in preparing the National Life Tables. The principles were set forth fully in Part I, "Introductory", of my Report upon the Life Tables, and were repeated in my paper, but it may be well to reproduce them once more. It was laid down that "It was desirable that a method should be employed, simple in theory, easy in application, and which would produce curves of smooth graduation, and curves which would adhere closely to the original data"; and, again, "In constructing the present Life Tables no correction has been attempted to counteract possible misstatements of age. An unbiased position has been assumed, and an effort has been made to construct the tables absolutely in accordance with the facts as recorded, and to allow those interested in the subject to pursue further investigations, and to make such corrections as may, in their opinion, be thought necessary."

As stated above, Dr. Stevenson, the Superintendent of Statistics

* It is unfortunate that the enumerated at ages of 100 and upwards are only given in a lump.

at Somerset House, alone of all the speakers referred to these principles, and he endorsed them. It was open to any critic, whether reviewer, or speaker at the discussion, to call in question the propriety of the laying down of such principles, and to suggest other principles which might have been followed, but no one did so, and therefore judgment must go by default, and it must be assumed that it is generally admitted that the course pursued is the correct one. It follows that the criticisms passed upon the Report on the Life Tables and upon my paper are, for the most part, beside the point, although many interesting questions were raised which are well worthy of debate and investigation, and I hope, as I have said on more than one occasion, that some of our younger members will take the matter up, and produce valuable papers to be read before the Institute.

The most important and interesting question discussed was that of the larger age misstatements, called by me "major deliberate errors", and by others "systematic errors." It will be remembered that I do not deny the existence of such errors, but frankly admit them, but that I add that in my opinion they do not appreciably affect the mortality tables until the old ages. That was not my original opinion, because when commencing the investigation I expected to find that considerable effect was produced throughout the tables by these major deliberate errors, and it was only after prolonged investigation that I was driven to the conclusion that the disturbance introduced by them into the tables was inappreciable, and might be neglected.

Dr. Stevenson called attention to Diagram 5 in Vol. vii of the Registrar-General's Report on the Census of 1911. That diagram shows the age distribution of the population for England and Wales separately, and for the United Kingdom as a whole, and it appears that the disturbance from ages 20 to 40, which some attribute to age misstatements, and others principally to migration, is more marked in England and Wales than when Scotland and Ireland are included, and hence it may be argued that at least part of the disturbance is caused by migrations from Scotland and Ireland into England. The diagram does not show the actual populations, but assumed populations to a given total number living. I have thought it worth while to complete, as it were, that diagram by taking out in the same way the age distribution of the populations for Scotland and Ireland separately, and these are given in Table A annexed, and are represented in Diagrams 6 and 7. The table shows the age distribution in each section of the United Kingdom for one million males and one million females living aged from 16 to 35 last birthday, while the diagrams assume one hundred thousand living. The figures for Scotland have been calculated by the Census method in exactly the same way as those for England and Wales, the central points of age being 21, 26, 31, 36, &c. For Ireland the results of the Census have not been published age by age, but after age 21 only in age groups, and therefore it was necessary to take as central points of age 22, 27, &c, but this slight difference in the method of calculation cannot appreciably affect the results. It is

Population

Diagram N° 6. Males.

Graduated Population at each age, out of 100,000 living aged 16 to 35 last birthday, inclusive.

England & Wales ———

Scotland.....

Ireland.....

6250

6000

5750

5500

5250

5000

4750

4500

4250

4000

Age → 16

18

20

22

24

26

28

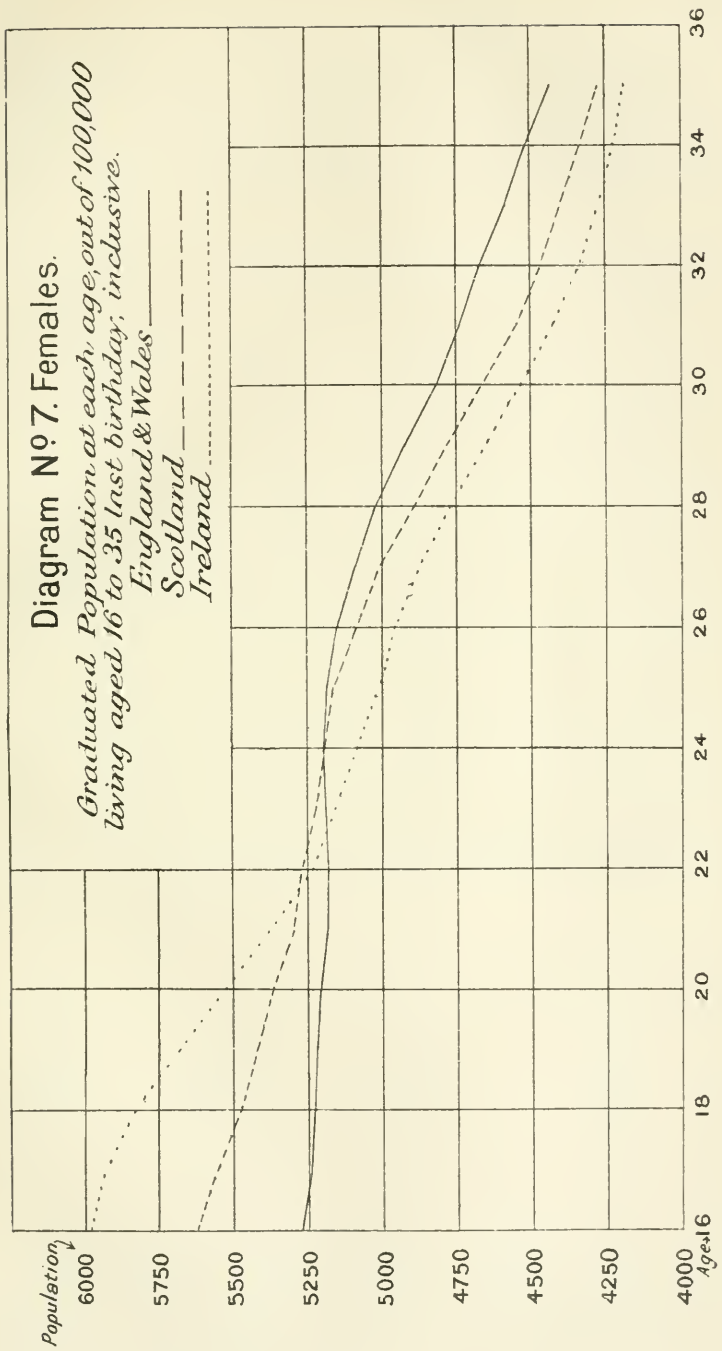
30

32

34

36

34819



seen from the diagrams that if, as some desire, a certain number enumerated in England and Wales between ages 20 and 30 are transferred to age group 30 to 40, then the divergence between England and Wales and Scotland or Ireland becomes more accentuated, and this rather tends to confirm the view that at least part of the disturbance in England and Wales is caused by migrations.

In my paper, in Diagram 3, I set forth the central death rate from age 15 to 55, and it appears that the death rate increases throughout steadily with age, whereas if a transfer were made of some of those living between 20 and 30 to the next decade there would be an increase in the death rate below age 30, and a diminution above age 30, and there would probably be an actual fall after age 30, with a minimum, and then a rise again. I argue that the steady progression of the curve proves that there is no appreciable disturbance from age misstatements, but Mr. Rowland takes the opposite view, and calls in the H^F and the O^F Tables in support thereof, alleging that these tables show a maximum female rate of mortality at about age 25, with a corresponding minimum in the thirties. This is a point well worth investigating. In these insurance tables Mr. Rowland says that the ages can be relied upon, and with this I agree, but he adds that the tables may be trusted "because the lives were not all subject to recent selection." Exactly what he means by the words "not all" I do not know, but certainly a great many of the lives were subject to recent selection. The particulars given in the Institute volume of mortality experience for female lives, although complete, are not in a form which lends itself to investigation without the expenditure of a good deal of trouble and time, which at present I cannot give, and, moreover, the Institute female experience is limited, and from the present point of view not of much value. We can, however, examine the O^F Table, and the following are the numbers at risk for quinquennial age groups from 15 to 39 :

Ages	O^F Table	$O^{F(5)}$ Table	Recent Lives
15-19	3,564	1,206	2,358
20-24	8,951	3,423	5,528
25-29	17,952	7,159	10,793
30-34	29,650	14,339	15,311
35-39	42,354	24,633	17,721
Total	102,471	50,760	51,711

The recent lives are those assured for less than five years, and they bear a very large proportion to the total. I have always understood that in tables constructed from assured lives, the selection was greatest just at this section of the table, but Mr. Rowland seems to think differently.

We get rid of a great part of the effects of selection by ignoring the first five years of assurance, and I have therefore graduated the $O^{F(5)}$ Table from age 15 to age 50, and the results are given in Table B annexed as far as age 39. The graduation was effected by the Census method, taking age groups 15-19, 20-24, &c., as that gives very little trouble, and occupies very little time, but it is good as shown by the comparison of actual with expected deaths. Table C annexed accentuates this point, and shows that in each of the five age groups the deviation is exceedingly small. The curve produced is a little undulating, but for present purposes that does not matter.

Looking at the column of q_x , it is seen that Mr. Rowland's statement is not borne out. There is no maximum at about age 25, and no corresponding minimum after age 30. It is true that there is a slight fall after age 38, with a minimum at age 41, when the rate begins to rise again, but this feature is very slight, and is beyond the range of ages we are discussing. My own opinion is that it is due to accidental causes, but others may differ from me. It thus appears that at the essential ages which have been discussed the $O^{F(5)}$ Table corresponds with the new National Tables, and therefore I am justified in repeating that the curve in my diagram No. 3 tends to show that misstatements of age have no appreciable effect on rates of mortality, and that therefore the disturbances in the populations must be largely due to migrations.

Mr. Henry paid attention more particularly to graduation, and to the tests which should be applied to ascertain the smoothness of a graduated table. He said that there is an inherent tendency in the summation method to reproduce accidental features and fluctuations of the ungraduated data, or, in other words, that summation formulas of graduation, while giving a smooth curve, adhere closely to the original data as recorded. In this I entirely agree, and that is the great merit of summation formulas. They show us what the curve really is, and then, if we think that bends are due to accidental causes, we can remove them. Mr. Henry, however, adds that my formula does very much to exaggerate this property of summation formulas, but there I differ from him. It is true that the Census method has a very slight cyclical tendency, but so slight that it cannot be discovered without close investigation. The cyclical tendency in the statistics from which the National Life Tables are formed is very much more marked than any cyclical tendency in the formula, and that very cyclical tendency in the formula was used towards the correction of the cyclical tendency in the data, the age groups being so selected as to have this effect. Any cyclical tendency that remains in the graduated tables is thus due, not to the formula employed, but to the actual data.

Mr. Henry also says that my formula for finding the pivotal values has very small graduating power. But the formula is not used for the purpose of graduation. Wide experience shows that it gives excellent pivotal values, and that is all that it is intended to do. For applying the osculatory method of interpolation, pivotal values are a necessity, and I searched for a long time to find a

formula that would produce them with little labour, and, at the same time, with good results. In my paper on a New Method of Constructing and Graduating Mortality and other Tables, *J.I.A.*, vol. xliii, p. 115, I point out that the formula is an embryo graduation formula, and is the central of five grouped curves. That is the way it was first arrived at, but afterwards I derived it by a more direct process, given in that paper, in my report to the Registrar-General, and elsewhere. If another short formula for pivotal values, giving better results, could be found, it would be all to the good.

In discussing the smoothness of graduation, Mr. Henry says he does not gather what criterion I would apply. In my paper, however, I give the information. A table is smooth if all the third differences are very small, or, if not small, if they follow a definite law, and I give an example of large differences which are smooth, and which follow a definite law. Mr. Henry added that if the radius of curvature be changing very rapidly, this indicates that the curve is not a smooth one considered purely from the point of view of the graduation of actuarial statistics. I do not, however, think that we can separate actuarial statistics from others, and certainly in the case of some actuarial statistics we have smooth curves with rapid changes of curvature. My paper on the Construction of Mortality Tables from Census Returns, *J.I.A.*, vol. xlii, p. 225, gives an example of this. Diagram No. 2 gives the Carlisle death curve, and that is a smooth curve mathematically, and yet the variations in its curvature are very remarkable.

It is difficult to take Mr. Gunningham's remarks seriously, because evidently they were made without sufficient consideration. Speaking of the sample populations dealt with in my report on the Graduation of Ages, he asks "were these populations for 1901 graduated by the same method as those for 1911", but he has failed to notice that in paragraph 13 of the report I say emphatically that all the tables there mentioned were graduated in exactly the same way. It is true that while the sample populations of 1901 are included in the tabular statement to which this remark applies, I do not actually mention them in speaking of the graduation, but had Mr. Gunningham looked up his references he would have found that these tables had never before been graduated, and had he spent a few minutes testing the figures he would have ascertained for himself that they were graduated by the Census method. All the data for such test were supplied to him.

Then again, Mr. Gunningham says that the four tables, total populations for England and Wales, and their sub-divisions into single, married, and widowed, were all graduated in the same way, and on this assumption he makes certain deductions derogatory to some of the results of my work. He has entirely overlooked paragraph 17 of my report on the Graduation of Ages, where it is stated that all the four columns were graduated in the same way only from age 39 onwards, and that for ages younger than 39 the graduation method was quite different for the columns relating to marital condition. Yet Mr. Gunningham assumes that the methods

were the same from age 14 onwards, and therefore his criticisms fall to the ground.

Lastly, Mr. Gunningham says that when the first review was written the ungraduated figures were the only ones available, and that there were no graduated figures, and that the ungraduated had to tell their own tale. This is bewildering, because the First Reviewer was reviewing the report on the Graduation of Ages, and every graduated figure is given there, and no graduated figures have since been prepared. It is difficult to divine what Mr. Gunningham had in his mind when he made that statement. Surely it would have been wise on his part before speaking to have read carefully the report that he was discussing, and the review thereof by the First Reviewer.

Dr. Sheppard, in a foot-note to his addendum to the discussion, expresses a wish to have the enumerated centenarians given age by age. He will find that information on page 37 of my report upon the Life Tables.

It was suggested in the discussion that in the formula $u_7 = \cdot 2w_5 - \cdot 008\Delta^2w_0$ the term $\cdot 008\Delta^2w_0$ is so small that it may be neglected, and that we may simply write $u_7 = \cdot 2w_5$. It seems to me, however, that this would be a retrograde step, because by neglecting the second difference correction there is a tendency to take the curvature out of the curve. An illustration of this is afforded by the Government Annuity Tables of the late John Finlaison. His early summation formulas had no correction for the second difference, with the result that the annuity values in his tables were understated, and a loss was incurred by the Government. The great merit of summation formulas as we now know them is that they supply the necessary correction, and produce accurate results. It is true that if mortality tables be *constructed* by what has been called the "Census method", the second difference correction in the formula for the pivotal values might sometimes without appreciable error be omitted. Pivotal values in the construction of a mortality table are obtained for the exposed to risk and the deaths separately, and often the second difference correction is of the same sign, although seldom of the same proportionate magnitude, and when that is the case the correction scarcely affects the mortality table. Sometimes, however, in all the tables that I have worked on the second difference correction is of opposite sign, and at these points an error, which might be appreciable, would be introduced. When, however, the formula is used to *graduate* a table, such as the columns of populations in the Census data, I do not think the second difference correction can be neglected, as the table would be to a certain extent distorted, and the formula would not reproduce in the graduated column the exact total of the ungraduated populations, as the full formula with the second difference correction does. In my recent papers on Summation Formulas, &c., I gave a longer formula including a fourth difference correction, the results of which were more exact still; but the fourth difference correction is so very minute that it is not worth while taking the trouble to include it, and by omitting it the tables are not in any way vitiated.

TABLE A.

Age Distribution of Populations. Graduated Population at each age, out of 1,000,000 living, aged 16 to 35 last birthday inclusive.

Age last Birthday	England and Wales	Scotland	Ireland	Age last Birthday
<i>Males.</i>				
16	56,709	61,470	61,734	16
7	55,769	60,187	61,257	7
8	54,658	58,616	60,337	8
9	53,505	56,917	59,097	9
20	52,435	55,247	57,629	20
1	51,576	53,766	56,032	1
2	50,985	52,471	54,403	2
3	50,577	51,259	52,587	3
4	50,266	50,128	50,516	4
25	49,968	49,082	48,426	25
6	49,597	48,119	46,551	6
7	49,161	47,262	45,127	7
8	48,718	46,510	44,266	8
9	48,255	45,831	43,814	9
30	47,757	45,195	43,596	30
1	47,211	44,574	43,445	1
2	46,644	44,031	43,191	2
3	46,065	43,588	42,896	3
4	45,434	43,145	42,670	4
35	44,710	42,602	42,426	35
<i>Females.</i>				
16	52,686	56,259	59,797	16
7	52,419	55,565	59,354	7
8	52,232	54,870	58,348	8
9	52,098	54,197	56,912	9
20	51,989	53,565	55,286	20
1	51,879	52,995	53,708	1
2	51,833	52,560	52,418	2
3	51,869	52,247	51,494	3
4	51,888	51,946	50,777	4
25	51,794	51,546	50,149	25
6	51,488	50,937	49,491	6
7	50,912	50,035	48,690	7
8	50,132	48,916	47,675	8
9	49,235	47,703	46,523	9
30	48,306	46,518	45,338	30
1	47,430	45,485	44,225	1
2	46,636	44,670	43,287	2
3	45,865	43,991	42,621	3
4	45,077	43,349	42,157	4
35	44,232	42,646	41,750	35

TABLE B.

O^{F(5)} Table. Graduated by Osculatory Interpolation.

Age	q_x	At Risk	Actual Deaths	Expected Deaths	Deviation	Accumulated Deviation	Age
15	·00763	137	2	1·0	1·0	1·0	15
6	·00718	177	1	1·3	·3	·7	6
7	·00675	224	2	1·5	·5	1·2	7
8	·00635	288	1	1·8	·8	·4	8
9	·00597	380	2	2·3	·3	·1	9
20	·00565	479	2	2·7	·7	·6	20
1	·00544	594	5	3·2	1·8	1·2	1
2	·00539	686	3	3·7	·7	·5	2
3	·00555	781	6	4·3	1·7	2·2	3
4	·00589	883	3	5·2	2·2	—	4
25	·00633	1,001	7	6·3	·7	·7	25
6	·00679	1,177	5	8·0	3·0	2·3	6
7	·00717	1,397	12	10·0	2·0	·3	7
8	·00746	1,653	15	12·3	2·7	2·4	8
9	·00772	1,931	13	14·9	1·9	·5	9
30	·00797	2,212	14	17·6	3·6	3·1	30
1	·00826	2,515	20	20·8	·8	3·9	1
2	·00864	2,822	38	24·4	13·6	9·7	2
3	·00916	3,193	29	29·2	·2	9·5	3
4	·00981	3,597	25	35·3	10·3	·8	4
35	·01049	4,075	30	42·7	12·7	13·5	35
6	·01107	4,469	55	49·5	5·5	8·0	6
7	·01147	4,905	57	56·3	·7	7·3	7
8	·01160	5,390	61	62·5	1·5	8·8	8
9	·01154	5,794	78	66·9	11·1	2·3	9

TABLE C.

Summary of Table B.

O^{F(5)} Table. Graduated by Osculatory Interpolation.

Ages	At Risk	Actual Deaths	Expected Deaths	Deviation	Accumulated Deviation	Ages
15-19	1,206	8	7·9	·1	·1	15-19
20-24	3,423	19	19·1	·1	—	20-24
25-29	7,159	52	51·5	·5	·5	25-29
30-34	14,339	126	127·3	1·3	·8	30-34
35-39	24,633	281	277·9	3·1	2·3	35-39

On the Relation between the Method of Moments and the Method of Least Squares. By J. F. STEFFENSEN, D.Phil.

THE following few remarks have, in the main, been suggested by two notices in this *Journal** by Mr. W. Palin Elderton on papers which I have published elsewhere.† In both notices, while doing full justice to my work, Mr. Elderton seems to assume in a general way—and this opinion is, in fact, shared by many actuaries—that the method of moments gives practically as good results as the method of least squares, if proper account is taken of the weights of the observations in *both* cases.

In order to see that this cannot be generally correct, and that it must at least be incorrect in that particular class of cases to which the method of moments was first successfully applied, we will begin by establishing the well-known equations from which the constants are determined.

We denote by σ_x the observation, by w_x its weight, by y_x the function assumed for graduating the observations, and by $a_0, a_1, \dots a_{n-1}$ the n constants on which y_x depends. For determining these constants we have according to the method of least squares

$$\Sigma w(y - \sigma)^2 = \text{a minimum}$$

or, for $r=0, 1, \dots (n-1)$,

$$\Sigma w(y - \sigma) \frac{\delta y}{\delta a_r} = 0 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The method of moments gives, on the other hand, when the same weights are used for the observations

$$\Sigma w(y - \sigma)x^r = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The most obvious and also most important case in which the two sets of equations (1) and (2) give identical values for the constants is when

$$\frac{\delta y}{\delta a_r} = x^r \quad . \quad . \quad . \quad . \quad . \quad (3)$$

for all values of r .‡ It follows that

* Vol. xlvii, p. 431, and vol. xlix, p. 172.

† *Proceedings of the Fifth International Congress of Mathematicians*, Cambridge, 1913, Vol. II, p. 389; and *Svenska Aktuarieforeningens Tidskrift*, 1915, p. 1.

‡ The generalization obtained by taking

$$\frac{\delta y}{\delta a_r} = b_0^{(r)} + b_1^{(r)}x + \dots + b_{n-1}^{(r)}x^{n-1}$$

is only apparent.

$$y = a_r x^r + \phi(x, a_0, a_1, \dots a_{r-1}, a_{r+1}, \dots a_{n-1})$$

for all values of r , whence

$$y = a_0 + a_1 x + \dots + a_{n-1} x^{n-1} + \psi(x) \quad . \quad . \quad (4)$$

where $\psi(x)$ is independent of the constants a_r .

We have in (4) the most important form of function for which the method of moments and the method of least squares, *both with weights*, produce identical results.

Now, it must be noted that, in spite of the arbitrary function $\psi(x)$ contained in (4), this formula, looked upon as a weapon of graduation, has a rather limited range. As $\psi(x)$ stands for a function *which does not depend on the unknown constants* but can otherwise be arbitrarily chosen, it is clear that graduating σ_x by (4) is equivalent to graduating $\sigma_x - \psi(x)$ by a polynomial of degree $n-1$. But the series of observations which can, after deduction of a *known* function, be graduated by a polynomial is rather limited. This is, at least, the case if the number of unknown constants is small in comparison with the number of observations, as ought always to be the case when a graduation, and not an interpolation, is contemplated. Everybody knows, for instance, that a function such as μ_x or a frequency-distribution such as E_x cannot, over a range of, say, fifty or sixty years, be adequately represented by a polynomial of the second degree.

In order to apply (4) with profit we must, therefore, do one of two things:

- (a). We may attempt to choose $\psi(x)$ so close to y_x , that the constants a_r become very small. In this case the polynomial is only to be considered as a correction applied to $\psi(x)$, but then the difficulty is to make a sufficiently correct guess at $\psi(x)$ from the outset.
- (b). We may confine the graduation to a comparatively short interval, in which case we may often at the same time with little loss of accuracy assume equal weights for the observations. This is, however, not to solve the real problem, which is to represent the whole series of data by one function. Also, by reducing the proportion of observations to constants we are liable to under-graduate (if the expression be allowed) the observations.

An objection to employing (2) which, to my mind, outweighs all other arguments is this, that even if it were true

that (2) yields practically the same values for the constants as (1), there would be no advantage in using (2) instead of (1). The great advantage of the *unweighted* method of moments, or the equations

$$\Sigma(y - \sigma)x^r = 0, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

is that these equations are sometimes comparatively easy to solve, while the method of least squares, even when the weights are omitted, leads nearly always to very difficult equations for the constants. But the weighted equations for the moments, or (2), are generally just as difficult to solve as (1).

Why, then, not prefer the strict equations of least squares to the weighted equations of moments, when these latter derive their questionable legitimacy from the former and are no easier to solve?

The great success that has attended the method of moments in one particular field of research, and hence led many investigators to believe that this method is a sort of universal instrument, is due to the quite opposite fact that in certain cases the *unweighted* equations of moments are identical with the *weighted* equations of least squares. Or, in other terms: for certain forms of the function y and the weights w_x the equations (5) and (1) are identical.*

In order that this shall take place, we may put

$$w \frac{\delta y}{\delta a_r} = x^r \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

As w may depend on the constants a_r , these equations are not so easy to deal with as (3). An important solution, how-

* At least as early as 1879 this fact was known to Danish actuaries, as appears from a book by J. P. Gram: "Om Rakkendviklinger, bestemte ved Hjælp af de mindste Kvadraters Methode", Copenhagen, 1879, p. 94. I quote the passage in translation, as it is not without historical interest.

"It is by researches into these 'skew frequency-curves' that I was originally led to the general theory which forms the basis of this paper. A considerable time ago Professor Oppermann suggested the use of the formula $y = (a_0 + a_1x + a_2x^2 + \dots)e^{-x^2}$ for representing the equation to such curves, and also pointed out that in order to determine the constants in this formula it will be necessary to employ equations of the form $\Sigma x^i \sigma_x = \Sigma x^i y_x$, where the latter can be replaced by the corresponding integral. This mode of determination remained, for some time, something of a puzzle to me, and only through various researches of a more general nature did I succeed in realizing the true meaning of the method as a graduation by the method of least squares according to the formula $y = (a_0 + a_1x + \dots)e^{-x^2}$, assuming the weights proportional to e^{x^2} ."

Another important case will be dealt with below, viz., formula (10), assuming the weights proportional to $\frac{1}{y}$. English actuaries will be familiar with this case through G. F. Hardy's book on the Construction of Tables of Mortality, p. 129.

ever, is obtained when y is a function of a polynomial whose coefficients are the unknown constants, that is

$$y = \phi(a_0 + a_1x + \dots + a_{n-1}x^{n-1}) \quad . \quad . \quad (7)$$

If y has this form, (6) will be satisfied provided

$$w = \frac{1}{\phi'(a_0 + a_1x + \dots + a_{n-1}x^{n-1})} \quad . \quad (8)$$

A still more general form is obtained if we put

$$y = f(p, x); \quad w = \frac{1}{\delta_p f} \quad . \quad . \quad . \quad . \quad (9)$$

where p denotes the above polynomial. It is, of course, understood that the constants a_r only occur in p and not in the component parts of f .

Now a remarkable fact, which has long been known, is that there are cases where (8) gives an approximately correct value for the weights. Let, for example,

$$y = e^{a_0 + a_1x + \dots + a_{n-1}x^{n-1}} \quad . \quad . \quad . \quad . \quad (10)$$

represent a *frequency-distribution*; in that case, when the number of observations is large, we shall have approximately

$w = \frac{1}{y}$, or the same value which is derived from (8). In other words:

If a frequency-distribution can be represented by an expression of the form (10), then, if the number of observations is large, we obtain practically the same values for the constants by determining them from the *unweighted* equations of moments as from the *weighted* equations of least squares.

Most frequency-curves can more or less approximately be represented by an expression of the form (10). This is probably the true reason why the method of moments has proved such a powerful instrument for determining the constants of frequency-curves. But the same consideration is a warning against using that method indiscriminately, with or without weights, outside its natural scope.

The cases where it is said that the *weighted* method of moments* has been successfully applied to curves (such as

* Yet with incorrect weights.

the force of mortality) which are *not* frequency-curves, will, on inspection, dissolve into cases where some artifice has been brought in (such as working directly on the exposures and deaths) by which the problem has been transformed into that of applying the *unweighted* method of moments to a *frequency-curve*. Hence the success.

LEGAL NOTES.

By WILLIAM CHARLES SHARMAN, F.I.A., *Barrister-at-Law*.

Policy effected for funeral expenses. Section 36, Assurance Companies Act, 1909. A CASE of considerable interest and importance to Industrial Assurance Companies is that of *Wolenberg v. Royal Co-operative Collecting Society*, 84 L.J., K.B. 1316. The case was an appeal to the Divisional Court against a decision of the County Court Judge, and deals with the question whether a person who has effected several policies with different companies under Sec. 36 (1) of the Assurance Companies Act, 1909, against any funeral expenses he may incur on the death of his mother, and on his mother's death is paid by one or more companies the full amount of such expenses, can maintain a further claim against another of the companies which has refused to pay the amount of the policy.

It was held that such a claim could not be maintained, and moreover, in the absence of fraud or mistake of fact the return of the premiums paid could not be obtained. It is interesting to note that in the course of the action an *obiter dictum* was expressed by the Judge that policies issued under Sec. 36 (1) of the Assurance Companies Act are policies of indemnity. The facts of the case are as follows:—On 11 December, 1911, the defendants, an industrial assurance society, issued a policy of insurance to the plaintiff by which she was insured against any funeral expenses to which she might be put on the death of her mother, Fanny Goldberg, to the extent of £19. Subsequently, Fanny Goldberg died. The defendants having refused to pay to the plaintiff the amount named in the policy, the plaintiff brought this action in the County Court, claiming £19, the sum assured, or in the alternative

the return of £4 17s. 6d., premiums paid by her in respect of such policy.

During the hearing of the action it appeared from the evidence that the plaintiff had taken out several other policies, similar to the one in question, with other assurance companies, and that on the death of her mother she had received some £70 from three of these other companies.

The County Court Judge held that the plaintiff was entitled to recover on the policy, but that the sum of £19 was excessive and unreasonable, and entered judgment for the plaintiff for £10.

The plaintiff now appealed from that decision on the ground that she should have been awarded the full amount of her claim—that is, £19—or in the alternative a further sum of £2 8s. 9d., being the amount of premiums paid by her over and above those payable on a policy for £10.

Lush, J., in delivering judgment said:— “The answer to
“the question which we have to decide in this case depends
“upon the construction we give to sub-section 1 of Section 36 of
“the Assurance Companies Act, 1909. Before proceeding to the
“consideration of the wording of the sub-section it is important
“to look at the object of section 36 and the evil it was designed
“to remedy. Before 1909 insurance societies had been in the
“habit of issuing policies in respect of funeral expenses to be
“incurred by the policyholders upon the death of certain
“relatives and there were a large number of these policies in
“existence. The question whether or not a person had an
“insurable interest in respect of such a policy depended on
“whether or not he was under a legal obligation to bury such
“relative. That resulted in constant difficulty. There was an
“insurable interest only when there was such an obligation, and
“a merely moral obligation was not sufficient to create an
“insurable interest. The object of section 36 was to remedy
“this. The section is divided into two sub-sections, sub-section 1
“dealing with policies to be made after the passing of the Act,
“and sub-section 2 with policies in existence at that date. As
“this policy was issued in 1911, it is sub-section 1 with which
“we are concerned. It is to be noted that in sub-section 1
“entirely different language is used from that in sub-section 2.
“There is no provision that the amount assured must be
“‘reasonable.’ The only effect of the section is to render valid
“policies which insure ‘money to be paid for the funeral expenses
“of a parent, grandparent, grandchild, brother, or sister.’

“ There are two views that may be taken of the sub-section.
“ One is that it treats these policies as in the category of life
“ policies, and that it gives to the relative an insurable interest
“ in respect of money he may be called upon to pay for the
“ funeral expenses of the relative. In that view we should
“ have to consider whether, at the time the policy was effected,
“ the policyholder came within the necessary degrees of
“ relationship; and, secondly, whether the policy could at the
“ time it was effected properly be regarded as a policy the only
“ purpose of which was to insure against funeral expenses. The
“ other view is that these policies ought to be treated as
“ policies of indemnity—contracts by which the assurance
“ societies undertake to indemnify the policyholder to the
“ extent of the money which the policyholders will ultimately
“ pay in respect of funeral expenses of their relatives.

“ It is not easy to determine which of these is the correct
“ view. I am myself inclined to think that the latter is
“ right, and that these policies must be treated as policies by
“ way of indemnity. It is a matter of very considerable
“ importance, but I do not think it is necessary to decide it in
“ this particular case. Whichever view is right, I am clearly of
“ opinion that, if a policyholder has effected several policies in
“ respect of a risk and has been paid on those policies by one
“ or more assurance societies the full amount of the expenses to
“ which he has been put, he cannot make a further claim
“ against any other society with which he has insured against
“ the same risk. The Legislature never authorised a relative to
“ take out a number of policies under this section and make a
“ profit from them. Such a transaction would be contrary to
“ the whole law of insurance, and is in no way justified by
“ section 36, sub-section 1. In my opinion, the defendants in
“ this case present a complete answer to the plaintiff's claim
“ when they show that when she brought this action she was
“ not a penny out of pocket as the result of her mother's death,
“ but, on the contrary, had actually received a large sum of
“ money and had made a considerable profit. The County
“ Court Judge should have held that the plaintiff had no cause
“ of action and no right to recover on the policy. I am aware
“ that many difficulties might arise in regard to these questions,
“ but in this case I feel no difficulty in saying that the plaintiff
“ cannot recover the sum claimed, nor can she obtain the return
“ of the premiums she has paid. She insured in respect of

“ funeral expenses and effected other policies against the same
 “ risk in other assurance societies. She has paid the premiums
 “ with the knowledge that the society took a risk during the
 “ currency of the policy. In the absence of fraud or mistake of
 “ fact I can see no ground for the contention that, because the
 “ society is not liable to pay the policy moneys, the premiums
 “ should be repaid. In my opinion, the judgment of the County
 “ Court Judge must be set aside and judgment entered for the
 “ defendants.”

Atkin, J., also delivered an assenting judgment.

Contract of
 re-insurance.
 Liquidation of
 original assur-
 ance society.
 Claim for benefit
 of re-insurances.

The case of *In re Law Guarantee Trust and Accident Society*, Godson's Claim, reported (1915) L.R. 1 Ch. 340, is of some interest, as it decided the way in which the moneys payable under a contract of re-insurance were to be appropriated in the liquidation of the original assurance society. The facts are briefly as follows: A brewery company made a debenture issue with a trust deed of which an assurance society were the trustees. By a contract indorsed on each debenture the society guaranteed the registered holder payment of the principal and interest secured by the debenture if the brewery company should make default. The society received for this guarantee a consideration in the nature of an annual premium, and also an annual fee for acting as trustees, the payment of which was provided for by the trust deed. The society then re-insured part of their risk under the guarantee with another insurance company. Subsequently both the brewery company and the society went into liquidation, and the debentures remained unpaid. A purchaser of all the debentures of the brewery company proved in the liquidation of the society for the amount due under the debentures after deducting the value of his securities, and his proof was admitted. He also claimed to be entitled, as against the general creditors of the society, to the benefit of all re-insurance effected by the society in connection with the debentures.

Neville, J., held that he was not so entitled. Although under the trust deed there was a fiduciary relation between the society and the debenture holders, there was no fiduciary relation between them under the contract by which the payment of the debentures was guaranteed. Consequently the society

were entitled to recover the money from their re-insurers and to apply it to their own use as part of their general assets, and the holder of the debentures had no special claim on it.

The question whether a married woman could effect a policy on her own life for the benefit of her children under section 10 of the Married Women's Property Act, 1870, was considered in the case of *In re Burgess's Policy, Lee v. Scottish Union and National Insurance Co.*, a report of which is to be found in the *Law Journal* 5 June 1915.

Married Women's
Property Act,
1870.

Policy effected by
a married woman
for the benefit of
her children.

On 2 August 1872 Mrs. Burgess effected with the defendant company a policy of insurance on her own life for £500. The policy contained the following recital: "Whereas M. E. Burgess, the wife of J. Burgess, being desirous to effect an insurance on her own life in terms of the provisions of the Married Women's Property Act, 1870, for the benefit of her children for the whole term thereof to the extent of £500. . . ."

Mrs. Burgess was referred to in the policy as "the assured" and provision was made for payment of the premium by her executors in the event of her death within 30 days after the day for payment. There was issue of the marriage eleven children, nine of whom were still living. The husband died in January 1901. By her will dated 28 August 1913, after appointing the plaintiffs executors and trustees, Mrs. Burgess bequeathed the policy and policy moneys to her four daughters in equal shares absolutely. The testatrix died on 19 November 1914, leaving her four daughters and five sons her surviving, all of whom were born before the date of the policy. A summons was taken out by the executors to have it determined whether the four daughters were entitled to the policy moneys.

Eve, J., said the policy was effected under section 10 of the Married Women's Property Act, 1870. Under that act the testatrix had power to effect an insurance on her own life for her separate estate, but she had no power to take out a policy for the benefit of her children. The nine children were not *cestuis que trust*. That was concluded by the case of *Cleaver v. Mutual Reserve Fund Life Association* (1892) 61 L.J., Q.B. 128; (1892) 1 Q.B. 147. The contract was with the assured, and the policy moneys belonged to her estate alone and went to the four daughters who were legatees under the will.

"Mortgagee in possession."

Courts (Emergency Powers) Act 1914.

The case of *Ziman v. Komata Reefs Gold Mining Company* was recently reported in these notes (*J.I.A.*, xlix, 287) and it may now be mentioned that a Bill has been presented to Parliament by the Solicitor-General, the object of which is to reverse the decision in this case by making it clear that the expression "mortgagee in possession" where used in the Courts (Emergency Powers) Act, 1914, does not include mortgagees of personal property.

Finance Act, 1915.

The Finance Act, 1915, is largely concerned with the question of the assessment of Life Insurance companies to Income Tax, and is of the greatest importance to those who have to deal with this vexed question. Considerable alterations are also made in the rebate allowed from Income Tax in respect of life assurance premiums.

In view of the fact that the Act represents a complete revision of the whole basis of taxation which has hitherto been adopted, it is perhaps inadvisable to attempt a summary of it here, and it has been thought better to print the clauses dealing with insurance business *in extenso*. The full text of these clauses is as follows :

11. Where an assurance company carries on life assurance business in conjunction with assurance business of any other class, the life assurance business of the company shall, for the purposes of the Income Tax Acts, be treated as a separate business from any other class of business carried on by the company.

12. In ascertaining for the purposes of section one hundred and one of the Income Tax Act, 1842, or of section twenty-three of the Customs and Inland Revenue Act, 1890, whether an assurance company has sustained a loss in respect of its life assurance business, any income of the company derived from the investment of its life assurance fund shall be treated as part of the profits of the company required in that business.

13. The amount of annuities which an assurance company carrying on the business of granting annuities is entitled, for the purposes of subsection (3) of section twenty-four of the Customs and Inland Revenue Act, 1888, to treat as having been paid out of profits or gains brought into charge to income tax shall not exceed the amount of the taxed income of its annuity fund.

14.—(1) Where an assurance company carrying on life assurance business or any company whose business consists mainly in the making of investments, and the principal part of whose income is derived therefrom, claims and proves to the satisfaction of the Special Commissioners that for any income tax year it has been charged to income tax by deduction or otherwise, and has not been so charged in respect of its profits in accordance with the rules under the first case in section one hundred of the Income Tax Act, 1842, the company shall be entitled to repayment of so much of the tax paid by it as is equal to the amount of the tax on any sums disbursed as expenses of management (including commissions) for that year :

Provided that—

(a) relief shall not be given under this section so as to make the income tax paid by the company less than the tax which would have been paid if the profits of the company had been charged in accordance with the said rules ; and

(b) the amount of any fines, fees, or profits arising from reversions in the case of an assurance company, and in the case of any other company the amount of any income or profits derived from sources not charged to income tax, shall be deducted from the amount treated as expenses of management for the year; and

(c) in calculating profits arising from reversions, the company may set off against those profits any loss arising from reversions for any previous year during which this section was in operation.

(2) Notice of any claim to the Special Commissioners under this section together with the particulars thereof shall be given in writing to the Surveyor of Taxes for the district within twelve months after the expiration of the income tax year in respect of which the claim is made, and where the surveyor objects to such claim the Special Commissioners shall hear and determine the same in like manner as in the case of an appeal to them against an assessment under Schedule D., and section fifty-nine of the Taxes Management Act, 1880 (which relates to the statement of a case on a point of law), and any rules made for the purposes of that section shall apply in the case of any such appeal.

(3) A company shall not be entitled to any relief under this section in respect of any expenses as to which relief may be claimed or allowed under section thirty-five of the Finance Act, 1894, or section sixty-nine of the Finance (1909-10) Act, 1910, as extended by section eight of the Finance Act, 1914, by which enactments relief is conferred in respect of the cost of maintenance, repairs, insurance, or management of land or houses.

15.—(1) Where an assurance company not having its head office in the United Kingdom carries on life assurance business through any branch or agency in the United Kingdom, any income of the company from the investments of its life assurance fund (excluding the annuity fund, if any), wherever received, shall, to the extent provided in this section, be deemed to be profits comprised in Schedule D. of the Income Tax Act, 1853, and shall be charged under the rules of the third case in section one hundred of the Income Tax Act, 1842.

(2) Such portion only of the income from the investments of the life assurance fund shall be charged under this section as bears the same proportion to the total income from those investments as the amount of premiums received in that year from policy holders resident in the United Kingdom and from policy holders resident abroad whose proposals were made to the company at or through its office or agency in the United Kingdom bears to the total amount of the premiums received by the company.

Provided that in the case of an assurance company having its head office in any British possession the Commissioners of Inland Revenue may, by regulation, substitute some basis other than that prescribed by this section for the purpose of ascertaining the portion of the income from investments to be charged under this section as being income derived from business carried on in the United Kingdom.

(3) The relief conferred by this Act in respect of expenses of management shall, in the case of a company charged to income tax under this section, be calculated by reference to a like proportion of the total expenses of management of the company for the year estimated in accordance with the provisions of this Act.

(4) Every assessment under this section shall be made by the Special Commissioners as though the company under the provisions of the Income Tax Acts had required the proceedings relating to the assessment to be had and taken before those Commissioners.

(5) Where a company has already been charged to income tax, by deduction or otherwise, in respect of its life assurance business, to an amount equal to or exceeding the charge under this section, no further charge shall be made under this section, and where a company has already been so charged, but to a less amount, the charge under this section shall be proportionately reduced.

16. Section five of the Finance Act, 1914 (which provides for the taxation of income in respect of foreign property), shall not apply to income arising from the sources specified in that section of an assurance company so far as that income arises from the investments of the foreign life assurance fund of the company, but a corresponding reduction shall be made in the relief granted under this Act in respect of expenses of management.

17.—(1) A person shall not be entitled under section fifty-four of the Income Tax Act, 1853 (as amended by any subsequent enactment), to deduct from profits or gains—

(a) In respect of any premium or other payment payable on a policy for securing a capital sum on death (whether in conjunction with any other benefit or not), more than seven per cent. of the actual capital sum assured; and

(b) In respect of any premiums or payments to which that section applies payable for securing any other benefits, more than one hundred pounds in all;

and the relief by way of repayment of tax under that section, or by way of deduction for the purposes of supertax under paragraph (b) of subsection (2) of section sixty-six of the Finance (1909-10) Act, 1910, shall be correspondingly limited.

(2) In calculating the deduction under this section in respect of any premium or other payment payable on a policy for securing a capital sum on death no account shall be taken of any sum payable on the happening of any other contingency or of the value of any premiums agreed to be returned or of any benefit by way of bonus or otherwise, which is to be or may be received either before or after death, either by the person paying the premium, or by any other person, and which is not the sum actually assured.

ACTUARIAL NOTES.

On the determination, by means of Bond-value Tables, of the rate yielded by a redeemable bond when Income Tax is taken into account.

A Bond of 1 repayable at par at the end of n years and bearing interest at the rate of g per annum, payable half-yearly, subject to tax at rate t per unit, is bought at a price of $1 + K$ where K may be $+$ or $-$. Find j , the rate per annum payable half-yearly subject to tax at the same rate, yielded by the purchase.

We must have $K = (g - j)(1 - t)^{\frac{1}{2}} \cdot a_{\frac{1}{2}n}^{j(1-t)}$, or dropping the subscript $2n$ for brevity and writing $K \div (1 - t) = K'$

$$\frac{K}{1-t} = K' = \frac{1}{2}(g-j)a^{\frac{1}{2}j(1-t)},$$

or $a^{\frac{1}{2}j(1-t)} = \frac{K'}{\frac{1}{2}(g-j)} \quad \dots \dots \dots (1)$

A first approximation, say j' , is evidently given by finding the rate corresponding to a price of $1 + K'$ in the tables. A closer approximation may be found as follows: Since j' is the rate corresponding to a price of $1 + K'$ when tax is not taken into account,

$$a^{\frac{1}{2}j'} = \frac{K'}{\frac{1}{2}(g-j')} \quad \dots \dots \dots (2)$$

From (1) and (2),

$$K' \left(\frac{1}{a^{\frac{1}{2}j'}} - \frac{1}{a^{\frac{1}{2}j(1-t)}} \right) = \frac{1}{2} (j - j') \quad \dots \quad (3)$$

Suppose that for a small variation, say x , in the rate, we have $\frac{1}{a^{i+x}} - \frac{1}{a^i} = \theta x$. Then from (3)

$$j - j' = K' \theta [j' - j(1-t)]$$

whence

$$j = j' \cdot \frac{1 + K' \theta}{1 + K \theta} \quad \dots \quad (4)$$

This formula, though in itself exact, can only be applied approximately because it involves θ which itself depends on the rate which is sought. It happens, however (*c.f.* Henderson, *T.A.S.A.*, vol. x, pp. 660-1), that θ varies very slowly with i , and for all practical rates and for terms not less than 3 years* nor over about 30 years, θ may be taken as $= \frac{2}{3}$ with a maximum error of about .06 and in general a much smaller error. Now a small error ϵ in the value of θ may be shown to lead to an error of approximately $\epsilon j' \frac{t}{1-t} \frac{K}{(1+K\theta)^2}$ in the result. Hence, if $\epsilon \geq .06$, $j \geq .05$, $\frac{t}{1-t} \geq \frac{1}{7}$ (corresponding to tax at 2s. 6d. in the £), and $K \geq .2$ numerically, the maximum error in the result will be $.06 \times .05 \times \frac{1}{7} \times \frac{.2}{(1.133)^2} = .00007$ or under 2d. per-cent if K be +, or $.06 \times .05 \times \frac{1}{7} \times \frac{.2}{(.867)^2} = .00011$ or 2½d. per-cent if K be -. Evidently this is sufficient for all practical purposes, especially considering the necessary but improbable assumption that t , the rate of tax, remains constant during the term of the bond.

If the value of $\theta = \frac{2}{3}$ be adopted, the formula becomes

$$j = j' \frac{1 + \frac{2}{3}K'}{1 + \frac{2}{3}K}$$

where j' is the first approximation, found from the Bond Tables by entering inversely with $1 + K' = 1 + K/(1-t)$.

* For terms under 3 years θ differs considerably from $\frac{2}{3}$, but that value may nevertheless be used without causing any considerable error in the result because K is necessarily small in such cases.

As a first example we may take that given in the *Text Book*, Part I (1st new Edition), p. 88, namely, $n=10$, $K=.15684$, $g=.06$, $t=.05$, true value of j , $.04$. Using Deghuée's Bond Value Tables the calculation is as follows:

$$K' = .15684 \times \frac{20}{19} = .16509 = 16.509\%$$

3.95 gives 16.801

$$\begin{array}{ccc} 4.00 & ,, & 16.351 \\ .05 & & .450 \end{array} \quad \therefore 16.509 \text{ is given by } 3.95 + \frac{.292}{.450} \times .05 = 3.982 = j'$$

$$\frac{2}{3}K = .105, \quad \frac{2}{3}K' = .110 \quad j = 3.982 \times \frac{1.110}{1.105} = \underline{4.000.}$$

As a second example take $n=20$, $g=.07$, $j=.05$, $t=.1$, $K=.23574$, $K'=.23574 \div .9 = .26193 = 26.193$ per-cent.

4.90 gives 26.581

$$\begin{array}{ccc} 4.95 & ,, & 25.839 \\ .05 & & .742 \end{array} \quad \therefore 26.193 \text{ is given by } 4.90 + \frac{.388}{.742} \times .05 = 4.926 = j'$$

$$\frac{2}{3}K = .157, \quad \frac{2}{3}K' = .175 \quad j = 4.926 \times \frac{1.175}{1.157} = \underline{5.003.}$$

An obvious alternative to the foregoing method is to use the interpolation formula $\frac{1}{1-t} \left[J' - \frac{g' - g(1-t)}{g' - g''} (J' - J'') \right]$, where g' , g'' are the tabular values of g between which $g(1-t)$ lies, and J' , J'' are the results of entering the g' and g'' tables inversely with $1+K$. If t is $< .005/g$, the higher of the two rates g' , g'' will usually be g , and in any case the interpolation may be made—without much loss of accuracy—between g and the tabular value next below $g(1-t)$. The interpolation formula then becomes

$$\frac{1}{1-t} \left[J - \frac{tg}{g-g'} (J - J') \right] \dots \dots \dots (5)$$

This formula gives a close approximation to the true rate, but requires two inverse entries in the tables. We can, however, deduce a formula which requires one such entry only. For, since

$$\frac{1}{1-t} \left[J - \frac{tg}{g-g'} (J - J') \right] = J - \frac{t}{1-t} \frac{Jg' - gJ'}{g-g'}$$

and J, J' are approximately proportional to $g - K/n$ and $g' - K/n$, so that $J' = J(g' - \frac{K}{n}) / (g - \frac{K}{n})$ approximately, it follows that the true yield will be nearly

$$J - \frac{t}{1-t} \cdot \frac{JK}{ng - K} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where J is the yield without allowance for tax.

Applying these formulas to the second example given above, we have

$$g(1-t) = .063$$

which lies between the tabular values .07 and .06.

For $g = .07$

5.10 gives 23.648

$$\frac{5.15}{.05} \quad ,, \quad \frac{22.930}{.718} \quad \therefore \quad 23.574 \text{ is given by } 5.10 + \frac{.074}{.718} \times .05 = 5.105$$

For $g = .06$

4.20 gives 24.194

$$\frac{4.25}{.05} \quad ,, \quad \frac{23.420}{.774} \quad \therefore \quad 23.574 \text{ is given by } 4.20 + \frac{.620}{.774} \times .05 = 4.240$$

Hence formula (5) gives

$$\frac{10}{9} \left[5.105 - \frac{.007}{.01} \times .865 \right] \text{ which } = 4.999.$$

And formula (6) gives

$$5.105 - \frac{1}{9} \times \frac{5.105 \times 23.574}{1.4 - 23.574} \text{ which } = 4.990.$$

G. J. L.

R. T.

On Osculatory Interpolation, where the given values of the function are at unequal intervals.

SUPPOSE we have given the four values u_o, u_h, u_{h+k} , and u_{h+k+v} , being values of a function which are not equidistant, and that it is required to interpolate between u_h and u_{h+k} in such a way that a smooth junction is obtained, when the

series is continued in each direction by a similar interpolation between the points u_0 and u_h , and u_{h+k} and u_{h+k+r} .

We will assume that the interpolation curve is of the third degree and can be written in the form

$$u_{h+x} = u_h + x\delta u_h + \frac{x(x-1)}{2}\delta^2 u_h + \frac{x(x-1)(x-2)}{6}\delta^3 u_h$$

where δu_h denotes differencing for interval unity.

This curve will pass through the points u_h and u_{h+k} .

Now in order that the junction between the curves passing through u_0 and u_h , and u_h and u_{h+k} , may be smooth, it is necessary that the slopes of the curves at the point u_h should be the same, and this is secured by equating the first differential coefficient, at the point u_h , of the curve of the third degree passing through u_h and u_{h+k} , to the first differential coefficient at the same point u_h of the curve of the second degree passing through u_0 , u_h , and u_{h+k} .

Similarly, the first differential coefficient, at the point u_{h+k} , of the curve of the third degree passing through the points u_h and u_{h+k} , must be equated to the first differential coefficient at the same point of the curve of the second degree passing through the points u_h , u_{h+k} , and u_{h+k+r} .

Now since the interpolation equation passing through u_h and u_{h+k} is

$$u_{h+x} = u_h + x\delta u_h + \frac{x(x-1)}{2}\delta^2 u_h + \frac{x(x-1)(x-2)}{6}\delta^3 u_h$$

$$\therefore \frac{du_{h+x}}{dx} = \delta u_h + \frac{1}{2}(2x-1)\delta^2 u_h + \frac{1}{6}(3x^2-6x+2)\delta^3 u_h$$

So that

$$\frac{du_h}{dx} = \delta u_h - \frac{1}{2}\delta^2 u_h + \frac{1}{3}\delta^3 u_h$$

and

$$\frac{du_{h+k}}{dx} = \delta u_h + \frac{1}{2}(2k-1)\delta^2 u_h + \frac{1}{6}(3k^2-6k+2)\delta^3 u_h.$$

Employing, now, the adjusted differences, applicable where the given values of the function are at unequal intervals (see *Ency. Brit.*, 11th Edition, vol. xiv, p. 710),* we have

* The values of δ , δ^2 , and δ^3 , can be deduced in terms of the normal (unadjusted) differences of u_0 , u_h , u_{h+k} , and u_{h+k+r} , but, in the resulting formulas, the coefficients of these differences (in terms of h , k , and r) are very complicated, and the formulas are much simplified by the adoption of the adjusted differences.

$$\Delta' u_0 = \frac{u_h - u_0}{h}, \quad \Delta' u_h = \frac{u_{h+k} - u_h}{k}, \quad \Delta' u_{h+k} = \frac{u_{h+k+r} - u_{h+k}}{r}$$

$$\Delta'^2 u_0 = \frac{\Delta' u_h - \Delta' u_0}{\frac{1}{2}(h+k)}, \quad \&c.,$$

$$\Delta'^3 u_0 = \frac{\Delta'^2 u_h - \Delta'^2 u_0}{\frac{1}{3}(h+k+r)}, \quad \&c.$$

It follows that (*loc. cit. supra*)

$$u_x = u_0 + x \Delta' u_0 + \frac{x(x-h)}{2} \Delta'^2 u_0$$

whence
$$\frac{du_x}{dx} = \Delta' u_0 + \frac{2x-h}{2} \Delta'^2 u_0$$

and
$$\frac{du_h}{dx} = \Delta' u_0 + \frac{h}{2} \Delta'^2 u_0.$$

Similarly
$$\begin{aligned} \frac{du_{h+k}}{dx} &= \Delta' u_h + \frac{k}{2} \Delta'^2 u_h \\ &= \Delta' u_0 + \frac{h+k}{2} \Delta'^2 u_0 \\ &\quad + \frac{1}{2} k \left\{ \Delta'^2 u_0 + \frac{h+k+r}{3} \Delta'^3 u_0 \right\} \end{aligned}$$

Also
$$\begin{aligned} \Delta' u_h &= \frac{u_{h+k} - u_h}{k} = \frac{(1+\delta)^k u_h - u_h}{k} \\ &= \delta u_h + \frac{k-1}{2} \delta^2 u_h + \frac{(k-1)(k-2)}{6} \delta^3 u_h. \end{aligned}$$

We, therefore, have the following three equations for δu_h , $\delta^2 u_h$, and $\delta^3 u_h$.

$$\delta u_h - \frac{1}{2} \delta^2 u_h + \frac{1}{3} \delta^3 u_h = \Delta' u_0 + \frac{1}{2} h \Delta'^2 u_0$$

$$\delta u_h + \frac{1}{2} (2k-1) \delta^2 u_h + \frac{1}{6} (3k^2-6k+2) \delta^3 u_h = \Delta' u_0 + \frac{h+2k}{2} \Delta'^2 u_0 + k \frac{h+k+r}{6} \Delta'^3 u_0$$

$$\delta u_h + \frac{k-1}{2} \delta^2 u_h + \frac{(k-1)(k-2)}{6} \delta^3 u_h = \Delta' u_0 + \frac{h+k}{2} \Delta'^2 u_0$$

so that

$$\delta u_h = \Delta' u_0 + \frac{h+1}{2} \Delta'^2 u_0 - \frac{(h+k+r)(k-1)}{6k} \Delta'^3 u_0 \quad . \quad (i)$$

$$\delta^2 u_h = \Delta'^2 u_0 - \frac{(h+k+r)(k-3)}{3k} \Delta'^3 u_0 \quad . \quad . \quad . \quad (ii)$$

$$\delta^3 u_h = \frac{h+k+r}{k} \Delta'^3 u_0 \quad . \quad . \quad . \quad . \quad . \quad (iii)$$

From these differences we can now interpolate between u_h and u_{h+k} , assuming third differences constant.

As an example of the working of this formula, let us take the infantile ages of the English Life Table No. 8 for Male and Female lives. In the case dealt with by Mr. King the values of q_x for ages 0 to 5 inclusive, and for 11, 16..... are given, and it is desired to find the intervening values for ages 6 to 10, and 12 to 15 inclusive, by interpolation, on the lines set forth above.

In order to obtain a smooth junction, and at the same time to use as much of the available data as possible, the above formula was applied twice, first to the interval 5 to 11, using values of $\log q_x$ at ages 1, 5, 11 and 16, and secondly to the interval 11 to 16, using the values at ages 5, 11, 16 and 21. The values of $\log q_x$ for Male lives, and of the unadjusted and adjusted differences, are set out in the following scheme:

x	$\log q_x$	Δ	Δ^2	Δ^3	Δ'	Δ'^2	Δ'^3
1	-1.4654891						
5	-2.3109132	-.8454241			-.2113560		
			+ .4141358			+ .0278949	
11	-2.7122015	-.4312883		+ .1721169	-.0718814		-.0018381
			+ .5862527			+ .0187044	
16	-2.5872371	+ .1549644		- .5903803	+ .0309929		-.0035380
			- .0041276			- .0001651	
21	-2.4364003	+ .1508368			+ .0301674		

Employing, now, formulas (i), (ii), and (iii) above, in the particular case, we have, for the values 1, 5, 11, 16, where $h=4$, $k=5$, $r=5$, the following equations:

$$\delta u_5 = \Delta' u_1 + 2.5 \Delta'^2 u_1 - 2.08 \dot{3} \Delta'^3 u_1$$

$$\delta^2 u_5 = \Delta'^2 u_1 - 2.5 \Delta'^3 u_1$$

$$\delta^3 u_5 = 2.5 \Delta'^3 u_1$$

whilst for the values 5, 11, 16, 21, where $h=6$, $k=5$, $r=5$, the resulting equations are

$$\delta u_{11} = \Delta' u_5 + 3.5 \Delta'^2 u_5 - 2.1 \dot{3} \Delta'^3 u_5$$

$$\delta^2 u_{11} = \Delta'^2 u_5 - 2.1 \dot{3} \Delta'^3 u_5$$

$$\delta^3 u_{11} = 3.2 \Delta'^3 u_5.$$

The following table shows, for both Male and Female lives, at ages 5 to 16 inclusive,

- (i) the resulting interpolated values of q_x ;
- (ii) the expected deaths, deduced by multiplying the estimated population at each age by $m_x = \frac{2q_x}{2 - q_x}$;
- (iii) the actual deaths per annum;
- (iv) the deviation of the expected deaths from the actual;
- (v) the accumulated deviation.

Age	MALE LIVES					FEMALE LIVES				
	Graduated Values of q_x	Ex-pected Deaths	Actual Deaths	Devia-tion	Accum-d. Devia-tion	Graduated Values of q_x	Ex-pected Deaths	Actual Deaths	Devia-tion	Accum-d. Devia-tion
5	·0048875	1,868	1,868	0	0	·0047989	1,825	1,825	0	0
6	·0035588	1,330	1,372	- 42	- 42	·0035328	1,322	1,371	- 49	- 49
7	·0027925	1,045	1,047	- 2	- 44	·0028021	1,049	1,053	- 4	- 53
8	·0023367	858	863	- 5	- 49	·0023675	873	880	- 7	- 60
9	·0020630	748	757	- 9	- 58	·0021066	763	754	+ 9	- 51
10	·0019015	684	727	- 43	- 101	·0019518	703	709	- 6	- 57
11	·0018105	637	671	- 34	- 135	·0018617	657	652	+ 5	- 52
12	·0018152	635	617	+ 18	- 117	·0018660	655	648	+ 7	- 45
13	·0019334	669	632	+ 37	- 80	·0019840	687	726	- 39	- 84
14	·0021312	732	701	+ 31	- 49	·0021762	751	784	- 33	- 117
15	·0023689	793	773	+ 20	- 29	·0023945	805	832	- 27	- 144
16	·0025868	870	874	- 4	- 33	·0025700	869	878	- 9	- 153
Total	...	10,869	10,902	+ 106	- 737	...	10,959	11,112	+ 21	- 865
				- 139					- 174	
				- 33	- 737				- 153	- 865

These results, although greatly superior to those deduced by Mr. King at these ages, are not satisfactory, the accumulated deviations being negative in sign at all ages, for both sexes. In the age group 11 to 16, the graduated values do not differ appreciably from those previously deduced (see pages 336 and 337) by Mr. King's formula for equal intervals 6, 11, 16, 21, taking the ungraduated value of q_6 ; but, for the earlier age group 6 to 10, the deviations from the actual deaths are, in the Male Table, throughout negative in sign, but only considerable in amount at ages 6 and 10, whilst, in the Female Table, the deviations are mainly negative in sign, with a large deviation at age 6. On the whole, therefore, the graduated values

previously deduced, based on equidistant ordinates, taking the ungraduated value at age 6, give the closest approximation which I have been able to obtain, and are, it is thought, sufficiently good for practical purposes. The examples now given are merely submitted as practical illustrations of the special method deduced in this note for the case of ordinates not equidistant.

It may be added that, in the case of equal intervals, where $h=k=r=t$ (say), $\Delta' u_0 = \frac{\Delta u_0}{t}$, $\Delta'^2 u_0 = \frac{\Delta^2 u_0}{t^2}$, and $\Delta'^3 u_0 = \frac{\Delta^3 u_0}{t^3}$, where Δ denotes differences for interval t ; and inserting these values in formulas (i), (ii), and (iii), we have

$$\delta u_t = \frac{\Delta u_0}{t} + \frac{t+1}{2} \cdot \frac{\Delta^2 u_0}{t^2} - \frac{t-1}{2} \cdot \frac{\Delta^3 u_0}{t^3}$$

$$\delta^2 u_t = \frac{\Delta^2 u_0}{t^2} - (t-3) \frac{\Delta^3 u_0}{t^3}$$

$$\delta^3 u_t = 3 \cdot \frac{\Delta^3 u_0}{t^3}$$

which are identical with Mr. King's formulas for equal intervals.

It may also be noted that the ordinary interpolation formulas, where four values at unequal intervals are given, and third differences are assumed to be constant, are as follows:

$$\delta u_h = \Delta' u_0 + \frac{h+1}{2} \Delta'^2 u_0 - \frac{(h+1)(h-1)}{6} \Delta'^3 u_0$$

$$\delta^2 u_h = \Delta'^2 u_0 + \frac{h-h+3}{3} \Delta'^3 u_0$$

$$\delta^3 u_h = \Delta'^3 u_0$$

which may be compared with formulas (i), (ii), and (iii), where osculatory interpolation is introduced. These formulas will sometimes be found more convenient than Lagrange's well-known expression, especially where it is desired to deduce a complete series, or several consecutive terms, of values of u_x .

In the case of equal intervals, where $h=k=r=t$, the

above formulas, expressed in normal values of Δ , Δ^2 , and Δ^3 , for interval t , become

$$\delta u_t = \frac{\Delta u_0}{t} + \frac{t+1}{2} \cdot \frac{\Delta^2 u_0}{t^2} - \frac{t^2-1}{6} \cdot \frac{\Delta^3 u_0}{t^3}$$

$$\delta^2 u_t = \frac{\Delta^2 u_0}{t^2} + \frac{\Delta^3 u_0}{t^3}$$

$$\delta^3 u_t = \frac{\Delta^3 u_0}{t^3}$$

which may be compared with Mr. King's formulas, given above, in which osculatory interpolation is introduced.

T. G. A.

REVIEWS.

Legal Risks incident to Investments in Reversionary Interests in Personalty. By A. H. WITHERS. (London: *Post Magazine Press*.)

LEGAL questions connected with reversions very frequently require the actuary's attention, perhaps more so in this branch of his professional work than in any other. Though the legal procedure employed in carrying out a purchase of a reversion, or loan upon it, is necessarily left to a solicitor, the actuary has to consider some of the legal risks in the preliminary stages of the negotiations and others after evidence bearing upon them has been collected by the solicitor. In the first category would fall such questions, among others, as those relating to the trustees' power of investment, to the inclusion of the reversioner as a trustee, or to the security for mortgages held by the trustees. Evidence relating to Notice, and to the circumstances attending an appointment, come within the second. In most insurance or reversionary companies with a long experience of reversions the actuary is familiar with the risks referred to, and has adopted certain well known principles on which to deal with them, but until the publication in 1910 of "Withers on Reversions", there was no legal text book specially dealing with this branch of law to which he could refer, or in which the points in question were treated in one volume. The work just mentioned has been of no small value, and its author has now rendered a further service to the actuarial profession by a useful little summary "for the use of persons who are not lawyers" of the legal risks which are, or may be, run by purchasers or mortgagees of reversions. This deserves the attention of actuaries, though many of the points discussed are, as suggested above, already well known to them, and some of them were dealt with in the pages of the *Journal* (vol. xlv, p. 392) a few years ago in an article by the same author, extracted from the *Solicitors' Journal*.

That the points discussed appear to present themselves in a different proportion from that in which they arise in actual practice is, we believe, not infrequently the general impression obtained by most laymen from a perusal of legal text books, and the same feature will be noticed in Mr. Withers's summary now under review. The author himself is, however, fully alive to the lawyer's difficulty in the way indicated, and makes the useful suggestion that an actuary should deal with the risks discussed collectively and individually from his own professional point of view. The treatment of the same points by men of two professions has, it is well known, often produced results of much greater practical value than are forthcoming from the somewhat one-sided point of view from which a member of either profession must necessarily regard them. Some notes, therefore, by Mr. R. R. Tilt, written in response to Mr. Withers's suggestion, form an appendix to the summary which will be welcomed as a contribution of considerable value to the subject as coming from a source of recognized authority.

Among the more important questions included by Mr. Withers among "legal risks" are those relating to mortgages forming part, or the whole, of a trust fund. These risks are well known to most actuaries, and precautions are taken to guard against them. The inability to place a *distringas*, where the fund is all invested in mortgages, is in many cases, apart from other reasons, considered as sufficient for declining to deal with a reversion. Experience shows that where fraudulent manipulation of the fund by the trustees' solicitors has occurred, it has nearly always been in connection with the mortgage investments in the trust fund. There are also difficulties in ascertaining at frequent intervals from the mortgagor what amount is owing by him and in providing for partial repayments being properly accounted for. One feature of these trust mortgages that is sometimes overlooked by a purchaser of a reversion, is that the security for the mortgages will not as a general rule receive the same close attention by the trustees or their solicitors as do the mortgage investments of an insurance company. The mortgages are often left far too long without review of the circumstances affecting the security, and steps are not taken in time in the way of calling for reduction or repayment of the principal. This sometimes arises from the fact that the mortgagor is a client of the trustees' solicitors, who are averse from bringing themselves into conflict with his interests by taking action that would be considered necessary by a prudent mortgagee. Mr. Tilt mentions another feature of trust mortgages that experience in reversions shows to be not uncommon. The trustees are too often content with surveyors' valuations that would not be accepted by an insurance or reversionary company; valuations that do not deal with the question of the facility of selling or letting the property, but are more in the nature of multiplication sums representing so many years' purchase of the rental.

As an illustration of one or two minor points which may appear more important to the lawyer than to the actuary, the special

difficulties connected with investments in the trust fund represented by shares or debentures in limited companies may be mentioned. In dealing with such investments it is the general practice, we believe, to decline a reversion if the securities (apart from mortgages and land) are not officially quoted and actively dealt in; or, if a small portion of the fund consists of such securities, to allow a large margin for contingencies. Of relatively small importance, too, is the question of apportionment of income and capital.

Mr. Withers has devoted two sections of his pamphlet to some special features of Life Interests and Expectancies. In the case of the former, the possible difficulties of proving the existence of the life tenant have been for many years thoroughly appreciated by actuaries, and the common practice is to decline to purchase these interests. From the point of view of both the investor and the life tenant there is seldom sufficient reason for arranging a transaction in the form of a purchase rather than a loan, where the investor is an insurance or reversionary company and will not require repayment before the policy falls in. In either case a substantial margin of income is generally left as a protection against the risk in question. But in this, as in many other of the risks attaching to reversions and life interests, the practical considerations bearing on the personal element weigh largely in the actuary's decision, and no life interest would be lent upon or purchased unless he were thoroughly satisfied at the outset that the special risk referred to could be ignored.

Expectancies were no doubt dealt in occasionally by insurance companies in the past, but have probably for some time been entirely outside their scope of investment and have been left to private investors. The author has, however, devoted some special remarks to this class of reversionary interest which summarize in a useful form a lender's position. An important case dealing with these interests was *re Dallas* (1904, 2 Ch. 385), which formed the subject of some remarks in the *Journal* (vol. xxxix, p. 252) a few years ago, when it was shown that an insurance company or reversionary interest society dealing with such an interest would have practically no security. Mr. Withers has, however, dealt with several points not discussed in the article above referred to, and in particular with a recent decision of the Court of Appeal in *re Lind* (1915, 1 Ch. 744), which should receive the actuary's attention. It was held in that case that a mortgage of an expectancy is not affected by the mortgagor becoming bankrupt and obtaining his discharge before the expectancy becomes an interest. Prior to this decision the judgment of the Court of Appeal in *Collyer v. Isaacs* (19 Ch. D. 342) in 1881 had always been taken to decide the contrary, and the discharge of the bankrupt was taken to release the assignor and the expectant interest from the covenant to assign. The removal of the risk facilitates dealings with expectancies, but it is extremely unlikely, in view of the other risks attaching to them, that these interests will even now be dealt in by insurance or reversionary companies. Mr. Withers suggests that it would seem prudent for

insurance and reversionary companies dealing largely with reversions to ask their solicitors whether they are aware of any titles upset by the decision. He states that unquestionably titles have been accepted freely on the footing that when a man has become bankrupt and has got his discharge, he is free to deal with his expectancies without regard to mortgages created by him before the bankruptcy; that titles have been so accepted principally in two cases—(1) when the discharged bankrupt has an expectant interest as the next of kin of a living person, and (2) where the discharged bankrupt before his bankruptcy has mortgaged his interests under appointments thereafter to be made in his favour, and the interest proposed to be dealt with arises under an appointment executed in his favour after he has got his discharge.

Mr. Tilt has pointed out that it is impossible to estimate on any scientific basis the monetary equivalent of the various risks described by Mr. Withers, and it would seem that the principle adopted by the actuary of taking percentage margins is probably the most satisfactory method of dealing with them. A useful contribution to the subject would, we think, be an investigation into the circumstances that have actually arisen on the falling in of a large number of reversions over a long period, showing, apart from the results of interest and mortality, what losses, if any, the insurance or reversionary company had made (1) in respect of such questions as notice, the reversioner's trusteeship, and others, for which no margin is usually taken, and (2) to what extent the margins taken for possible depreciation of the trust mortgages had been sufficient to meet any losses, and (3) the financial effect of other incidents that had arisen of the nature discussed by the author. Although it might still be impossible to arrive at a scientific basis for dealing with these risks, such an investigation would, we think, give a clearer idea of the proportion in which they should be viewed, than is obtained from the general impressions of a more or less indefinite character on which the practice is, we believe, at present based.

J. R. H.

Improved Four-Figure Logarithm Table. By GEORGE C. McLAREN, F.F.A.

(Cambridge University Press).

THERE is a well-known, and no doubt untrue, story of a certain Chancellor of the Exchequer, who, after examining for a time some financial statistics expressed in decimals, which had been submitted to him by a Treasury official, enquired *what on earth the d—d dots meant*. We confess to having made the same enquiry mentally on opening Mr. McLaren's Improved Four Figure Logarithm Table and finding some of the logarithms followed by either a full stop or a colon. The answer, in the case of our enquiry, is that "the single dot is a contraction for one-third, and the double dot for two-thirds", by which we understand the full stop to mean that the remainder

of the characteristic lies between $\cdot 00001\dot{6}$ and $\cdot 0000\dot{5}$, and the colon that the remainder lies between $\cdot 0000\dot{5}$ and $\cdot 00008\dot{3}$. The dots are intended to be added and subtracted "just as if they were thirds." The device is an ingenious and interesting one, and although its introduction in a table bearing the sub-title "Multiplication and Division Made Easy," and intended partly for persons not very familiar with the use of logarithms, may seem a rather curious way of making logarithms "easier to understand and easier to handle", this, in itself, would not matter if it rendered the table more useful to experienced computers. Naturally it admits of greater accuracy than can be obtained by ordinary four-figure logarithms, and Mr. McLaren states that it involves no appreciable increase in work. But the latter statement would apply equally, and the former in a greater degree, to the introduction of fourths, or fifths—or of a fifth figure. Personally, having regard to the inconvenience—especially in entering the table inversely with the logarithm—of working in a combination of two scales of notation, one the common scale and the other a scale of dots, we should prefer the fifth figure. A more important point is that the introduction of the dots practically precludes the use of the table as an ordinary four-figure table. One would need to be an exceptionally strong-minded computer to neglect deliberately the single dot and to put up the last figure when followed by a double dot, and even then the dots would tie one in knots in the process of taking out the antilogarithm. Yet there are many calculations in which four figure logarithms are all that is wanted, and it is just in those cases that the ordinary four-figure table, by removing temptation out of the computer's way, is so useful. Of course, as the present Astronomer Royal of Ireland pointed out in his article on Aids to Calculation (*J.I.A.*, vol. xlv, p. 193), "if any importance attaches to the precise figure in the last place, too rigid economy is being practised, and the calculation ought to be made with tables to an extra place."

We observe that Mr. McLaren compares his Four-and-a-Third Figure Table with the four-figure *card*—naturally to the disadvantage, as regards accuracy, of the latter—but it would seem to be more strictly comparable with Hannyngton's well-known Four-Figure Table. Except for the dots in Mr. McLaren's Table, and the anti-logarithms in Hannyngton's—and it is only fair to add, a considerable difference in price—these two tables are very similar. The same numbers appear at an opening in both, and both are thumb-indexed. As some compensation for the absence of an anti-logarithmic section Mr. McLaren gives the logarithm of the initial figure, as well as the figure itself, on the thumb-index, but this, although an assistance, will not be a complete solatium to those computers who, not being astronomers, are unable to "sacrifice the anti-logarithms entirely without regret." (*J.I.A.*, vol. xlv, p. 194.) On the other hand, some computers may find Mr. McLaren's nicely-graded spaces more helpful to the eyes than Hannyngton's feint and heavy rules.

CORRESPONDENCE.

LIMITED PAYMENT POLICIES.
MORTALITY AND SPECIAL RESERVES.

To the Editors of the Journal of the Institute of Actuaries.

SIRS,—In the discussion on Mr. Todhunter's paper submitted in March there were remarks by various speakers, particularly Mr. R. G. Salmon, on the variations in the mortality rates under different classes of assurance, and Mr. Salmon called in question the value of certain figures I had supplied to Mr. Todhunter, of which only the final results and not the details were given. As the matter is one which appears to be creating a good deal of interest at the present time, it is possible that further details of the investigation, which I made three years ago, may be considered of sufficient value to be inserted in the *Journal*, and I accordingly send short summaries of the actual and expected deaths, grouped in different ways. The figures are based on the experience of the Scottish Life Assurance Company, and although the numbers involved are small when compared with those included in modern investigations, yet the results seem to me to be remarkably regular throughout the groups, and therefore to be worthy of confidence. The experience was extracted by policies—or rather by medical examinations—and included only male lives insured at the ordinary rates under participating policies in the various classes effected during the thirty years 1881 to 1910, and it terminated with the anniversaries in 1911 of existing policies. Reassurances from other Offices were excluded, as it was found that the rates of mortality amongst them were considerably higher than under direct policies. The expected deaths were based throughout on the O^[M] Table.

In the summaries arranged by ages at entry and durations the percentages become increasingly good as the class of policy proceeds from whole life to limited payment and to endowment assurance. Mr. Salmon referred in the discussion to the high ratios found in the whole life class at the older ages in another investigation upsetting the comparison of the total results, and while this will no doubt always be the case to a certain extent where a larger body of whole life policyholders reaches the higher ages, the figures in the third summary, arranged by ages attained, indicate throughout heavier percentages for the whole life class. The differences are not great up to age 40, but thereafter the other classes, especially the endowment assurances, seem to show a greater staying power. Another point made in the discussion was that the better mortality shown amongst limited payment and endowment assurance policies might be due to these classes being more recent, and accordingly showing more the modern improved mortality amongst policyholders generally. This may, perhaps, have had something to do with the very light mortality shown above in the endowment assurances, but as regards the limited payment class, the policies were on the average older than the

I.—Summary arranged by Ages at Entry.

Ages at Entry	WHOLE LIFE			LIMITED PAYMENTS			ENDOWMENT ASSURANCES		
	Actual Deaths	Expected Deaths	Percent-ages	Actual Deaths	Expected Deaths	Percent-ages	Actual Deaths	Expected Deaths	Percent-ages
15-34	57	98.79	58	61	115.05	53	130	246.09	53
35-54	101	125.76	80	59	87.46	67	53	109.25	49
55 and over	31	31.13	100	8	6.34	126	1	.51	197
Total	189	255.67	74	128	208.85	61	184	355.85	52

II.—Summary arranged by Durations.

Durations	WHOLE LIFE			LIMITED PAYMENTS			ENDOWMENT ASSURANCES		
	Actual Deaths	Expected Deaths	Percent-ages	Actual Deaths	Expected Deaths	Percent-ages	Actual Deaths	Expected Deaths	Percent-ages
0-4	51	72.51	70	27	45.19	60	81	149.78	54
5-9	45	63.03	71	30	49.79	60	49	110.75	44
10-14	37	48.24	77	25	43.69	57	31	58.43	53
15-19	29	38.21	76	26	34.95	74	16	25.69	62
20-24	19	24.93	76	14	26.83	52	6	10.05	60
25-29	8	8.75	91	6	8.40	71	1	1.16	86
Total	189	255.67	74	128	208.85	61	184	355.85	52

III.—Summary arranged by Ages attained.

Ages Attained	WHOLE LIFE			LIMITED PAYMENTS			ENDOWMENT ASSURANCES		
	Actual Deaths	Expected Deaths	Percent-ages	Actual Deaths	Expected Deaths	Percent-ages	Actual Deaths	Expected Deaths	Percent-ages
Up to 29	9	17.28	52	7	13.90	50	36	74.63	48
30-39	27	48.69	55	28	49.98	56	70	138.77	50
40-49	45	64.46	70	38	66.28	57	58	96.26	60
50-59	50	61.76	81	39	55.93	70	19	41.29	46
60-69	33	41.71	79	14	19.30	73	1	4.89	20
70-79	22	18.87	117	2	2.84	70
80-89	3	2.91	10362
Total	189	255.67	74	128	208.85	61	184	355.85	52

corresponding whole life policies, owing to a large proportion of the business of the office in its early years having been done under the former class.

I had not an opportunity before the date of the meeting of

looking fully into the method suggested by Mr. Todhunter for making the special reserve required for limited payment policies, as compared with that proposed by me in the *Transactions of the Faculty of Actuaries* (vol. vi, page 93) to which Mr. Todhunter refers in the footnote at page 263, and I may, perhaps, be allowed to make the following remarks.

The formula given by me for the requisite addition to the net premium reserve was (using Mr. Todhunter's notation)

$$\{P'_x(1-\kappa)a_{x+n}^{(j)} - P_x a_{x+n}\} - \{ {}_tP'_x(1-\kappa)a_{x+nt-n}^{(j)} - {}_tP_x a_{x+nt-n}\}$$

which, making obvious substitutions, is practically identical with Mr. Todhunter's "commutation-basis" formula. The main difference is that Mr. Todhunter restricts his formula to the case of the office premiums being valued by the rates of mortality and interest used in obtaining the limited payment from the whole life premiums, while I showed that the formula was perfectly general when the office premiums are valued by the rates of mortality and interest expected to be experienced by the office in the future.

The Text-Book formula gives reserves much too high throughout, and it seems almost a pity that Mr. Todhunter departs from his original formula and recommends a modification which, although very ingenious, not only entails the use of hypothetical office premiums, but reproduces the Text-Book formula after the premiums are paid up. This weighs heavily on policies by a small number of payments, and particularly (as Mr. Todhunter himself points out) on single payment business, the strain in the latter case being 4 per-cent to 5 per-cent of the sum assured, as shown in the tables appended to my note. The use of the correct formula involves a certain amount of additional work at a valuation, but in these days of classified registers and calculating machines, the time taken is in my opinion much more than counterbalanced by the truer results obtained.

Yours faithfully,

ALEX. FRASER.

19 *St. Andrew Square, Edinburgh,*

8 *September 1915.*

[With reference to the concluding paragraph of Mr. Fraser's letter, Mr. Todhunter states that the "commutation-basis" formula, given in his paper as representing on certain assumptions the true value of a limited-payment policy, was intended to be used, not as a practical valuation formula, but as a standard of comparison—with due regard to the validity of the underlying assumptions—for the other formulas.

The formula merely expresses the fact that a limited-payment

policy may be regarded as a combination of two contracts, viz.: an ordinary whole-life policy and a premium-commutation contract, and although it could no doubt be used for valuation purposes as suggested by Mr. Fraser in his Transactions of the Faculty note, a question would arise as to the basis on which the premium-commutation contract should be valued. That basis should be a "future experience" basis—not necessarily the basis employed in commuting the premium—and it might be considered that the only course consistent with the principles on which the general valuation is made would be to employ the basis used in valuing other (non-profit) contracts. But the effect of this would, in general, be to reproduce the Text-Book formula.—EDS. *J.I.A.*]

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, 1915.

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EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART I).

First Paper.

1. A country contains a native population and an alien population. The native population has, during a period of five years, remained stationary, both in total numbers and in distribution between the sexes. The alien population, however, has increased in geometrical progression at the rate of 1 per-cent per annum, while the proportion of women to men has risen from unity to 1.15. The proportion of native men to alien men at the beginning of the period is equal to the proportion of native women to alien women at the end. The excess of women over men in the total population, native and alien, was 1,000,000 at the beginning of the period, and 1,070,000 at the end.

Find the numbers in each class at the beginning and end of the period.

2. Find an expression for the number of homogeneous products of r dimensions that can be formed with n letters, where each letter may be repeated any number of times.

How many different sets of letters can be formed from the letters of the word *facetious*, using each letter once only,

(a) in which no two vowels are together?

(b) in which not more than two vowels are together?

3. How many sums of money can be paid in exactly 500 ways by means of £1 notes and postal orders for 10s. 6d. each, and what is the greatest such sum?

*4. Find the sum to n terms of the series

$$(1) \frac{4 \cdot 6}{1 \cdot 3 \cdot 7 \cdot 9} + \frac{6 \cdot 8}{3 \cdot 5 \cdot 9 \cdot 11} + \frac{8 \cdot 10}{5 \cdot 7 \cdot 11 \cdot 13}$$

$$(2) 1 \cdot 2^2 \cdot 3 + 2 \cdot 3^2 \cdot 4 + 3 \cdot 4^2 \cdot 5 + \dots$$

or

*5. Explain what is meant by "convergency" of series, and discuss the convergency of the following:

$$(1) \frac{1 \cdot 3}{2 \cdot 4}x + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^2 + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}x^3 + \dots$$

$$(2) 2 - \frac{5}{4} + \frac{10}{9} - \frac{17}{16} + \frac{26}{25} - \dots$$

6. Find the equation whose roots are the roots of

$$x^4 - 3x^3 - 36x^2 + 68x + 240 = 0$$

each diminished by 3.

7. If n be any integer and if the expansion of $(1+x)^{2n}$ be $1 + a_1x + a_2x^2 + a_3x^3 + \dots$, prove that the sum of n terms of the series $a_2 - a_3 + a_4 - a_5 + \dots$ is

$$2n - 1 + \frac{(-1)^{n+1}(2n-1)!}{(n+1)!(n-2)!}$$

8. A bag contains 8 counters, numbered 1 to 8. Four are drawn at random. Find the chances that

(1) The sum of the numbers on the four counters amounts to at least 17.

(2) The counters numbered 2 and 3 are among the four.

(3) The four counters contain at least 2 of the 3 counters numbered 3, 5 and 7.

9. If a coin be tossed 15 times, what is the probability of getting heads exactly as many times in the first 10 throws as in the last 5?

10. A particular Army Corps consists of 24 Battalions, each consisting of 1,000 men. It is to be divided into 4 Divisions, each to consist of 6 Battalions, and it is equally likely that any particular Battalion will be allotted to any one of the Divisions. What is the probability that 100 men, known to be serving in the Army Corps, who were only eligible to enlist in 2 particular Battalions out of the 24, and each of whom is equally likely to have enlisted in either of the 2, will all be in the same Division?

How much, if any, of the information given above has no bearing on the probability in question?

* Questions (4) and (5) are alternatives. Only one of them is to be answered.

Second Paper.

1. What is denoted by each of the following symbols?

$$\Delta^n u_x, \left\{ \frac{\Delta^m u_x}{(\Delta x)^m} \right\}_0, \Delta^5 0^5.$$

Find $\Delta^{20} \frac{6x+29}{(3x+1)(3x+4)(3x+7)(3x+10)}$

2. Given $u_0 = 70795$, $u_1 = 72444$, $u_2 = 74131$, and $u_6 = 81283$. find u_3 , u_4 , and u_5 .

3. Prove Lagrange's formula for interpolation, and explain the assumptions it involves.

Apply the formula to find u_5 and u_6 , given that

$$u_1 = 2, u_2 = 4, u_3 = 8, u_4 = 16 \text{ and } u_7 = 128,$$

and explain why the results differ from those obtained by completing the series of powers of 2.

4. Prove that approximately

$$\int_{-\frac{1}{2}}^{+\frac{1}{2}} f(x) dx = \frac{1}{24} \{f(-1) + 22f(0) + f(1)\}$$

and find thereby an approximate value for

$$\int_{\frac{1}{2}}^{1\frac{1}{2}} 10,000 \times 4^{-x} \times 2^{3x} dx.$$

5. Find the conditions that a given value of x may make $f(x)$ a maximum or a minimum.

What are the dimensions of the largest rectangular box on a square base the area of whose surface does not exceed 12 square feet?

6. Evaluate

$$(1) \int \frac{27x^2 - 171x + 256}{2(x-3)^3(x+4)} dx.$$

$$(2) \int_0^1 \frac{x^3}{(2x+1)^5} dx.$$

7. Explain what is meant by "integration by parts."

Show how a form of Maclaurin's series, expressing $f(n)$ in terms of $f(0)$, $f'(0)$, $f''(0)$. . . can be obtained by repeatedly integrating by parts the integral

$$\frac{x^n}{(n-1)!} \int_0^1 (1-t)^{n-1} f^{(n)}(xt) dt.$$

8. In a besieged garrison, the rate of decrease in the population at any time due to

(1) Violent death is proportional to the population existing at that time ;

(2) Disease and other causes is proportional both to the population existing and to the duration of the siege.

Assuming that the effect of the first cause is to reduce the population at the rate of 1 in 10,000 a day, and that the effect of the second is to reduce the population at the rate of

1 in 10,000 the first day,
2 in 10,000 the second day,
3 in 10,000 the third day,
and so on,

in how many days from the commencement of the siege will the population be reduced by 50 per-cent ?

Third Paper.

1. What is meant by "force of interest" ?

If the force of interest after time t be $a + bt$, find expressions for

(i) The amount of a unit after time t

(ii) The effective rate of interest in the n th year.

2. If a fund is increasing in such a way that the rate of increase at any time is proportionate to the amount of the fund at that time, and if it has increased from £5,000,000 to £6,000,000 in the last 10 years, by how much will it increase in the next 10 years ?

*3. A money lender makes small advances of £5 for which he is to receive instalments of 5s. a week until they amount to £6 5s. in all when the loan is considered to be repaid. Out of each 100 borrowers one fails to pay the first instalment of 5s., one fails to pay the second, one the third, and so on, and 75 pay all the instalments. Assuming that in these cases of default the lender is unable to obtain any more money from the borrower and that his expenses amount to 2s. 6d. for each loan, find the effective rate he realises.

*4. The leaseholder of certain premises at a rent of £500 a year, payable quarterly (next payment due 24 June 1915) wishes to obtain an extension of his lease which expires on 24 June 1925. It is estimated that after June 1925 the rental value of the premises will be £1,500 a year. The leaseholder is willing to pay £2,000 on 1 May 1915 and a quarter's rent on 24 June 1915, and also to increase his rent to £600 for the year ending 24 June 1916, £700 for the next year, £800 for the next and £900 a year thereafter. For what period should his lease be extended, on the basis of interest at the rate of 5 per-cent per annum convertible quarterly?

*5. Certain $4\frac{1}{2}$ per-cent Bonds are redeemable at par by annual drawings by means of a cumulative sinking fund of 1 per-cent per annum commencing 1 January 1905. The market price of undrawn Bonds on 1 January 1916 is 105, and a holder of Bonds for £100,000 just before the drawing on that date wishes to insure the payment of the difference between the market price and the redemption price in respect of any of his Bonds which may then be drawn. What is the value of the risk?

*6. A Government Stock bearing interest at the rate of 5 per-cent per annum payable half-yearly and redeemable at 102 at the end of 7 years is quoted at 108. On what terms should the holders of the stock be allowed to exchange into stock bearing interest at the rate of 4 per-cent per annum payable quarterly and redeemable at par at the end of 20 years, so as to give the same yield for the two stocks until their respective dates of redemption?

*7. A man invests £1,200 in a perpetual debenture yielding 6 per-cent per annum payable half-yearly. Each half-year when he receives the interest he sells one sixth of the original investment for £199, and re-invests this sum and all the interest he receives in an investment yielding 5 per-cent per annum convertible half-yearly. What rate of interest does he obtain in each year on his investment and at what effective rate per annum would the £1,200 have had to be invested to produce the same final result?

*8. A local authority borrowed, on 24 June 1909, a sum of £1,000, repayable by way of annuity in 25 years on a 4 per-cent basis, by equal yearly payments, due on the 24 June in each succeeding year. On 18 September 1914, a sum of £50 is accepted as a special repayment on account of principal, subsequent instalments being reduced, but the original term of the loan remaining unaltered. Draw up a Schedule showing the amounts paid by the authority on account of principal and interest (irrespective of income tax) down to and including that due on 24 June 1915.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE

(PART II).

First Paper.

1. Prove that ${}_x p_x = \frac{{}_x l_x}{l_x}$ exactly

$$= {}_x p_x + \frac{1}{2} - \frac{1}{12} \frac{d_x}{l_x} \text{ approximately.}$$

2. Find approximately the probability in the case of three persons, A, B and C, all aged x :

- (1) That all will die in the t^{th} year from the present time.
- (2) That the order of death will be A, B, C.
- (3) That A will die in the t^{th} year from the present time and B will survive him by n years and C will survive the year in which B dies by n years.

3. Define q_x and μ_x , and explain why it is that μ_x lies between q_x and q_{x-1} at the younger ages but not at the older ages.

4. If the probability at age 32 of dying within 10 years is $\cdot 1$, and the probabilities at various ages of dying within 1 year are as follows:—

Age	Probability
32	$\cdot 00820$
33	$\cdot 00846$
42	$\cdot 01100$
43	$\cdot 01157$

find the probability that a life now aged 33 will die in the tenth year from now, leaving a life now aged 34 surviving him.

*5. A company's staff consisting of 1,000 men, who all enter at age 20, has reached a stationary condition.

Assuming :

- (1) In the first three years of service 15 per-cent of those attaining age 21 resign at that age ; similarly that 10 per-cent resign at age 22, and 5 per-cent at age 23, after which there are no resignations.
- (2) 10 per-cent of those attaining age 55 retire at that age ; similarly 40 per-cent retire at age 60 and 100 per-cent at age 65.
- (3) H^M mortality, find :
 - (a) The number resigning each year.
 - (b) The number of pensioners.

*6. A publishing firm issues a book suitable for boys. Immediately after publication, copies of the book are sold as follows :

To boys between the ages of 13 and 14 at the rate of 1 per 1,000 living.

"	"	"	14	"	15	"	2	"
"	"	"	15	"	16	"	3	"
"	"	"	16	"	17	"	2	"
"	"	"	17	"	18	"	1	"

Subsequent sales are such that one per-cent of the boys who attain age 13 purchase a new copy on their birthday ; two per-cent of those who attain age 14, either purchase a copy or have already done so ; in the same way, three per-cent of those who attain age 15, two per-cent of those who attain age 16, and one per-cent of those who attain age 17 either purchase or have previously purchased a new copy of the book.

Find the number of books sold in the first instance and the number sold in each succeeding year.

The book circulates among a stationary male population supported by 127,283 annual births, and the purchasers of the book, in common with the general population, are subject to H^M mortality.

7. If the number of members of an annuity fund at the beginning of a year at any age x is given by $k x^p a^x$ and the number of members at the end of the year at any age x is given by $l (x-1)^q b^x$ find an expression for the force of mortality, assuming that no new members join and that there are no withdrawals from the fund except by death.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Second Paper.

1. Express, in terms of assurances which determine at the first death, an assurance payable at the death of the survivor of x and y if such survivor die before z and within n years.

Explain fully how you would obtain the value of such an assurance by means of a method of approximate summation.

*2. A man aged 30 effects an endowment assurance for 30 years. He is going to an unhealthy climate and the extra risk is such that the rate of mortality is '05 in the first year, '03 in the second year and subsequently the same as for a normal life which had entered at age 50. Calculate the level net premium by $O^{[NM]}$ 3 per-cent. Is there any objection to using it?

3. Find an expression for the annual premium for an annuity to a child aged 3 to commence at age 16 or at the end of the year of previous death of the child's father (aged 30) and to cease when the child reaches age 21 or at previous death. The premiums are to be returned with 3 per-cent compound interest on the death of the child before the annuity commences.

$$4. \text{ Prove } a_x^{(m)} = \bar{a}_x \left(1 - \frac{\delta}{2m} \right) + \frac{\delta}{12m^2} \bar{a}_{x:1}$$

*5. An office which grants annuities on the basis of the Carlisle 3 per-cent Table is asked to grant an immediate annuity during the life of a man aged 60 (payable yearly and apportionable) with a return in the event of death during the first 20 years equal to two-thirds of the difference between the purchase money accumulated at 3 per-cent compound interest and the annuity payments similarly accumulated. If the question of loading be ignored, what rate of annuity can be given?

*6. A company grants whole-life assurances, subject to the payment of H^M 3 per-cent premiums, with a loading of 25 per-cent, and issues policies on the lives of persons, one of whose parents has died of phthisis, with a proviso that a deduction of 20 per-cent will be made from the sum assured in the event of death within the first twelve months; the deduction being reduced by one-twentieth in respect of every year that the assured survives.

The rate of mortality experienced by such persons can be represented by an addition of 10 per-cent to the d_x column of H^M Table between the ages of 20 and 30, and 5 per-cent between the ages of 30 and 40.

A man aged 20 wishes to pay an increased premium in order to secure the full sum assured, whenever he may die. Find the annual premium that should be charged.

7. A whole-life assurance is to be effected by single premium on the life of (x) by three men aged (y) , (w) and (z) and a corporation "A", on the understanding that the survivor or survivors of the three lives at the death of (x) shall equally divide the sum assured and that if neither (y) , (w) or (z) survive (x) then the corporation "A" will be entitled to the policy.

Show how to arrive at the premium to be paid by each of the interested parties.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Third Paper.

1. Find at rate i the annual premium at age (x) for a pure endowment payable at the end of n years under which the premiums received are to be returned with compound interest at rate j in the event of death before age $(x+n)$.

2. Assuming there are a large number of each of the following classes of policies, draft schedules for their valuation, and show exactly how you would classify them.

(1). Immediate annuities.

(2). Deferred annuities by annual premiums.

(3). Sinking fund assurances.

3. Explain in detail how you would construct and verify select tables of joint-life annuities.

*4. Express ${}_nV_x^{(m)}$ as a function of ${}_nV_x$ and calculate the values by H^M 3 per-cent Table, taking quarterly and annual premiums respectively, of policies effected at age 30 which have been 20 years in force.

5. Demonstrate the relation

$${}_nV_x = 1 - (1 - {}_1V_x)(1 - {}_1V_{x+1}) \dots (1 - {}_1V_{x+n-1})$$

In what circumstances will ${}_nV_x$ decrease as i increases?

6. A whole-life policy on (x) in favour of (y) under which the sum assured is payable in twenty equal annual instalments (first payable at end of year of death) has been ten years in force.

If premium payments are now discontinued, what paid-up policy, (payable in instalments as above) can be granted, and to what extent would such instalments be reduced if it be provided that they shall continue during the life of (y) if the latter live more than twenty years after the death of (x) .

*7. Calculate by the $O^{(NM)}$ Table with 3 per-cent interest, the net yearly premium, limited to 10 payments, for a policy under which £100 will be paid yearly for 5 years, the first payment falling due at the end of ten years, subject to the condition that, in the event of a life aged 35 dying during the ten years, the premiums will cease and an annuity of £50 will be paid yearly in advance for the remainder of the ten years in addition to the annuity of £100 for 5 years from the end of the ten years.

*8. Find by the H^M Table with 3 per-cent interest, the net cost of an immediate annuity, payable by half-yearly instalments and apportionable, till the death of the last survivor of three lives aged respectively 65, 59 and 51 the amount of the annuity being £100 a year till the second death and £70 a year for the remainder of the lifetime of the last survivor.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Fourth Paper.

1. Give a short account of the general Discount and Loan business of the Bank of England.

2. Under what authority can loans be raised by the following bodies, and what is the general nature of the security which each of these bodies can offer?

London County Council.
Metropolitan Water Board.
Port of London Authority.

3. Explain the following terms :

Refunding bonds.
Collateral trust bonds.
Income debentures.
Participating preference shares.
Bonus shares.

4. How are the various items of the National Debt classified in the annual parliamentary return? Give a brief description of each class.

5. A proprietary life assurance company

(1) Writes down its Stock Exchange securities in the life assurance fund by £50,000.

- (2) Transfers £50,000 to life fund investment reserve.
- (3) Has a valuation surplus of £230,000, after giving effect to (1) and (2).
- (4) Apportions £180,000 of the foregoing surplus to its participating policies, £20,000 to its shareholders, and £30,000 in setting up annuities in respect of existing pensions chargeable on the life fund.

What Journal entries are necessary to give effect to the above transactions?

6. The following is the outline Balance Sheet of a life assurance company at 31 December 1913:

	£		£
Life Assurance Fund ...	3,530,000	Mortgages	1,500,000
Claims admitted or in-		Investments	2,000,000
timated but not paid ...	60,000	Outstanding Premiums ...	40,000
Outstanding Commission	10,000	Interest accrued	50,000
		Cash	10,000
	<u>£3,600,000</u>		<u>£3,600,000</u>

The cash items received and paid during 1914 are as follows:

<i>Receipts.</i>	£	<i>Payments.</i>	£
Premiums	250,000	Claims	300,000
Interest	140,000	Surrenders	30,000
		Commission	20,000
		Expenses of Management	25,000

The outstanding items on 31 December 1914, are:

	£		£
Premiums	42,000	Claims	70,000
Interest	53,000	Commission	12,000

Prepare a Revenue Account and Balance Sheet for the year 1914.

7. A company purchases for £100,000 leasehold premises for use as an office which are subject to a ground rent of £2,000 per annum for an unexpired term of 90 years, and spends £5,000 on furniture and fittings. The company passes through its accounts 4 per-cent on the purchase money as rent (in addition to the ground rent), $3\frac{1}{2}$ per-cent as interest and $\frac{1}{2}$ per-cent as sinking fund. It also writes off at the end of each year 5 per-cent for depreciation of furniture and fittings.

What were the necessary entries in respect of the purchases, and what further entries must be made in the first year after purchase to give effect to the above provisions?

8. What entries should be made in the books of a Life Assurance Company in respect of the following transactions?

1914.

5 November. Bought £10,000 $2\frac{1}{2}$ per-cent Consols at a total cost of £6,870.

10 December. Purchased Municipal Term Annuity of £500 (payable by equal half-yearly instalments due 1 January and July, last instalment due 1 July 1930) at a price to pay 4 per-cent on the investment, viz, £5,971.

1915.

1 January. Received instalment of the above Term Annuity.

1 January. Sold £10,000 5 per-cent Debenture Stock (Interest 31 March and 30 September) for £9,500. The Stock stood in the Company's books at £9,800.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW

(PART III).

First Paper.

1. Given the age next birthday at entry, calendar year of entry, calendar year of exit and number of premiums paid, show how you would construct select and aggregate tables of mortality. The following cases, which entered at age 30 next birthday in 1910 and were amongst the "exits" in 1914, form part of the experience. Explain how each would be dealt with.

No.	Number of Premiums paid	Cause of Exit
1	4	Dead.
2	$4\frac{1}{2}$	Surrendered.
3	$4\frac{1}{2}$	Dead.
4	5	Surrendered.
5	5	Existing on 31 December 1914.

2. If you had been asked last August to estimate for naval and military purposes the number of unmarried males in Great Britain and Ireland between the ages 18 and 35, how would you have proceeded to answer the question?

3. A Society with a stationary membership of 5,000, entrants being confined to males aged 20 to 25, desires a full report on its mortality experience for the five years ending 31 July 1915 by the end of August. The secretary can supply you on 3 May with cards—written up to 30 April—showing for each member dates of birth and of entry, and in the case of cancelments, date and cause of cancelment. He promises to supply full details of any changes in membership up to 31 July by the 2nd of August.

How would you proceed with such an investigation with the view to completing it in the period mentioned? Assume entrants are selected by medical examination and all impaired cases are rejected, also that the members are engaged in similar occupations.

*4. A graduation has been made for a select and ultimate table, and you have to decide whether the graduation of the force of mortality is smooth. Explain generally what you would do and use the figures in Table XIX of the Examination Tables as an example of the methods you indicate.

5. Describe briefly the processes adopted in constructing combined Marriage and Mortality Tables relating to Peerage males.

6. Describe the methods by which the Manchester Unity Whole Society Rates of Sickness for the first six months' sickness were analysed into the rates applicable for each week of the first six months.

7. Give two well-known formulas for graduation by summation. If you decide to adopt one of them in practice, which would you use, and why?

8. Give a short account of, and the conclusions to be drawn from, a recent investigation made into the mortality of assured lives who had a family history of consumption.

or

9. What Mortality Tables have been published in connection with the 1911 census and what are their characteristics?

Questions (8) and (9) are alternatives. Only one of them is to be answered.

* A Short Collection of Actuarial Tables will be supplied for use in answering this question.

Second Paper.

1. State concisely how you would proceed to analyse the trading profits of a Life Office into the various sources of profit from the information published in the Board of Trade Returns.

2. A Mutual Life Office has been valuing by O^M and $O^{M(5)}$ $2\frac{1}{2}$ per-cent with O^M $2\frac{1}{2}$ per-cent net premiums. Since its last valuation 5 years ago, when it gave a compound reversionary bonus of £1 12s. 6d. per-cent per annum, it has done a rapidly increasing new business and its expense ratio has gone up from 13 per-cent of the premium income to $15\frac{1}{2}$ per-cent by $\frac{1}{2}$ per-cent per annum. Depreciation is 7 per-cent of the whole fund; net rate of interest earned is $4\frac{1}{4}$ per-cent before allowing for depreciation; valuation loadings 22 per-cent of gross premiums, and miscellaneous (including mortality) profit 0.6 per-cent of the fund annually. The cost of £1 per-cent compound bonus for the quinquennial period is 6 per-cent of the fund at the end of the period. The annual premium income is roughly 10 per-cent of the fund. Discuss the position of the office and whether you would advise a change in the valuation basis. Neglect special considerations due to the war.

3. An Office has issued a large number of Endowment Assurances maturing at certain ages. Under some of these the number of premiums payable is

Maturity age – age next birthday at entry + 1

and under the others,

Maturity age – age next birthday at entry.

In practice, the office either dispenses with the last payment or only charges a proportion of it on each policy in the first group, and in a valuation it assumes that on such policies the final premium will be proportionate to the period from date of last payment to date of maturity.

Explain clearly how you would value these policies.

4. An Industrial Assurance Company issues a very large number of whole life assurances at monthly premiums. The sum assured is constant at all ages for a given monthly premium but the term for which the premiums are payable varies with the age at entry. There is also an option to convert into an endowment assurance for the same sum assured by paying the premiums for an additional period. How would you classify these policies for valuation and what formula would you use?

5. What methods can be conveniently used for giving annual bonuses on industrial policies? State which method you prefer, and why.

6. A Life Office valuing by O^M and $O^{M(5)}$ 3 per-cent and deriving 35 per-cent of its surplus from favourable mortality, gives each policyholder the option of taking Bonuses either (a) on the Compound Bonus System, or (b) by way of reduction of all future premiums, the method selected at the first distribution to be adhered to at subsequent quinquennial distributions. Assuming the

majority of policyholders select (*a*) and a 30s. per-cent per annum compound bonus to be maintained, how would you calculate the equivalent bonus in the cases in which (*b*) has been chosen, and how would you treat such policies in a valuation?

7. In the case of a Life Office making an annual valuation but only distributing profits quinquennially, outline a method by which the increase in the surplus disclosed by the valuation can be analysed into its different sources, such as profit from mortality, &c.

8. An Office declares its Bonus quinquennially in the form of a Compound Reversionary addition to the sums assured.

What machinery would you set up in order to estimate the cost of the new bonus with a minimum of labour, and how would you deal in the valuation with policies which guarantee the profits of other offices?

Would your machinery be such as to give the cost of one year's bonus at the end of the first year of the quinquennium, two years' bonus at the end of the second year and so on? If not, what modification would you make to enable the cost of the bonus to be estimated each year during the quinquennium?

Third Paper.

1. How would you calculate premiums for a whole-life policy without profits, 50 per-cent of the premiums paid to be returned at age 50 and 50 per-cent of subsequent premiums at the end of each subsequent 10 years? How should such policies be treated in a valuation?

2. On what basis would you calculate annual premiums for the following assurances, and what reserves would you make for (*a*) and (*b*)?

(*a*) Pure endowments at 60 on lives aged 25 to 40 with return of premiums paid without interest in the event of death.

(*b*) Deferred annuities without return of premiums in the event of death.

(*c*) Without profit endowment assurances.

Mention any points you would have specially in mind in fixing the rates to be charged.

3. A policy is required to cover the risk of the survivor of two brothers aged 13 and 19 years, respectively, predeceasing their father aged 50.

The brother, aged 19, would be liable under a whole-life policy to an extra of 2 per-cent per annum for 5 years. How would you construct and load the annual premium for the risk?

4. How would you calculate the office annual premium

$$P_{x:\overline{y}:z}^1?$$

During what status would you require the premiums to be payable, and why?

5. A firm, having a large number of employees in Northern Rhodesia, Jamaica, India and Southern Brazil, has decided to arrange for endowment assurances at 50 to be effected on their lives when they are first sent abroad at age 25, the extra risk being paid for by a single premium. What would you charge (a) for each place mentioned, (b) as a general single extra premium in all foreign cases assuming that an equal number of employees is sent to each place?

What reserves should be made?

6. State the various ways which have been adopted in Great Britain for contrasting the relative mortality rates of persons in different occupations. Can any weight be attached to the results of the past investigations for assessing premiums for life assurance? State reasons.

*7. It is suggested that certain occupation extra premiums on whole-life assurances effected at ages under 35 should be paid by a single premium, returnable at age 50 in the event of survival to that age. State the methods and basis you would adopt to work out the single premiums, and give, for age 30, a numerical example showing your method.

* A Short Collection of Actuarial Tables will be supplied for use in answering this question.

Fourth Paper.

*1. Table I gives the whole business on 31 December 1914 of an office which values by O^M 3 per-cent. Table II gives the revenue account for 1914 excluding depreciation, which amounts to £7,000. Complete the valuation of the business in Table I, making any special reserves you consider necessary. Is the office solvent? If so, do you anticipate that it will in future be able to pay a bonus, and of what amount?

NOTE.—All your workings, which must be in tabular form, should be shown. Assume that the annuities in Table XIV are O^M Annuities.

TABLE I.

Exact Age attained	Sums Assured	Bonuses	Office Annual Premiums	OM 3 % Net Premiums
	£	£	£	£
25	29,000	1,000	600	440
45	72,500	7,500	2,500	1,600
65	30,000	10,000	1,200	900
85	1,000	1,000	50	35
Total	132,500	19,500	4,350	2,975

TABLE II.

Revenue Account.

Fund on 1 Jan. 1914 ...	£42,000	Claims ...	£1,975
Premiums (including war extras of £750) ...	4,800	Surrenders ...	975
Interest (less tax) ...	1,800	Commission ...	325
		Expenses ..	325
		Fund on 31 Dec. 1914 ...	45,000
	<u>£48,600</u>		<u>£48,600</u>

*2. An office distributes its surplus by a modified contribution method, allotting to each policy the interest profit on the reserve held at the beginning of the quinquennium, and the balance of profit being divided in proportion to the ordinary premiums paid during the quinquennium, the bonus being declared as a reversionary addition to sum assured.

The particulars of the business are shortly as follows:—

All Ordinary Whole Life—over five years in force.

Total Reserves held at beginning of quinquennium in respect of business in force at end of quinquennium
£242,000.

Ordinary premiums paid during quinquennium on policies in force at end of quinquennium £138,184.

Valuation basis—H^M 3 per-cent with usual adjustment for early payment of claims.

Surplus to be divided £31,644.

Rates of interest earned during quinquennium (net)—

£3 18s. 7d., £3 19s. 3d., £4 1s. 0d., £4 0s. 6d.,
£4 2s. 3d.

Calculate the actual reversionary bonuses, allotted to the following policies, shewing each step in your calculations :

Age at Entry	Present Valuation Age.	Sums Assured	Bonuses	Office Yearly Premiums		
		£	£	£	s.	d.
20	45	1,000	318	21	7	0
25	55	300	45	7	6	6
30	65	1,500	470	42	2	8
35	75	750	—	24	12	0

*†3. A purely life office, having an annuity business of £80,000 per annum, which was valued at the beginning of 1912 on the British Offices Annuitants 3 per-cent basis, giving effect to selection, at £700,000, grants new annuities on a British Offices Annuitants $3\frac{1}{2}$ per-cent select basis, loaded $2\frac{1}{2}$ per-cent. The amount of new business was as follows :—

1912.	£5,000	per annum ;	purchase money	...	£40,000
1913.	5,500	„ ;	„	...	60,000
1914.	8,000	„ ;	„	...	80,000

The mortality was as expected for 1912, 1913 and 1914. The rate of interest realised was 4 per-cent net. Depreciation at the end of 1914 was 5 per-cent on the fund for the period. Discuss the position of the fund, and the effect of the new business, making suitable allowance for initial expenses.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

† For the purposes of the question it may be assumed that two-thirds of the business is on the lives of females, and that the British Offices Annuitants Female life annuities are the same as ONM with 5 years off the age, and male life annuities as ONM with 2 years off the age, and that one-half per-cent in the rate of interest alters the present value of an annuity by 5 per-cent.

EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART IV).

First Paper.

1. For what purposes can a tenant for life of settled land create a mortgage thereon under his statutory powers?

2. Distinguish between a negotiable instrument and an ordinary *chose in action* with special reference to the rules affecting their assignment.

3. Application is made for the surrender value of a policy. What title would you require to be shown, or what enquiries would you make when the applicant is

- (a) Assignee under a voluntary assignment?
- (b) Trustee under a marriage settlement?
- (c) Mortgagee?

4. What is a shifting use and how may it arise?

In what circumstances can a similar interest in personal property arise?

A policy is effected under the Married Women's Property Act by a man on his own life for benefit of his wife in the event of her surviving him. The policy moneys are expressed to be payable to

“(the wife) in the event of her surviving the Assured, in which event she shall be trustee of the policy moneys for her own separate use, but otherwise to the executors, administrators, or assigns of the Assured.”

Criticise the wording of the policy and suggest any alternative that seems to you preferable.

5. What precautions would you take in paying moneys to Trustees

- (a) in England
- (b) „ Scotland

and why?

6. In 1865 A executed a marriage settlement and assigned to trustees a life policy on his life. The settlement gave a life interest to the wife and the reversion after the death of the wife to the children of the marriage.

In 1910 A applies to surrender the policy and states that all his children are of age, and that they and his wife are willing to concur in the surrender. The settlement makes no provision for the question of surrender.

State whether and, if so, on what terms you would be willing to accede to the application, and indicate what enquiries you would make before proceeding.

Would your attitude be affected if the date of the settlement were 1875?

7. What are the essential conditions under which the Court will decree specific performance of a contract?

Will the following contracts be specifically enforced?

- (a) Contract for the sale of stock and shares.
- (b) Contract for the sale and purchase of an annuity.

8. What are the principal alterations in the Bankruptcy Laws of this country that were effected by the new acts recently passed?

9. What obligation is there on (a) the mortgagor, (b) the mortgagee in the matter of giving notice for repayment of a mortgage debt?

A mortgagor gives notice of intention to repay, but at expiration of period he is not in a position to meet his obligation.

What were the legal remedies available to the mortgagee if he wished to enforce payment,

(a) In December 1913; (b) In December 1914?

Second Paper.

1. State concisely what special legislation affecting soldiers there is in

(a) The Friendly Societies Act, 1896.

(b) The Wills Act.

(c) National Insurance Acts.

2. What special points arose for consideration in calculating the ordinary "reserve values" and "transfer values" for males under the National Insurance Scheme, and how were they dealt with?

3. A Widows' Fund is enabled to grant annuities much in excess of the amounts supported by the members' contributions, &c., through having an income from an extraneous source which may be regarded as a perpetuity. Would you, in a valuation, take into account the capitalized value of the perpetuity and place an estimate on the liability which may be incurred in respect of new entrants, or in what alternative manner would you deal with the perpetuity? Assume the number of new entrants to be constant each year.

How would you deal with the question of withdrawals in the valuation?

4. Describe, and criticize from an actuarial standpoint, the scheme of arrears under the National Insurance Act, 1913.

5. A deficiency is shewn in the valuation of a pension fund in which retirement may take place at any age between 60 and 65. It is proposed to rectify the deficiency by:

(a) the employers undertaking the liability for pension payments up to age 65 in all cases of retirement before that age, and

(b) an increase in the rates of members' contributions.

How would you ascertain the relief afforded by (a) and the required increase in the rates of contributions? Assume pensions to be calculated on the "average salary" basis and contributions to be a percentage of salary.

6. Draft a clause providing for the payment of policy moneys direct to the Inland Revenue Authorities in payment of estate duties, and state clearly what your procedure would be on a claim arising under the policy.

7. What are the usual methods of rectifying an error in the age of the assured when the discrepancy is discovered (*a*) while the policy is in force, (*b*) after it has become a claim? What alternative methods have been suggested as being more equitable?

Third Paper.

1. Discuss concisely the various measures taken by the Government after the outbreak of war dealing with

- (*a*) Bills.
- (*b*) Currency.

2. An assurance company holds, amongst others, the following investments:

- (*a*) Imperial Continental Gas Association Capital Stock.
- (*b*) Cunard Steamship Co. $4\frac{1}{2}$ per-cent Mortgage Debenture Stock.
- (*c*) Belgian 3 per-cent Loan, 1914.
- (*d*) Brazil 4 per-cent Loan, 1911.

At the end of 1914 a report was desired by the Board of Directors upon such securities. To what points should attention be drawn?

3. An assurance company A proposes to acquire a proprietary assurance company B by paying £*a* for its shares and £*b* for expenses, &c. It is suggested that the purchase should be carried through either as an amalgamation in which the B policies are treated as a separate series, or as a purchase by A of the large majority of the shares of B. (1) What formalities are necessary in each case? (2) How will the accounts of A appear after the completion of the transaction?

4. The directors of an assurance company have agreed to make an advance on security of a large agricultural estate in England, subject to the receipt of a satisfactory valuation. What information would you ask the surveyor to furnish in his report, and why?

5. What statement and certificate regarding the value of investments are now required to be given at the foot of insurance companies' balance sheets?

How often, and by whom, must these statements and certificates be given in the case of a company transacting

- (*a*) Life Assurance business only?
- (*b*) Life Assurance and other classes of insurance business?

6. Sketch briefly the course of the New York Exchange rate on London during the period August 1914 to December 1914, and mention any steps which were taken with the object of regularising the rate. Was the prevailing rate during this period an advantage or otherwise to this country?

7. A block of American Railway Bonds is offered to your company as an investment. Indicate the nature of the investigation you would make before coming to a decision as to their suitability.

8. For several years past the trade statistics of Canada show that the imports each year exceed in value the corresponding exports, whereas, in the case of somewhat similar countries, as *e.g.* Argentina and Australia, the exports exceed in value the imports. What conclusions do you draw from these statistics, more particularly in relation to the condition of Canadian finances at the present time and in the near future?

NOTE.—In answering this question the candidate is not to take into account the effect of the present War.

Fourth Paper.

*1. A, aged 50, is entitled to a life interest in the undermentioned Fund, which after his death is to be divided between B and C, in the following proportion: a first charge to B of £5,000 free of all duties and expenses, the residue to be payable to C. It is proposed to apportion equitably the fund by mutual consent. Calculate the respective shares of each of the parties. Assume income tax at 2s. 6d. in £ and death duties at 10 per-cent.

Fund—£5,000 New Zealand 4 per-cent Inscribed Stock, 1929.
 £5,000 Grand Trunk Railway Guaranteed 4 per-cent Stock.
 £500 Madras Railway Annuity, Class B, 1956.

*2. A is entitled to the absolute reversion to the following fund on the death of a lady aged 70, the fund being subject to the payment of an annuity of £100 per annum to a lady aged 60 and £150 per annum to a lady aged 55.

Fund—£5,000 War Loan 1925-1928.
 £200 East Indian Railway Company, A Annuity, 1953.
 £5,000 on a well secured Mortgage yielding $3\frac{1}{2}$ per-cent.
 A leasehold house, unexpired term 40 years, ground rent £15 per annum, let at £150 per annum.
 Value A's interest.

3. In dealing with reversions submitted for purchase, explain what difficulties might arise in the following cases and state what special precautions, if any, you would take to deal with them :

- (1) A reversion where the reversioner is one of the trustees.
 - (a) When the reversioner is entitled to the whole fund.
 - (b) When the reversioner is only entitled to a share of the fund.
- (2) A reversion to land held in trust for persons in succession and not subject to a trust for conversion.
- (3) A contingent reversion.

4. You are requested to value the assets of a reversionary company which consist entirely of reversions and life interests. These assets have not been valued since the 31 December, 1908, when a valuation was made on a $4\frac{1}{2}$ per-cent basis, the Carlisle table being used for the reversions. Draft a report to the company dealing specially with the following points :

- (a) The rate of interest and table of mortality to be adopted in your valuation distinguishing, if you think fit, between the reversions included in the valuation made at the 31st December, 1908, and the new purchases since that date, which may be assumed to form one-third of the total investments.
- (b) The prices at which the Stock Exchange investments included in the funds in reversion should be valued.

*5. A, aged 55, is entitled to a life interest in the following fund and to the reversion to the whole fund on the death of his wife, aged 53, without issue,

£5,000 $2\frac{1}{2}$ per-cent Consols.

£5,000 War Loan 1925-1928.

£5,000 Midland Railway $2\frac{1}{2}$ per-cent Debenture Stock.

What is the value of his interest ?

*6. A whole-life policy for £10,000, with reversionary bonuses of £5,600, in a first class office which, at its last quinquennial valuation at the end of 1913, gave a 30/- compound reversionary bonus, is offered for sale. The life assured is aged 70 and the annual premium is £250. Interim bonuses are given at the rate last declared. What is the market value of the policy if a premium is now due and unpaid ?

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

PROCEEDINGS OF THE INSTITUTE.—SESSION 1914-1915.

First Ordinary Meeting, 30 November 1914.

The President (Mr. ERNEST WOODS) in the Chair.

The President delivered an Inaugural Address.

Second Ordinary Meeting, 21 December 1914.

The President (Mr. ERNEST WOODS) in the Chair.

Mr. Henry Edwin Jones, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "Canadian Mortgages regarded as a field for the investment of the Funds of British Life Assurance Companies, with some general notes on Canadian indebtedness," was read in abstract by the Author, Mr. A. D. Besant.

The following gentlemen took part in the discussion:—Sir Thomas Whittaker and Mr. F. W. Hirst (visitors), Messrs. Owen Kentish, G. H. Lawton, S. G. Warner, Geoffrey Marks, and the President.

Third Ordinary Meeting, 25 January 1915.

The President (Mr. ERNEST WOODS) in the Chair.

A paper entitled "The Analysis of Life Office Expenses," was read in abstract by the Author, Mr. C. H. Maltby.

Mr. H. J. Rietschel and Mr. S. G. Warner spoke on the paper.

Fourth Ordinary Meeting, 22 February 1915.

The President (Mr. ERNEST WOODS) in the Chair.

A paper entitled "Practical Points in connection with the Formation and Valuation of Pension Funds, with a Note on Group Assurances," by Messrs. J. Burn and F. P. Symmons, was read in abstract by Mr. Symmons.

The following gentlemen took part in the discussion:—Messrs. T. Tinner, E. C. Thomas, V. Marr, J. Bacon, H. H. Austin, C. R. V. Coutts, T. G. Ackland, and the President.

Fifth Ordinary Meeting, 29 March 1915.

The President (Mr. ERNEST WOODS) in the Chair.

A paper entitled "Two Notes on Questions of Office Practice," by Mr. R. Todhunter, was submitted.

The following gentlemen took part in the discussion:—Messrs. R. C. Simmonds, C. C. Monkhouse, C. R. V. Coutts, R. G. Salmon, and W. Palin Elderton.

Sixth Ordinary Meeting, 26 April 1915.

The President (Mr. ERNEST WOODS) in the Chair.

A paper entitled "The New National Life Tables," was read in abstract by the Author, Mr. George King.

The following gentlemen took part in the discussion:—Dr. A. Newsholme, Dr. T. H. C. Stevenson, and Dr. E. C. Snow (visitors), Sir Alfred Watson, Messrs. S. J. Rowland, A. Henry, T. G. Ackland, and S. J. Gunningham.

The Sixty-Eighth Annual General Meeting, 10 June 1915.

The President (Mr. ERNEST WOODS) in the Chair.

The proceedings at the Annual General Meeting will be found on page 415.

REPORT, 1914–1915.

The Council have the pleasure to report to the Members upon the progress of the Institute during the Session of 1914–1915, the sixty-seventh year of its existence.

There has been an *increase* of 35 in the total number of members, as compared with the previous year. At the end of the official year in which the Institute was incorporated by Royal Charter the number of Members was 434; twenty years later, at 31 March 1905, it was 881. Since that time the numbers have been as follows:

On 31 March	Fellows	Associates	Students	Corresponding Members	Total
1906	232	301	367	22	922
1907	248	303	383	22	956
1908	253	313	421	22	1,009
1909	254	325	400	19	998
1910	259	335	348	21	963
1911	267	339	308	20	934
1912	278	354	268	20	920
1913	282	355	252	19	908
1914	295	358	238	19	910
1915	304	361	263	17	945*

* This includes a certain number of Members (most of whom are on military or naval service) whose names have been retained on the list although their subscriptions have not been paid.

The following schedule shows the additions to, and the changes and losses in the membership which have occurred during the year ending 31 March last:

Schedule of Membership, 31 March 1915.

	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1914 .	295	358	238	19	910
ii. Withdrawals by					
(1) Death	5	4	1	2	14
(2) Resignation or otherwise	2	
	290	352	237	17	896
iii. Additions to Membership					
(1) By Election	1	49
(2) By Examination	38	...	
(3) By Re-instatement	4	6	...	
	290	357	281	17	945
iv. Transfers					
(1) By Examination:					
<i>from Associates</i>	...	14
<i>to Fellows</i>	14
	304	343	281	17	945
(2) By Examination:					
<i>from Students</i>	18
<i>to Associates</i>	18
v. Number of Members in each class on 31 March 1915 .	304	361	263	17	945

There are also 188 candidates admitted as Probationers, and 72 as Students conditionally on their passing Part I of the Examination. These are not included in the above Schedule of Membership. The numbers in these two classes since 31 March 1909 have been as follows:

On 31 March	Probationers	Conditional Students	On 31 March	Probationers	Conditional Students
1910	141	42	1913	197	55
1911	160	58	1914	200	67*
1912	181	59	1915	188	72

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of seven Fellows, Sir G. F. Hardy, K.C.B., Messrs. F. Addiscott, H. P. Calderon, J. Chisholm, A. Hewat, H. W. Manly, and T. B. Winser; six Associates, Messrs. E. Corble, M. Fox, R. Jerman, J. Kearry, F. C. Mann, and H. J. Pearce; one Student, Mr. E. W. Cox; and two Corresponding Members, Dr. J. Klang and M. Paul Guieysse.

By the death of Sir George Hardy actuarial science has sustained a severe loss. He had been a member of the Institute for nearly forty years, and was President in 1908-10. Few members of the profession have left a more memorable record of original work, or have produced so deep an impression on contemporary actuarial thought.

Mr. Manly, who was President in 1898-1900, had been a member of the Institute for fifty-four years, and during that time rendered valuable service to the profession. He was a frequent contributor to the *Journal*, and was a constant attendant at the meetings. In later years he devoted much of his time to original work in connection with Staff Pension Funds.

The Council greatly regret that two Members, 2nd Lieut. Frederick C. Mann and Private Edgar Corble, have been killed in action.

At this great crisis in the history of the Empire the actuarial profession has responded loyally to the call for personal service. Over 180 Members and Probationers of the Institute in the United Kingdom and the Colonies—representing a very large proportion of those of military age—are known to have joined the Army or Navy. The Council have given instructions that a list of Members serving with the Forces shall be compiled for future publication.

The Annual Subscriptions and the Entrance Fees appearing in the Revenue Account amounted to £2,111. 11s. 0d., as compared with £2,064. 6s. 0d., received in the previous year. The Income and Expenditure for the year were £2,612. 14s. 10d., and £2,125. 17s. 7d. respectively.

The following papers were submitted at the sessional meetings of the Institute, namely :

- 30 *November* 1914.—Inaugural Address by the President, Mr. Ernest Woods.
- 21 *December* 1914.—“Canadian Mortgages regarded as a field for the Investment of the Funds of British Life Assurance Companies, with some general notes on Canadian Indebtedness.”—Mr. A. D. Besant.
- 25 *January* 1915.—“The Analysis of Life Office Expenses.”—Mr. C. H. Maltby.
- 22 *February* 1915.—“Practical points in connection with the formation and valuation of Pension Funds, with a note on Group Assurances.”—Messrs. J. Burn and F. P. Symmons.
- 29 *March* 1915.—“Two Notes on Questions of Office Practice.”—Mr. R. Todhunter.
- 26 *April* 1915.—“The New National Life Tables.”—Mr. George King.

For the Examinations held in the United Kingdom and the Colonies on 19, 20, 21, 22, 23 and 24 April 1915, 143 entries were received, namely :

65	for Part	I.
2	„	I, (§) 3.
35	„	II.
22	„	III.
19	„	IV.

The results will be duly announced. The Council warmly acknowledge the valuable services of the Board of Examiners, and also those of the Honorary Supervisors.

The Council have appointed a Committee to consider the question of formulating a uniform scheme of Pension Fund notation.

At the Annual Meetings of 1913 and 1914 references were made to the proposed formation of a Research Bureau. The Council have to report that further consideration of the proposal has been deferred until after the conclusion of the War.

As announced at the January meeting, the International Actuarial Congress, which was to have been held at Petrograd this year, has, owing to the War, been indefinitely postponed.

Dr.

Revenue Account for the

1914.			1915.		
£	s.	d.	£	s.	d.
9,523	7	9	Amount of Funds at the beginning of the year—		
			General Fund (including Stock of Publications, other than <i>Journal</i>)	9,922	0 3
385	10	6	Messenger Legacy Fund	397	1 9
331	4	8	Brown Prize Fund	341	3 5
10,240	2	11		10,660	5 5
			Subscriptions—		
873	12	0	Fellows	915	12 0
720	6	0	Associates	740	5 0
258	6	0	Students	282	9 0
109	4	0	Probationers	100	5 6
1,961	8	0		2,038	11 6
6	6	0	Fines on Reinstatement	2	2 0
1,967	14	0		2,040	13 6
			Entrance Fees—		
12	12	0	Associates	8	8 0
42	0	0	Students	36	15 0
42	0	0	Probationers	25	14 6
96	12	0			70 17 6
127	1	11	Balance of Publications Account		131 19 6
			Dividends and Interest—		
338	11	8	General Fund	347	1 5
11	11	3	Messenger Legacy Fund	11	18 3
9	15	9	Brown Prize Fund	10	4 8
360	1	8			369 4 4
£12,791	12	6		£13,273	0 3

Publications Account for the

£	s.	d.	£	s.	d.
343	8	8	Stock (excluding <i>Journal</i>) at the beginning of the year	252	16 6
31	18	5	Cost of Index to Transactions of Seven International Actuarial Congresses	...	
23	2	6	Cost of Reprint of Text-Book, Part I	...	
20	1	7	Binding and Advertising	14	15 4
127	1	11	Balance	131	19 6
£545	13	1		£399	11 4

Balance Sheet.

£	s.	d.		LIABILITIES.	£	s.	d.	£	s.	d.
9,922	0	3	General Fund					10,386	14	7
233	9	2	Messenger Legacy Fund		233	9	2			
163	12	7	Accumulated Dividends		175	10	10			
397	1	9						409	0	0
200	0	0	Brown Prize Fund		200	0	0			
141	3	5	Accumulated Dividends		151	8	1			
341	3	5						351	8	1
								11,147	2	8
122	17	0	Examination Fees for year 1915					80	17	0
47	6	1	Sundry unpaid Accounts					12	10	1
£10,830	8	6						£11,240	9	9

year ending 31 March 1915.

			Cr.		
1914.			1915.		
£	s.	d.	£	s.	d.
707	12	4	561	6	2
86	5	0	86	5	0
793	17	4	647	11	2
323	14	0	215	6	4
470	3	4			
67	15	3			432 4 10
76	10	8			46 10 5
334	1	7			59 5 3
214	4	0	329	2	0
119	17	7	274	1	0
333	7	6			55 1 0
249	7	6	253	1	0
84	0	0	127	1	0
600	0	0			126 0 0
393	14	0			
78	8	3	600	0	0
17	1	2	416	7	0
30	9	11	85	1	11
134	18	2			
44	3	5			
4	5	4			
1,303	0	3			
10	0	0			
...					
10,660	5	5			

Examined and found correct, 1 May 1915.

12,791	12	6	W. G. TITMUSS,	} Auditors.	£13,273	0	3
			ARTHUR TAYLOR,				
			ROBT. S. B. SAVERY,				

year ending 31 March 1915.

£	s.	d.		£	s.	d.
292	16	7	Sales (excluding Journal)	197	8	11
252	16	6	Stock (excluding Journal) at the end of the year	202	2	5

Examined and found correct, 1 May 1915.

£545	13	1	W. G. TITMUSS,	} Auditors.	£399	11	4
			ARTHUR TAYLOR,				
			ROBT. S. B. SAVERY,				

31 March 1915.

£	s.	d.	ASSETS.	£	s.	d.
2,370	0	0	£3,000 Natal 3 per-cent Inscribed Stock	2,370	0	0
1,005	0	0	£1,200 Metropolitan Railway 3½ per-cent Debenture Stock	1,005	0	0
1,980	0	0	£2,000 Great Eastern Railway 4 per-cent Debenture Stock	1,980	0	0
835	0	0	£1,000 Great Northern Railway Preferred Ordinary Stock	835	0	0
1,431	0	0	£1,350 Great Western Railway 4½ per-cent Debenture Stock	1,431	0	0
937	8	0	£1,000 Dominion of Canada 3½ per-cent Registered 1930-50 Stock	937	8	0
890	0	0	£1,000 New South Wales 3½ per-cent Inscribed 1930-50 Stock	890	0	0
480	16	0	£600 Belgian Government 3 per-cent Sterling Loan of 1914	480	16	0
252	16	6	Stock of Publications (excluding Journal) in hand	202	2	5
...			Cash on Deposit Account	500	0	0
645	8	0	Cash on Current Account and in hand	417	3	4
...			Subscriptions in Arrear	189	0	0

Examined and found correct, 1 May 1915.

10,830	8	6	W. G. TITMUSS,	} Auditors.	£11,240	9	9
			ARTHUR TAYLOR,				
			ROBT. S. B. SAVERY,				

During the Session the honour of Knighthood was conferred upon Sir A. W. Watson. The Council have conveyed, by resolution, the congratulations of the profession to their distinguished colleague.

The stock in hand of the Institute publications on 31 March was as follows :

No. of Copies	Description of Work
25,557	Parts of <i>Journal</i> .
746	Index to Vols. 1 to 40.
193	<i>Text-Book</i> , Part II (Second Edition).
636	Government Joint-Life Annuity Tables
739	Select Life Tables.
448	A Short Collection of Actuarial Tables (New Edition).
1,117	Frequency-Curves and Correlation (W. P. Elderton).
40 in cloth } 2,337 in paper }	(Lectures on Finance and Law (Clare and Wood Hill).
1,533	Lectures on the Companies Acts (A. C. Clauson).
1,213	Lectures on the Law of Mortgage (W. G. Hayter).
716	Lectures on the Measurement of Groups and Series (A. L. Bowley).
1,454	Lectures on the Construction of Tables of Mortality, &c. (Sir G. F. Hardy, K.C.B.).
953	Lectures on Stock Exchange Investments (J. Burn).
1,533	Lectures on Friendly Society Finance (Sir A. W. Watson).
323	South African War Mortality (F. Schooling and E. A. Rusher).
267	Life Assurance Law (A. R. Barrand).
679	British Offices' Valuation Tables.
646	Transactions of the Second International Congress of Actuaries.
801	Index to Transactions of Seven International Actuarial Congresses.
1,500	Examination Questions, 1911-14.

The *Text-Book*, Part I, is under revision, and will be republished at an early date.

11 May 1915.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, 1915.

Examinations were held on the 19th, 20th, 21st, 22nd, 23rd, and 24th of April 1915, in the United Kingdom, the Colonies, and India, at London, Liverpool, Edinburgh, Dublin, Melbourne, Sydney, Perth, Wellington, Montreal, Toronto, Ottawa, Winnipeg, Bombay, Calcutta, with the following results.

The successful candidates are placed in two classes only, the names being printed in alphabetical order in each class.

PART I.

Sixty-five candidates sent in their names, of whom fifty-six presented themselves (thirty-three in the United Kingdom, and twenty-three in the Colonies), and thirteen passed, namely:—

Class I:

None.

Class II:

Bray, J. F. L.
Innes, F. F.
Keene, H. P.
Kitson, E. W.
Mabon, J. B.
McLean, J. V.
Marriott, A.

Rao, B. L.
Shrewsbury, A. H.
Snowdon, S. F.
Takeshita, S.
Thorpe, A. H.
Williams, E.

PART II.

Thirty-five candidates sent in their names, of whom twenty-nine presented themselves (twenty-four in the United Kingdom, and five in the Colonies), and eleven passed, namely:—

Class I:

None.

Class II:

Brown, S. P.
Clegg, C.
Fielden, G. S.
Fox, G. A.
Grantham, G. H.
Kerr, A. P. T.

Klagge, O. C. J.
Little, R.
McConney, E. M.
Sheehan, P. F.
Thompson, F. A.

PART III.

Twenty-two candidates sent in their names, of whom twenty presented themselves (sixteen in the United Kingdom, and four in the Colonies), and thirteen passed, namely:—

Class I:

None.

Class II:

Brown, P. G.
 Carey, N. L.
 Fortington, H. A.
 Harrington, E. W.
 McConaghy, C. A.
 McCornack, P. H.
 Marshall, A. W.

Menzler, F. A. A.
 Ruddie, F.
 Tayler, H. H.
 Underwood, R. E.
 Wallis, H.
 Wilton, H. G.

PART IV.

Nineteen candidates sent in their names, of whom eighteen presented themselves (fifteen in the United Kingdom, and three in the Colonies), and ten passed, namely:—

Class I:

None.

Class II: •

†Barrett, W. G.
 †Bennett, S.
 †Clarke, H. T.
 †Craig, W. G.
 †Jackson, H. M.

†Peter, J. C.
 †Stocks, J.
 †Sturgeon, R. W.
 †Williamson, W. W.
 †Wisdom, S. H.

Those marked (†) have now completed the Examination for the Class of Fellow.

PART I, § 3.

(COMPOUND INTEREST AND ANNUITIES.)

Two candidates, who had already passed, or been exempted from Part I of a Syllabus prior to 1908, entered for this section alone and presented themselves in the United Kingdom, neither of whom passed.

By Order of the Council,

W. PALIN ELDERTON,
Chairman of Board of Examiners.

A. D. BESANT,
 J. BURN,

Joint Honorary Secretaries.

28 June 1915.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Sixty-eighth Annual General Meeting of the Institute was held at Staple Inn Hall on Thursday, 10 June 1915, the President (Mr. Ernest Woods) being in the Chair.

The Report of the Council having been taken as read,

The PRESIDENT, in moving the adoption of the Report and Accounts, said that no President of the Institute, since its foundation, had ever had to propose such a resolution at a more eventful or momentous time. With the minds of the Members fixed on the great fight for liberty, humanity, and civilization in which the nation was taking part—wounded at heart as so many were by the loss of those near and dear—it was more than difficult, it was practically impossible, to concentrate attention on the comparatively smaller matters concerning the Institute. They were, however, more fortunate than some of their Continental friends, in the fact that they had been able to carry on their work throughout the Session.

The date of the present meeting had been fixed just a year ago at a later date than usual so as to permit many of the Members, who would otherwise have been prevented, to take part in the Eighth International Congress of Actuaries which was to have been held last week in Petrograd. That Congress, as was already known, had been indefinitely postponed, to the disappointment of all, and he asked permission to read a message which he had received within the last few days from the Russian Actuarial Society—“We desire to inform you that we deplore the impossibility of receiving you as our guests at the time appointed, but we associate ourselves with you in heart and spirit in the glorious cause of justice and of civilization which our countries are defending. Assured that the near future will ensure the triumph of justice and will enable us to hold our projected Congress, we, though far away, send you our best wishes for your success and prosperity.” (Applause.)

He had also received a letter from their esteemed friend, M. Quiquet, the General Secretary of the Institut des Actuaire Français, who stated that that body had been unable to hold its usual meetings and that its activities had been confined to the issue, by M. Quiquet, of letters to his colleagues giving information as to the whereabouts of their Members, and, alas, reporting deaths and wounds. M. Quiquet sent his best wishes to the Institute for the happy return of such of its Members as had gone to the Front, for they testified by their number to the noble effort that all social classes in the United Kingdom were making in the fight which was the fight of both nations. (Applause.)

Their friend, M. Bégault, also wrote that he and Mme. Bégault had felt it their duty to remain in Brussels, as had also MM. Lepreux, Hankar, and Maingie. M. Bégault hoped that we in this country were not passing through the horrible time they had had to endure for ten months. “Assuredly our cause is lovely—England has acted nobly against those who have attacked us, but in the moments of solitude we think of those who are constantly risking their lives for the liberty of the world.” (Applause.) He was sure that the Members would wish him in their name to write to his correspondents letters of greeting conveying best wishes for their prosperity and the early victory of the united cause.

The list of losses by death which the Institute had suffered during the year was a long one, and it included the names of men who had done pre-eminently good work for the Institute and the profession. First among them was that of Sir G. F. Hardy, K.C.B.—to whose memory he had paid an inadequate tribute in his Presidential address. Before next session he hoped that there might be seen in the Hall a bust of Sir George, which the Council had asked a distinguished sculptor to complete. It had been partly executed in their dear friend's lifetime. All the Members would have received a circular as to another memorial—of a more personal and intimate

nature—to which, he heard, there had been that ready response which all must have anticipated—and as to which Sir Gerald Ryan, the Chairman of the Committee which had the matter in hand, would make a statement before the meeting adjourned.

Mr. Manly had been a Past-President, and he had had the opportunity of referring to the good work which that gentleman had done for the profession, at the meeting in January. Mr. Hewat had been President of the Faculty of Actuaries in Scotland, and among the other names were those of 2nd Lieut. Frederick C. Mann, Private Edgar Corble, and Private Elliott, names which would, hereafter, have attached to them, in letters of gold, “killed in action”—the first of their members, but he feared not the last, to fall in their country’s cause.

The Report, which was in all the Members’ hands, was, in the main, one of progress, though slow. They had continued their customary meetings, and some of the discussions to which the papers had given rise had been of a very interesting character. Whether, if the war continued through next winter, it would not be wiser to omit some of the monthly meetings, would be a question which would come up for decision by the Council in due course. The Hon. Secretaries informed him that papers were still wanted for next session, and he would ask the Members to keep in mind the new regulations recently issued by the Council for the conduct of the monthly meetings—and more especially the suggestions as to the preparation of papers.

The number of Members in every class, Fellows, Associates, and Students, had increased. The number of Probationers alone had decreased, and that was, no doubt, partly due to so many young men having joined the forces. The only set-back was a fall in the number of candidates for examination, 143 this year against 277 last year—a feature which was inevitable in the circumstances. There was one item in the accounts to which he would specially refer, namely, the donation to H.R.H. the Prince of Wales’ National Relief Fund, which he was sure few in the Hall would have wished to see omitted. (Hear, hear.)

Towards the end of the Report would be noticed a paragraph stating that the further consideration of the proposal to form a Research Bureau had been deferred until after the conclusion of the war, and in that connection he desired to call serious attention to proposals, which he understood were being put forward, as to the registration of the causes of deaths. He had heard it stated that it had been proposed that the cause of death should be omitted from the death certificate and returned confidentially by the medical attendant to the central office—the suggestion being that by that means a more accurate return of the real causes of deaths would be obtained for the use of the public. Such a procedure seemed to him most inadvisable. Quite apart from the comparatively small question of the assessment of premiums for life assurance, which might thereby be rendered more uncertain, he could not but think that the proposal would have the effect of obstructing very materially many most useful investigations by competent observers. It would certainly affect most seriously such investigations as were contemplated by the Institute’s proposed Research Bureau, where the medical examination and detailed data of the history of the lives under observation formed a unique basis for biological enquiry.

The concluding words of the Report referred to Sir A. W. Watson’s knighthood—might he live long and happily in the enjoyment of his well earned honour. (Cheers.) He would conclude by expressing the hope that the next meeting in November might find them at the end of the war, with the assurance of a lasting peace. (Applause.) He begged to move the adoption of the Report and Accounts.

Mr. W. P. PHELPS, in seconding the motion, said that all the Members’ thoughts were with their colleagues who were on active service. (Hear, hear.) The list of deaths in the report for the present year was considerably

longer than usual, and it included the names of three Members of the Institute who had fallen at the front. He thought the response the Members had made to the call of their country had been very satisfactory, and it could not be otherwise expected than that a few of them would lose their lives. It was very sad, indeed, to think of those fine young lives—among the bravest and best in the country—falling in the great cause, but he hoped before long there would be seen a peace which would make everyone realise that their valuable lives had not been given in vain. (Applause.)

The motion for the adoption of the Report and Accounts was then put and carried unanimously.

Sir GERALD RYAN said he was glad to receive the President's permission to break the thread of the ordinary routine of the meeting in order to make a statement which he was sure would be of interest to the members. Every Fellow and Associate was aware that steps had been taken recently with the view of perpetuating the name and fame of their dear friend, the late Sir George Hardy. He believed the movement was initiated by some of Sir George's personal friends—by some whom he might not irreverently call Sir George's disciples. Immediately the movement became known it spread very fast, and there was scarcely any—in fact he might say there was no—member of the profession of Hardy's day who did not immediately show a great desire and anxiety to take his share in the matter. A Committee had been formed out of elements thus brought together, and he had been invited to become Chairman; and he could assure the members he had appreciated no compliment in his life more highly than that. He understood that the Committee would like him to take the present opportunity of explaining the grounds upon which the movement had been set on foot. In the first place, they were very anxious that it should not be thought that they considered anything was necessary to keep alive the name and high achievements of Sir George Hardy. The work he had done and the services he had rendered to the profession would live certainly as long as any memorial which they might put up. Nor, in the second place, did the Committee think that the scheme was a means by which the present generation of actuaries might discharge their obligations to their departed colleague. On neither ground had the movement been started; on neither ground could it be justified. He thought it must rather be looked upon as a spontaneous expression of the high esteem in which Hardy and his work were universally held by the men of his own time, and as an affectionate token of regard from comrades and colleagues and those who had benefited by his teaching. It could truly be said that on no previous occasion had such honour been done to a distinguished actuary by his contemporaries; but he need scarcely say that the Committee, in commemorating Hardy, intended no slur or slight upon other eminent men who had filled the chair of President of the Institute, and who in due time had been gathered to their fathers, leaving a record of good work behind them. Recent pages of the *Journal* bore eloquent testimony to the almost unique character of the work which Hardy had done for the profession of the actuary. He was inclined to think that Hardy's powers were on a higher plane than his achievements. It seemed hard to believe that, with such unerring judgment, such keen insight, such high intellectual power, given an ampler field Hardy would not have made for himself in the world a name as great as he had done in the actuarial profession. But it was useless, in fact it was unnecessary, to repine about the splendid work Hardy might have done, but did not do. It was sufficient for them to remember that his services to the profession were of such a character as could never be forgotten. Hardy came upon the scene at a time when actuarial proceedings and literature were getting a little dull and somewhat commonplace, and he introduced a brilliance of invention and a stimulating impulse from which actuarial science had received a fresh life.

It was for those reasons that he did not think the Committee needed any

justification for having so exceptionally sought to do honour to the memory of Sir George Hardy. He would like to explain that the response to the circular which had been issued had been of a most gratifying character. Aided by generous donations from Lady Hardy and another member of the family, already more than £500 had been received, and the list was still open. There must be many members of the Institute who had not yet signified their desire to assist in the cause, who would wish to know that the opportunity would be afforded them for another month or so. Moreover, from abroad—from America, Japan and other countries—there would, he felt sure, come a response from many actuarial students who had learned to honour, and had derived a great benefit from, the work and the services of their friend. With regard to the particular form which the memorial would assume, a discussion had taken place in Committee, and it was unanimously felt that the proceeds of the collection should be devoted to the encouragement and assistance of younger members of the profession. It was the general conviction that that was a method of helping actuarial work which would lie close to Hardy's own heart, and by which means might be found to encourage merit—for the benefit, not only of the happy individual himself, but of the profession as a whole. The Council would be asked to undertake the administration of the fund, and, aided perhaps by suggestions from the Committee, to settle the lines upon which the annual proceeds of the fund should be awarded. He felt confident there was no one who had taken part in promoting this memorial to a great actuary who would not be proud of having done so for the rest of his life, and who could not congratulate himself on having participated in an action which would redound to the credit of the profession which owed so much to their departed friend. (Applause.)

The PRESIDENT said he did not think it was necessary for him to add anything, except that the Institute would accept the trust and do its best to carry out the wishes of the subscribers.

ELECTION OF OFFICERS.

The PRESIDENT then announced the result of the ballot for the election of officials and members of Council as follows :

President.

ERNEST WOODS.

Vice-Presidents.

WILLIAM PEYTON PHELPS, M.A.
LEWIS FREDERICK HOVIL.

ROBERT RUTHVEN TILT.
RALPH TODHUNTER, M.A.

Council.

THOMAS GANS ACKLAND.
HENRY JAMES BAKER.
ARTHUR DIGBY BESANT, B.A.
JOSEPH BURN.
FREDERICK TIMOTHY MASON
BYERS.
CHARLES RONALD VAWDREY
COUTTS.
WILLIAM PALIN ELDERTON.
DUNCAN CUMMING FRASER, M.A.
JAMES ROBERT HART.
LEWIS FREDERICK HOVIL.
*OWEN KENTISH.
GEORGE KING.
ABRAHAM LEVINE, M.A.
GEOFFREY MARKS.

VYVYAN MARR.
ALFRED MOORHOUSE.
HARRY ETHELSTON NIGHTINGALE.
WILLIAM PEYTON PHELPS, M.A.
SIR GERALD HEMMINGTON RYAN.
RICHARD GEORGE SALMON.
JOHN SPENCER.
*ALFRED CHARLES THORN.
ROBERT RUTHVEN TILT.
RALPH TODHUNTER, M.A.
HAROLD MOLTKE TROUNCER, M.A.
SAMUEL GEORGE WARNER.
*SIR ALFRED WILLIAM WATSON.
*JAMES DOUGLAS WATSON.
ERNEST WOODS.
*FRANK BERTRAND WYATT.

Treasurer.

GEOFFREY MARKS.

Honorary Secretaries.

ARTHUR DIGBY BESANT, B.A. | JOSEPH BURN.

On the motion of Mr. LATTA, seconded by Mr. BARRETT, Messrs. Arthur Taylor, R. S. B. Savery and G. H. Lawton were unanimously elected Auditors for the ensuing year.

Mr. A. G. HEMMING, in proposing a vote of thanks to the President, Vice-Presidents, Council, Officers, Examiners and Honorary Supervisors, for their services during the past year, remarked that he had more than once heard, and he could well believe, that the summer holidays of a President-elect were utterly spoilt by his anxious search for a suitable subject for his opening address. If the members studied the very interesting financial review which had been prepared for them by the President this year, he thought they would acknowledge that the patience and research required for a paper of that kind had been no inconsiderable task of itself. That was only one example of the many duties which had been so ably discharged by the President during the past year. He thought a special debt of gratitude was due to the Board of Examiners. They could hardly look for any thanks from the Candidates—except, perhaps, from the successful ones.

Mr. E. C. THOMAS, in seconding the motion, observed that, at a time such as that through which the country had been passing of late, it said a great deal for the skill and tact with which the Institute had been managed, that the sequence of meetings had been maintained unbroken, and that the level of the papers submitted and the ensuing discussions thereon had been up to the standard which had become a tradition of the Institute. He would like to say a few words with regard to the loss the Institute had sustained by deaths during the current year. He referred more particularly to the deaths of Sir George Hardy and Mr. Manly. Of Sir George Hardy it would be presumptuous for him to say much, particularly after the eloquent tribute paid by Sir Gerald Ryan. He had the most grateful recollections of Sir George as a tutor, both in regard to his qualities as a man and his genius as a mathematician and actuary. But he did not feel himself qualified to attempt to render adequate justice to Sir George's many-sided and brilliant qualities. With regard to Mr. Manly, he felt himself in a somewhat different position. There was no need to say anything about Mr. Manly's actuarial attainments. His work was there and spoke for itself, and everybody who knew him by his contributions would gratefully acknowledge the debt under which he laid the profession. (Hear, hear.) As he had been associated with Mr. Manly over a long series of years he felt his loss personally and in a dual sense. Mr. Manly had been a man who had inspired deep affection in the minds of all who knew him intimately, and there must be many in the profession who would cherish his memory for many kindnesses received, for opportunities of honourable work provided, for wise and friendly counsel freely given, and for generous recognition of honest effort. (Hear, hear.)

The PRESIDENT returned thanks.

Mr. MONKHOUSE, in proposing a vote of thanks to the Auditors, for their services during the past year, said it always seemed to him the auditors deserved a special acknowledgment, because theirs was a rather thankless task. It was rather like being a Special Constable—nothing ever happened.

Mr. BAKER seconded the motion, which was carried unanimously.

The PRESIDENT in adjourning the meeting to Monday, the 29th November, said that the vote of thanks to the officials did not include the name of Mr. Jarvis, who had done his work excellently under depressing circumstances during the session. The Institute could not have a better Assistant Secretary. (Applause.)

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1914:

*By whom presented
(when not purchased).*

Accountants and Auditors, Society of Incorporated.

List of Members, &c., 1914-15.

The Society.

Accountants, Institute of Chartered, in England and Wales.

List of Members, 1915.

The Institute.

Actuarial Society of America.

Transactions, 1914-15.

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Containing *inter alia*—

“Health Insurance from a theoretical and practical aspect”, by J. D. Craig.

“Mortality Experience of the Provident Life and Trust Company under paid-up insurance”, by M. A. Linton.

“Mortality and Re-marriage Tables for valuation of compensation to widows and other dependents”, by M. M. Dawson.

“A table of mortality according to height and weight”, by J. F. Little.

“Life Insurance Book-keeping and Annual Statements”, by R. D. Murphy.

“Participating Annuities”, by C. C. Ferguson.

“A method of determining Pure Premiums for Workmen’s Compensation Insurance”, by H. E. Ryan.

“Practical treatment of under-average lives”, by A. Hunter.

“Valuation of the Death Benefits provided by the Workmen’s Compensation Law of New York”, by E. Olifiers.

“A Staff Pension Fund”, by D. A. Walker.

Actuarial Society of Sweden.

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Actuaries, Faculty of.

Transactions, 1914-15.

The Faculty.

Containing *inter alia*—

“The Amortisation of Bonds and the valuation of Assets of a Life Assurance Company”, by H. Moir.

“On the construction of Salary Scales for use in the valuation of Pension Funds, and the valuation of prospective Pensions based on accumulated contributions”, by J. J. McLauchlan.

“On Annual Valuations and Distributions of Profit, with some remarks on the construction of Office Premiums”, by W. Denham and C. K. Granger.

“The Expectation of Life in Scotland in the year 1911”, by Dr. J. C. Dunlop.

American Mathematical Society.

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Elliptic Functions. An elementary Text-Book for
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“Industrial disease under the Workmen's Compensation Act, 1906, from a legal aspect”, by T. D. H. Brown.

“Medico-social problems from an insurance point of view”, by Sir T. Oliver, M.D.

“Some recent legal problems affecting insurance practice”, by H. M. Cohen.

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“Traumatic hysteria and neurasthenia,” by Dr. J. W. G. Grant.

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 Experience of Lives enrolled at each age. (MS.)
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A Text Book on the method of Least Squares. 8th edit.
 8vo. 1903.
 A list of writings relating to the method of Least
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Lady Hardy.

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half-yearly interest. La. Svo. 1889.

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Consolidated Fund, 1914.

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*The Government
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Twenty-third Annual Report by the Board of Trade. Fol. 1914.	}	
East India (Railways).		
Administration Report on the Railways of India for 1913-14.		
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Building Societies.		<i>Purchased.</i>
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Account to 31 December 1914.		
Income Tax.		
Reports of H.M. Representatives abroad respecting graduated Income Taxes in Foreign States. Fol. 1913.	}	<i>Purchased.</i>
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Life Assurance Companies. Statements of Accounts and Abstracts of Actuarial Reports. Simla. Fol. 1915.	}	<i>H. G. W. Meikle.</i>
Irish National Teachers' Pension Fund.		
Report of the Actuaries (G. F. Hardy and T. Tinner) as to the cost of the proposed new scheme. Fol. 1914.	}	<i>Purchased.</i>
National Debt.		
Return showing Liabilities and Assets at the close of each Financial year from 1835-36 to 1914-15, both inclusive.	}	<i>Purchased.</i>
Savings Banks and Friendly Societies. Account to 20 November 1914.		
National Insurance Acts, 1911-1914.	}	<i>The National Health Insurance Joint Committee.</i>
Part I. Health Insurance. Report for 1914-15 on their administration. 8vo. 1915.		
Report of the Departmental Committee on Sickness Benefit Claims, with Appendix, Minutes of Evidence, &c. Fol. 1914.	}	<i>Purchased.</i>
Public Trustee.		
Seventh Annual Report, 1915.		<i>Purchased.</i>
Registrar-General. England.		
Supplement to 75th Annual Report on Births, Deaths, and Marriages, 1901-1910. Part I, Life Tables. Report by G. King. 1914.	}	<i>The Registrar-General.</i>
Seventy-sixth Annual Report of Births, Deaths, and Marriages, 1913.		
Annual Summary of Marriages, Births, and Deaths in England and Wales, and in London. 1914.		
Registrar-General. Ireland.		
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Registrar-General. Scotland.		
Fifty-eighth and Fifty-ninth detailed Annual Reports of Births, Deaths, and Marriages. (Abstracts of 1912 and 1913.)	}	<i>Purchased.</i>
Workmen's Compensation.		
Statistics of Proceedings under the Acts of 1897 and 1900, and the Employers' Liability Act, 1880, during the years 1912 and 1913.		

*By whom presented
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| Frequency-Curves. 4to. 1893. | } | <i>Lady Hardy.</i> |
| Regression, heredity, and panmixia. 4to. 1896. | | |
| Skew variation in homogeneous material. 4to. 1895. | | |
| Supplement to a memoir on Skew Variation. 4to. 1901. | | |
| On the influence of natural Selection on the variability and correlation of organs. 4to. 1902. | | |
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| On the probable errors of Frequency Constants and on the influence of random selection on variation and correlation. 4to. 1898. | } | <i>Lady Hardy.</i> |
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| Accountants' Magazine. | } | <i>Purchased.</i> |
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